Name:______________________________

Princeton University

ORF 363/COS 323
Midterm Exam, Fall 2016

October 27, 2016, from 1:30 pm to 2:50 pm

Instructor:
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AIs:
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PLEASE DO NOT TURN THIS PAGE. START THE EXAM ONLY WHEN YOU ARE INSTRUCTED TO DO SO.

1. You are allowed a single sheet of paper, double-sided, hand-written or typed.
2. No electronic devices are allowed (e.g., cell phones, calculators, laptops, electronic cigarettes, electronic marijuana to deal with exam stress, etc.).
3. All questions should be answered in the booklet. Please write out and sign the following pledge on the booklet "I pledge my honor that I have not violated the honor code during this examination."
4. Make sure you write your name on the first page of these questions and return the questions to us at the end of the exam. Please don't forget to write your name on the booklet as well.
5. You are allowed to cite results proved in lecture or lecture notes without proof.

Each question has 20 points. You need to justify your answers to receive full credit.

Problem 1: Find all the local minimizers, local maximizers, global minimizers, and global maximizers of the following function over $\mathbb{R}^2$ (or argue if some do not exist):

$$f(x_1, x_2) = \frac{1}{3}x_1^3 - \frac{1}{2}x_1^2 + 2x_1x_2 + \frac{1}{2}x_2^2.$$

Problem 2: Consider the following unconstrained optimization problem

$$\max_{x_1,x_2}. \quad -x_1^2 - 4x_2^2 + ax_1x_2 + 5,$$

where a is a scalar parameter. What is the range of values for a for which problem this problem has

(a) an optimal solution?

(b) a unique optimal solution?

Problem 3: True or False? (Provide a proof or a counterexample.)

(a) If a function $f : \mathbb{R}^n \to \mathbb{R}$ is convex, then f^2 is convex.

(b) If a function $f : \mathbb{R}^n \to \mathbb{R}$ is affine, then f^2 is convex.

Problem 4: Suppose you are given a (not necessarily symmetric) matrix $A \in \mathbb{R}^{n\times n}$ with linearly independent columns and a vector $b \in \mathbb{R}^n$. You are interested in solving the linear system $Ax = b$. All you have at your disposal is a blackbox that given a strictly convex quadratic function returns its unconstrained global minimum. Explain how you can use this blackbox to solve your linear system.

Problem 5: Consider the problem

$$\begin{array}{ll} \min\limits_{x\in\mathbb{R}^n}. & \dfrac{c^T x + d}{e^T x + g} \\ \text{subject to} & Ax \leq b, e^T x + g \geq 1, \end{array}$$

where $c, e \in \mathbb{R}^n$, $d, g \in \mathbb{R}$, $A \in \mathbb{R}^{m\times n}$, and $b \in \mathbb{R}^m$. We note that this is not a convex optimization problem in general.

Suppose we know the optimal value f^* of this problem is between 0 and 100. Explain how you can solve less than 10 convex optimization problems to come up with an approximate optimal value $\hat{f} \in \mathbb{R}$ that satisfies $|\hat{f} - f^*| \leq 0.5$.