Name:________________________________

Princeton University

ORF 523

Midterm Exam 1, Spring 2019

March 12, 2019, from 1:30 pm to 2:50 pm

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PLEASE DO NOT TURN THIS PAGE. START THE EXAM ONLY WHEN YOU ARE INSTRUCTED TO DO SO.

1. You are allowed a single sheet of A4 paper, double sided, hand-written or typed.
2. No electronic devices are allowed (e.g., cell phones, calculators, laptops, etc.).
3. All questions should be answered in the booklet. Please write out and sign the following pledge on the booklet: "I pledge my honor that I have not violated the honor code during this examination."
4. Each problem has 25 points. You can cite results shown in lecture or on problem sets without proof.
5. Good luck!

For True/False questions, a proof or a counterexample is required.

Problem 1:
Specify whether each of the following statements is true or false.

(i) The set of points that are closer to a point $\bar{x} \in \mathbb{R}^n$ than a set $S \subseteq \mathbb{R}^n$ is convex.

(ii) The set of points that are farther from a point $\bar{x} \in \mathbb{R}^n$ than a set $S \subseteq \mathbb{R}^n$ is convex.

(iii) The set of points that are closer to a set $S \subseteq \mathbb{R}^n$ than another set $T \subseteq \mathbb{R}^n$ is convex.

Problem 2:
Let $\Omega \subset \mathbb{R}^n$ be a compact convex set and $f : \mathbb{R}^n \to \mathbb{R}$ be a convex (continuous) function. Consider the following two optimization problems:

$$\begin{array}{ll} \text{minimize} & f(x) \\ \text{subject to} & x \in \Omega, \end{array} \quad (1) \qquad \begin{array}{ll} \text{maximize} & f(x) \\ \text{subject to} & x \in \Omega. \end{array} \quad (2)$$

Specify whether each of the following statements is true or false.

(i) There is an optimal solution to (1) at an extreme point of Ω.

(ii) There is an optimal solution to (2) at an extreme point of Ω.

Hint: If needed, you can use the fact (without proof) that a compact convex set has at least one extreme point and equals the convex hull of its extreme points.

Problem 3:
Let $\alpha_1, \ldots, \alpha_n$ be nonnegative integers. Give a simple necessary and sufficient condition based on these numbers for the monomial function $x_1^{\alpha_1} x_2^{\alpha_2} \ldots x_n^{\alpha_n}$ to be convex.

Problem 4:
Show that any closed convex set $\Omega \subseteq \mathbb{R}^n$ can be written as $\Omega = \{x \in \mathbb{R}^n \,|\, g(x) \leq 0\}$ for some convex function $g : \mathbb{R}^n \to \mathbb{R}$.