Investigating Group Behavior in Dance: An Evolutionary Dynamics Approach
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Social Decision Making in Animal & Human Groups

Nest Site Selection  Foraging Behavior  Flock Logic

Decision making as an Individual or Group
Motivation

Broadcast Feedback of Stochastic Cellular Actuators Inspired by Biological Muscle Control

The International Journal of Robotics Research November 2007 vol. 26 no. 11-12 1251-1265

Broadcast Feedback Control of Cell Populations Using Stochastic Lyapunov Functions with Application to Angiogenesis Regulation

Levi B. Wood, Anusuya Das, and H. Harry Asada, Fellow, ASME
A Group Dance Performance: *There Might be Others*

- Dancers choose the order of the modules.
- Maximum 2/3 modules are allowed simultaneously on stage.
- No leadership assignment (any dancer can introduce a new module).
A Group Dance Performance: *There Might be Others*
There Might be Others

Dancers Choose

- Diversity
  - 0.6
  - 0.5
  - 1
  - 0.35

Flow

- 0.6
- 0.5
- 0
- 0.65

Diversity vs. Exploitation

Flow vs. Exploration
Evolutionary Dynamics to Study Group Behavior

Replicator Mutator Model: \[ \dot{x}_i = \sum_{j=1}^{N} x_j f_{ji} q_{ji} - \phi x_i \]

Fitness: \[ f_i = \sum_{j=1}^{N} b_{ji} x_j \]

Average Fitness: \[ \phi = \sum_{i=1}^{N} f_i x_i \]

Mutation:
\[ q_{ij} = \frac{\mu b_{ij}}{\sum_{j \neq i} b_{ij}} \]
\[ q_{ii} = 1 - \mu \]

Previous Studies

Analysis for Two Strategies:


Analysis for Three Strategies:

Evolutionary Dynamics to Study Group Behavior

Model modification motivated by the observations

\[
\dot{x}_i = \sum_{j=1}^{N} x_j f_j q_{ji} - \phi x_i
\]

Understanding the social decision making dynamics driven by artistic Explore-Exploit tension
Interpretation of Model for Dance Group Behavior

Perceived Dominance:

$$\dot{x}_i = \sum_{j=1}^{N} x_j f_j q_{ji} - \phi x_i$$

$$\dot{\eta}_i = K \left( S(x_i) - \alpha \right)$$

$$w_i = \frac{1}{1 + e^{-\eta_i}}, \quad i \in 1, 2, \ldots, N$$

Perceived Dominance Threshold : $\alpha \in [0, 1]$
Awareness of Dominance for Two Strategies

Replicator Mutator Dynamics:
\[
\dot{x}_1 = x_1 \left( b + (1 - b)x_1 \right) \left( 1 - \mu - x_1 \right) + \left( 1 - x_1 \right) \left( 1 + (b - 1)x_1 \right) \left( \mu - x_1 \right)
\]

Awareness of Dominance:
\[
\dot{w}_i = K \left( x_i - \alpha \right) w_i \left( 1 - w_i \right), \quad i = 1, 2 \quad K > 0
\]

\[
\begin{align*}
\mu \geq (1 - b)/4, \quad & \lim_{t \to \infty} w_i(t) = \begin{cases} 
0 & \text{if } \alpha \in (0.5, 1] \\
1 & \text{if } \alpha \in [0, 0.5)
\end{cases} \\
\mu < (1 - b)/4, \quad & \lim_{t \to \infty} w_i(t) = \begin{cases} 
0 & \text{if } \alpha \in (v_u, 1] \\
0 & \text{if } \alpha \in (v_l, v_u) \text{ and } x_i(0) < 0.5 \\
1 & \text{if } \alpha \in (v_l, v_u) \text{ and } x_i(0) > 0.5 \\
1 & \text{if } \alpha \in [0, v_l)
\end{cases}
\end{align*}
\]

where \( v_u = 0.5 + \sqrt{0.25 - \mu/(1-b)} \) and \( v_l = 0.5 - \sqrt{0.25 - \mu/(1-b)} \).

\[ w_i \to 0 \]
\[ w_i \to 1 \]
\[ w_i \to 1, w_j \to 0, \quad \text{if } x_i(0) > x_j(0) \]
Loop Closure-Feedback Controlled Bifurcation

\[ \dot{x}_i = \sum_{j=1}^{N} x_j f_j q_{ji} - \phi x_i \]

\[ \mu = 1 - \max_{i \in \{1, 2, \ldots, N\}} (w_i) \]

\[ \{w_i\}_{i=1}^{N} \]

\[ \{x_i\}_{i=1}^{N} \]

\[ w_i = K (x_i - \alpha) w_i (1 - w_i) \]
Closed Loop Behavior (Phase Portraits)

Forward Dynamics:
\[ \dot{x}_1 = x_1 \left( b + (1 - b)x_1 \right) \left( \max(w_1, w_2) - x_1 \right) + (1 - x_1) \left( 1 + (b - 1)x_1 \right) \left( 1 - \max(w_1, w_2) - x_1 \right) \]

Feedback Dynamics:
\[ \dot{w}_i = K \left( x_i - \alpha \right) w_i \left( 1 - w_i \right), \quad i = 1, 2 \quad K > 0 \]

- \( \alpha = 0.25 \)
- \( b = 0.04 \)
- \( K = 2 \)
Loop Closure for $K \gg I$: Time-scale Separation

**Slow Dynamics: Replicator-Mutator**

$$
\dot{x}_1 = x_1 (b + (1 - b)x_1) \left( \max(w_1, w_2) - x_1 \right) + (1 - x_1) (1 + (b - 1)x_1) \left( 1 - \max(w_1, w_2) - x_1 \right)
$$

**Fast Dynamics: Awareness of Dominance**

$$
\epsilon \dot{w}_i = (x_i - \alpha) w_i (1 - w_i), \quad \epsilon \equiv \frac{1}{K} \ll 1
$$

$$
\lim_{t \to \infty} x_1(t) = \begin{cases} 
1 & x_1(0) > \max(\alpha, 0.5) \\
0.5 & \alpha > x_1(0) > 1 - \alpha \\
0 & x_1(0) < \min(1 - \alpha, 0.5)
\end{cases}
$$

Let, $x_i(0) \geq x_j(0)$, where $i, j \in \{1, 2\}$ and $i \neq j$. Then, $\lim_{t \to +\infty} x_i(t) \geq \lim_{t \to +\infty} x_j(t)$.

- $x_1 \to 1, \quad x_2 \to 0$
- $x_1 \to 0, \quad x_2 \to 1$
- $x_1 \to 0.5, \quad x_2 \to 0.5$
Conclusion and Future Work

Conditions for Exploration Phase

\[ \dot{x}_i = \sum_{j=1}^{N} x_j f_j q_{ji} - \phi x_i \]

\[ \dot{w}_i = K(S(x_i) - \alpha)w_i(1 - w_i) \]

Diversity vs. Exploitation

Flow

Exploitation

Exploitation?
Thank You