1. You are allowed a single sheet of A4 paper, double sided, hand-written or typed.

2. No electronic devices are allowed (e.g., cell phones, calculators, laptops).

3. All questions should be answered in the booklet. Please write out and sign the following pledge on the booklet: “I pledge my honor that I have not violated the honor code during this examination.”
**True or False?** Statements 1-7 have 14 points each and require a proof or a precise counterexample. Statement 8 has 2 points and does not need any justification. You are allowed to cite results proved in lecture without proof, but nothing else.

**S1:** If \( f: \mathbb{R}^n \to \mathbb{R} \) is a convex function and \( \Omega \subseteq \mathbb{R}^n \) is a convex set, then the set of global minimizers of \( \min_{x \in \Omega} f(x) \) is convex.

**S2:** A closed convex set in \( \mathbb{R}^n \) coincides with the intersection of all halfspaces that contain it.

**S3:** Suppose \( P \in \mathbb{R}^{n \times n} \) is a matrix with nonnegative entries whose columns each sum up to one. Then, there exists \( x \in \mathbb{R}^n \) such that \( Px = x, x \geq 0, \) and \( \sum_{i=1}^{n} x_i = 1. \)

**S4:** Consider a semidefinite program (SDP) in standard form

\[
\begin{align*}
\min_{X \in S^{n \times n}} & \quad \text{Tr}(CX) \\
\text{subject to} & \quad \text{Tr}(A_i X) = b_i, \ i = 1, \ldots, m, \\
& \quad X \succeq 0,
\end{align*}
\]

where \( C, A_i, b_i \) all have only rational entries. If there is an optimal solution, then there is an optimal solution whose entries are all rational.

**S5:** Consider the SDP of the previous statement. If \( n = 2 \), then the SDP can be written as a second order cone program.

**S6:** The convex hull of a closed set is closed.

**S7:** If the convex hull of a set is compact, then the set itself must be compact.

**S8:** If S8 is false, then I deserve two points.