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PRINCETON UNIVERSITY

ORF 523
Midterm Exam, Spring 2016

MARCH 3, 2016, FROM 1:30 PM TO 2:50 PM

Instructor:	*AI:*
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PLEASE DO NOT TURN THIS PAGE. START THE EXAM ONLY WHEN YOU ARE INSTRUCTED TO DO SO.

1. You are allowed a single sheet of A4 paper, double sided, hand-written or typed.
2. No electronic devices are allowed (e.g., cell phones, calculators, laptops, hoverboards to run away, etc.).
3. All questions should be answered in the booklet. Please write out and sign the following pledge on the booklet: "I pledge my honor that I have not violated the honor code during this examination."

You are allowed to cite results shown in lecture without proof, but nothing else. Throughout the exam, we have $x \in \mathbb{R}^n$.

Question 1: True or False? (60 pts)
(You need to provide a proof or a counterexample to justify your answers.)

(a) The sum of two quasiconvex functions is quasiconvex.

(b) A convex homogeneous polynomial $p(x)$ of degree $d \geq 2$ is nonnegative.
(Recall that a polynomial is homogeneous of degree d if all of its monomials have degree exactly d, and that $p(x)$ is nonnegative if $p(x) \geq 0$ for all $x \in \mathbb{R}^n$.)

(c) A quadratic function $f(x) = x^T Q x + b^T x + c$ is convex if and only if it is quasiconvex.
(You can use the fact that f is convex if and only if $Q \succeq 0$ if you need to.)

(d) If $f : \mathbb{R}^n \to \mathbb{R}$ is convex on a convex set $S \subseteq \mathbb{R}^n$, then f is continuous on S.

Question 2: Detecting uniqueness of solutions to an LP (25 pts)
Consider the LP

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & c^T x \\ \text{subject to} \quad & Ax = b \\ & x \geq 0, \end{aligned} \tag{1}$$

where A is $m \times n$, all data (i.e., A, b, c) is rational, and the feasible set is non-empty and bounded. Suppose you have a black box that, when given any LP with a non-empty and bounded feasible set and rational data, outputs its optimal value. How could you make a polynomial number of calls to this black box to decide if the optimal solution to (1) is unique?

Question 3: Computing the clique number for special graphs (15 pts)
A clique in a graph is a subset of the nodes where each node is connected to every other node. Show that for complements of bipartite graphs, the size of the largest clique can be computed by linear programming. (Recall that the complement G' of a graph G is a graph on the same node set as G, but where two nodes are connected if and only if they were not connected in G.)