# Topics in Operations Research

***Event Structure:***
The participants will work in teams of two. The event will take 50 minutes.

***Description:***
This event will test students on their knowledge of probability theory, mathematics, statistics, and optimization. It will also test on how well students can apply that knowledge to a variety of problems in resource management, finance, transportation, economics, etc. The exam assumes some prior knowledge of: single variable calculus, statistics, probability theory, linear programming, and basic game theory.

***Preparation:***
To perform well in the event, the student should be comfortable with the following topics:

- Single variable calculus: differentiation and integration
- Statistical Inference: confidence intervals and hypothesis testing using normal and t-distributions. Chi-square test for goodness of fit/homogeneity/independence
- Probability Theory: combinations and permutations, Bayes' Rule, conditional probability, independence of random variables, discrete distributions (Bernoulli, Binomial, Geometric, Poisson), continuous distributions (Normal, Uniform, Exponential, Gamma), expectation/conditional expectation, variance/conditional variance, and how to apply each of them to real-world problems
- Linear Programming: Formulating a problem as a linear program and finding the dual of a linear program.
- Basic Game Theory: Two-person zero-sum games, rational decision-making based on payoffs.

***Sample Problems:***

1. Bob is interested in looking at statistics for the mean SAT math score. To this end, he surveys 72 high school students and found that they had an average math score of 496.75 with a standard deviation of 14.11. Assume that SAT math scores follow a normal distribution.
   a) Compute a (two-sided) 99% confidence interval for the mean SAT math score. ***Answer****: [492.5, 501.0]*
   b) Say we want to evaluate the claim that the mean SAT math score is greater than or equal to 500. Formulate a hypothesis testing problem for this claim with significance level 5%.
   ***Answer****: $H_0$: mean >= 500, $H_A$: mean < 500, p-value: 2.5%, reject $H_0$*
2. A random variable *X* is said to be uniformly distributed over the interval (*a, b*) if its probability density function is given by
   $$\mathrm{f(x)} = \frac{1}{\mathrm{b-a}} \text{ if } \mathrm{a < x < b}, 0 \text{ otherwise}$$
   Say the random variable *X* is uniformly distributed over (3,5).
   a) Calculate the expectation and variance of this distribution. ***Answer:*** *E[X] = 4, Var[X] = ⅓*
   b) Calculate the expectation of $\mathrm{Xe^X}$. ***Answer:*** *E[$Xe^X$] = 276.74*
3. A plane has 10 seats. For a particular flight, there are 13 reservations made. Each one of those 13 reservations has probability 0.7 of actually arriving in time for the flight, and you can assume that the passengers all behave independently of each other. What is the probability that there are no angry customers i.e. that everyone who shows up for the flight has a seat? ***Answer:*** *79.75%*
4. Consider the following linear programming program:
   $$\text{Maximize } Z=2x_1+6x_2+9x_3$$

subject to $x_1+x_3<=3$
and $x_2+2x_3<=5$
and $x_1>=0$, $x_2>=0$, $x_3>=0$
Construct the dual problem for this primal problem.

***Answer:***
*Minimize $3y_1+5y_2$*
*subject to $y_1>=2$*
*and $y_2>=6$*
*and $y_1+2y_2>=9$*
*and $y_1>=0$, $y_2>=0$*

***Contact:***
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