Name:______________________________

Princeton University

ORF 363/COS 323
Midterm Exam, Fall 2017

October 24, 2017, from 1:30 pm to 2:50 pm

Instructor:
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AIs:
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PLEASE DO NOT TURN THIS PAGE. START THE EXAM ONLY WHEN YOU ARE INSTRUCTED TO DO SO.

1. You are allowed a single sheet of paper, double-sided, hand-written or typed.
2. No electronic devices are allowed (e.g., cell phones, calculators, laptops, etc.).
3. All questions should be answered in the booklet. Please write out and sign the following pledge on the booklet "I pledge my honor that I have not violated the honor code during this examination."
4. Make sure you write your name on the first page of these questions and return the questions to us at the end of the exam. Please don't forget to write your name on the booklet as well.
5. You are allowed to cite results proved in lecture or lecture notes without proof.

Each question has 20 points. You need to justify your answers to receive full credit.

Problem 1: Find all the local minimizers, local maximizers, global minimizers, and global maximizers of the following function over $\mathbb{R}^2$ (or argue if some do not exist):

$$f(x_1, x_2) = x_1^2 x_2 - 2x_1 x_2^2 + 4x_1 x_2 - 8.$$

Problem 2: You are stuck in a place with no electronic devices allowed and asked to compute $\sqrt{15}$. How can you use Newton's method for root finding to approximate $\sqrt{15}$ using only the operations $+, -, \times, \div$? Apply two iterations of Newton's method starting from $x_0 = 5$. Report your final estimate of $\sqrt{15}$ as a rational number.

Problem 3: Consider a homogeneous polynomial $p(x) = p(x_1, \ldots, x_n)$ of even degree $d \geq 2$. Show that if p is quasiconvex, then it is nonnegative; i.e., $p(x) \geq 0$, for all $x \in \mathbb{R}^n$. (Note: a polynomial is homogeneous of degree d if all of its monomials have degree exactly d. The degree of a monomial $x_1^{\alpha_1} \ldots x_n^{\alpha_n}$ is equal to $\alpha_1 + \cdots + \alpha_n$.)

Problem 4: True or False? (Provide a proof or a counterexample.)

(a) Any homogeneous, nonnegative polynomial $p(x) = p(x_1, \ldots, x_n)$ is convex.

(b) The function $f(x_1, x_2) = \max\{e^{(x_1 - x_2)}, (x_1 + 5x_2)^2\} + 2x_1^2 + 5x_2^2 - 6x_1 x_2$ is convex.

(c) If $f : \mathbb{R} \to \mathbb{R}$ is convex and $g : \mathbb{R} \to \mathbb{R}$ is convex, then $f(g(x))$ is convex.

Problem 5: Find the optimal value and the optimal solution(s) of the following convex optimization problem. Show your reasoning and derivation.

$$\begin{aligned} \min_{x_1, x_2} \quad & 2x_1^2 + 5x_2^2 - 6x_1 x_2 + x_2 \\ \text{s.t.} \quad & x_1^2 + x_2^2 \leq 4. \end{aligned}$$