Name:________________________________

Princeton University

ORF 523
Midterm Exam 1, Spring 2017

March 16, 2017, from 1:30 pm to 2:50 pm

Instructor:
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AIs:
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PLEASE DO NOT TURN THIS PAGE. START THE EXAM ONLY WHEN YOU ARE INSTRUCTED TO DO SO.

1. You are allowed a single sheet of A4 paper, double sided, hand-written or typed.
2. No electronic devices are allowed (e.g., cell phones, calculators, laptops, etc.).
3. All questions should be answered in the booklet. Please write out and sign the following pledge on the booklet: "I pledge my honor that I have not violated the honor code during this examination."
4. Each problem has *20 points*. You can cite results shown in lecture or on problem sets without proof.

Problem 1: Difference of convex decomposition

Show that any quadratic function can be written as the difference of two convex functions.

Problem 2: Unconstrained quadratic minimization

Consider the following unconstrained optimization problem

$$\max_{x_1,x_2}. \; -x_1^2 - 4x_2^2 + ax_1x_2 + 5,$$

where a is a scalar parameter. What is the range of values for a for which

(a) the problem has an optimal solution?

(b) the problem has a unique optimal solution?

Problem 3: Convexification

Explain how you can find an *optimal solution* to the problem of minimizing a continuous function $f : \mathbb{R}^n \to \mathbb{R}$ over a compact set $\Omega \subseteq \mathbb{R}^n$ by making a polynomial (in n) number of calls to only two blackboxes: one that takes convex hulls of sets, and one that returns the optimal value of any finite-dimensional optimization problem whose objective function and feasible set are convex. (Ignore representation issues.)

Problem 4: True or False? (Give a proof or a counterexample.)

(a) The infimum of a lower bounded (multivariate) polynomial is achieved.

(b) The infimum of a lower bounded homogeneous (multivariate) polynomial is achieved.

Problem 5: An LP relaxation for the maximum stable set problem

A *stable set* in a graph is a subset of the nodes no two of which are connected by an edge. For a graph $G(V, E)$, let us denote the size of its largest stable set by $\alpha(G)$. Observe that the optimal value of the following LP gives an upper bound on $\alpha(G)$:

$$\begin{aligned} LP_{OPT} := \max & \sum_{i=1}^{|V|} x_i \\ \text{s.t. } & x_i + x_j \leq 1, \text{ if } \{i, j\} \in E, \\ & 0 \leq x_i \leq 1, \; i = 1, \ldots, |V|. \end{aligned} \tag{1}$$

1. Produce a family of graphs for which $LP_{OPT} - \alpha$ is as large as you want.
2. Show that if G is bipartite, then $\alpha(G) = LP_{OPT}$.
3. Given an example of a non-bipartite graph for which $\alpha(G) = LP_{OPT}$.