Name:______________________________

PRINCETON UNIVERSITY

ORF 363/COS 323
Midterm Exam, Fall 2021

OCTOBER 14, 2021, FROM 1:30 PM TO 2:50 PM

Instructor:
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AIs:
Becker, Chaudhry, Dibek, Silin, Yu

PLEASE DO NOT TURN THIS PAGE. START THE EXAM ONLY WHEN YOU ARE INSTRUCTED TO DO SO.

1. You are allowed a single sheet of paper, double-sided, hand-written or typed.
2. Cell phones should be off or in airplane mode. No other electronic devices are allowed.
3. All questions should be answered in the booklet. Please write out and sign the following pledge on the booklet "I pledge my honor that I have not violated the honor code during this examination."
4. Make sure you write your name on the first page of these questions and return the questions to us at the end of the exam. Please don't forget to write your name on the booklet as well.
5. You are allowed to cite results proved in lecture or lecture notes without proof.

Each question has 25 points. You need to justify your answers to receive full credit.

Problem 1: Find all the local minimizers, local maximizers, global minimizers, and global maximizers of the following function over $\mathbb{R}^2$ (or argue if some do not exist):

$$f(x_1, x_2) = -\frac{5}{2}x_1^2 + 2x_1x_2 - \frac{1}{2}x_2^2 + x_1 - x_2 + 3.$$

Problem 2: Recall the Fermat-Weber facility location problem in two dimensions:

$$\min_{x \in \mathbb{R}^2} \sum_{i=1}^{m} \|x - z_i\|,$$

where $z_1, \ldots, z_m \in \mathbb{R}^2$ are given as input.

(a) Specify an input for which the optimal solution is not unique. If this cannot be done, argue why.

(b) Specify an input for which there are precisely two optimal solutions. If this cannot be done, argue why.

Problem 3: Consider a function $f: \mathbb{R}^2 \to \mathbb{R}$ and the following two statements:

(a) $f(x_1, x_2)$ is convex.

(b) For every fixed $\overline{x}_1 \in \mathbb{R}$, the univariate function $g(x_2) = f(\overline{x}_1, x_2)$ is convex, and for every fixed $\overline{x}_2 \in \mathbb{R}$, the univariate function $h(x_1) = f(x_1, \overline{x}_2)$ is convex.

Does (a) imply (b)? Does (b) imply (a)? Why or why not?

Problem 4: Give an example of a (feasible) convex optimization problem which has a finite optimal value, a linear objective function, a closed feasible set, and no optimal solutions. (Hint: you need at least two decision variables.)