Name:________________________________

PRINCETON UNIVERSITY

ORF 523
Midterm Exam 2, Spring 2017

APRIL 25, 2017, FROM 1:30 PM TO 2:50 PM

Instructor:
A.A. Ahmadi

AIs:
G. Hall, C.Y. Liu

PLEASE DO NOT TURN THIS PAGE. START THE EXAM ONLY WHEN YOU ARE INSTRUCTED TO DO SO.

1. You are allowed a single sheet of A4 paper, double sided, hand-written or typed.
2. No electronic devices are allowed (e.g., cell phones, calculators, laptops, etc.).
3. All questions should be answered in the booklet. Please write out and sign the following pledge on the booklet: "I pledge my honor that I have not violated the honor code during this examination."
4. Each problem has *25 points*. You can cite results proven in lecture or on problem sets without proof.

Problem 1: LP, SOCP, and SDP

Can the semidefinite program

$$\begin{aligned} \min_{x,y} \quad & x + y \\ \text{s.t.} \quad & y \leq 2x \\ & y \geq \frac{1}{2}x \\ & \begin{bmatrix} 1+x & y \\ y & 1-x \end{bmatrix} \succeq 0 \end{aligned}$$

be written as a linear program and/or a second order cone program? Why or why not?

Problem 2: Minimizing the Frobenius norm

Let $A(x) = A_0 + x_1 A_1 + \cdots + x_n A_n$, where $A_1, \ldots, A_n$ are $p \times m$ real matrices. Formulate the problem

$$\min_{x \in \mathbb{R}^n} ||A(x)||_F^2$$

as a semidefinite program (your SDP need not be in standard form).

Problem 3: Stability of a pair of symmetric matrices

We say that a real $n \times n$ matrix A is stable if $\rho(A) < 1$, where $\rho(A)$ is the spectral radius of A, i.e., the maximum of the absolute values of its eigenvalues. Recall that we call a pair of real $n \times n$ matrices $\{A_1, A_2\}$ stable if $\rho(\Sigma) < 1$, for any finite product Σ out of A_1 and A_2. (For example, Σ could be $A_2A_1, A_1A_2, A_1A_1A_2A_1$, and so on.) Show that if A_1 and A_2 are symmetric, then $\{A_1, A_2\}$ is stable if and only if A_1 and A_2 are individually stable.

Problem 4: Second largest eigenvalue

Let $\lambda_2(X)$ denote the second largest eigenvalue of a symmetric real $n \times n$ matrix X. Given rational $n \times n$ matrices A_i, rational $n \times 1$ vectors b_i, and an integer k, show that testing whether the constraints

$$\begin{aligned} & \text{Tr}(A_i X) \leq b_i, i = 1, \ldots, m \\ & X \succeq 0 \\ & \lambda_2(X) \leq k \end{aligned} \tag{1}$$

are feasible is NP-hard.

Hint: You may want to consider a reduction from STABLE SET.