Russell-Myhill and Grounding

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Abstract. The Russell-Myhill paradox (RMP) puts pressure on the Russellian structured view of propositions (structurism) by showing that it conflicts with certain prima facie attractive ontological and logical principles. I describe several versions of RMP and argue that structurists can appeal to natural assumptions about metaphysical grounding to provide independent reasons for rejecting the ontological principles used in these paradoxes. It remains a task for future work to extend this grounding-based approach to all variants of RMP.

Philosophers disagree on how finely propositions are individuated. Near the fine-grained end of the spectrum we find *structured views* (*structurism*). The Russell-Myhill paradox (RMP) shows that structurism is classically inconsistent with prima facie attractive ontological principles (Russell 1996: 527, Myhill 1958). This observation can be used to argue that structurism should be rejected to avoid the paradox. Structurists can defang this argument by providing another solution to RMP that is consistent with structurism. They would not need to argue that their solution is the best possible one, but merely that it is no worse than the solution that consists in rejecting structurism. That would suffice to show that RMP provides no strong reason to abandon structurism. I will sketch part of such a defence of structurism about Russellian propositions.[1]

After describing structurism and RMP (§1), I will introduce assumptions about metaphysical grounding and argue that they yield a unified solution to many versions of RMP, by providing independent reasons to reject their underlying ontological assumptions (§2). However, there is another variant of RMP to which this solution cannot be applied (§3). While I believe that the grounding-based approach can be extended to this version, it is a task for another occasion to show this. Adopting different solutions to different versions of RMP might seem unattractively disunified. However, I will argue (§3) that my approach does not obviously yield a less unified view than the anti-structurist solution to RMP.

1. Structurism and RMP

Structurism. Structurists holds that propositions have structures analogous to those of sentences and that identity of propositions requires sameness of structure and constituents. More precisely, structurism is not a single claim but a family of theses like those below. (***p***, ***q*** are singular and ***pp***,

[1] The distinctive feature of Russellian propositions is that they have the entities and pluralities they are about as constituents. My discussion will be restricted to such propositions.

qq plural propositional variables.[2] $\boldsymbol{v_1}, \ldots, \boldsymbol{v_n}, \boldsymbol{v_1'}, \ldots, \boldsymbol{v_n'}$ are any variables, $\boldsymbol{v_i}$ being of the same type as $\boldsymbol{v_i'}$. $\mathbf{X^n}, \mathbf{X^{n\prime}}$ are n-place predicate variables. I will use = type-ambiguously to express identity or its analogue for types other than the type of individuals. $\boldsymbol{\bigwedge pp}$ expresses the conjunction of *pp*. Where S is a sentence or name for a sentence, $\ulcorner \langle S \rangle \urcorner$ designates the proposition expressed by that sentence.)

(Predication-Structure) $\forall X^n \forall X^{n\prime} \forall v_1 \ldots \forall v_n \forall v_1' \ldots \forall v_n' \big((X^n v_1 \ldots v_n = X^{n\prime} v_1' \ldots v_n') \rightarrow ((X^n = X^{n\prime}) \;\&\; (v_1 = v_1') \;\&\; \ldots \;\&\; (v_n = v_n'))\big)$

(Atomic propositions are identical only if they ascribe the same property or relation to the same sequence of entities.)

(Conjunction-Structure) $\forall pp \, \forall qq \, \big((\bigwedge pp = \bigwedge qq) \rightarrow (pp = qq)\big)$

(Conjunctions are identical only if they have the same conjuncts.)

(Existential-Structure) $\big((\exists v \, A) = (\exists v \, B)\big) \rightarrow \forall v \, (A = B)$

(Propositions $\langle \exists v A \rangle$ and $\langle \exists v B \rangle$ are identical only if, for every v, propositions $\langle A \rangle$ and $\langle B \rangle$ are identical.)

(Atomic-Complex-Structure) $A \neq C$, A atomic, C complex

(No atomic proposition is identical with a complex proposition.)

RMP rests on a version of Cantor's theorem. I will explain the theorem before describing RMP.

Cantor's Theorem. Let variable $\boldsymbol{g}$ range over any domain, whose members I will call *groupables*. Let $\mathbf{G_1}$, $\mathbf{G_2}$ range over some kind of groups (such as sets, pluralities, properties or compounds) of groupables. Read $\mathbf{I}(\boldsymbol{g}, \mathbf{G_1})$ as 'G_1 includes g'. If Group-Plenitude is valid, then there are more groups than groupables, i.e., there is no surjective partial function from groupables to groups, nor any formula $\varphi(g, G_1)$ that could define such a function.

(Group-Plenitude) $\exists G_1 \forall g \, (I(g, G_1) \leftrightarrow A)$, $\mathbf{G_1}$ not free in A

(Some group includes exactly those groupables g such that A.)

More precisely:

[2] Plural propositional quantification is a relatively new device. See Hall 2021: 473ff. for some applications.

Cantor. For any formula $\varphi(g, G_1)$, the following sentences are jointly classically inconsistent with the schema Group-Plenitude.

(Functionality-of-$\varphi(g, G_1)$) $\quad \forall g\, \forall G_1\, \forall G_2\, ((\varphi(g, G_1)\ \&\ \varphi(g, G_2)) \rightarrow G_1 = G_2)$

(For each groupable g, there is at most one group G_1 such that $\varphi(g, G_1)$.)

(Surjectiveness-of-$\varphi(g, G_1)$) $\quad \forall G_1\, \exists g\, \varphi(g, G_1)$

(For each group G_1, there is some groupable g such that $\varphi(g, G_1)$.)

Proof. I will use underlined variables as constants designating values of these variables (e.g., $\underline{\boldsymbol{g}}$ is a constant designating a groupable). Suppose for *reductio* that Group-Plenitude is valid and, for some formula $\varphi(g,G_1)$, Functionality-of-$\varphi(g,G_1)$ and Surjectiveness-of-$\varphi(g,G_1)$ are true. By Group-Plenitude, there is a group $\underline{G_1}$ – the *diagonal group (relative to $\varphi(g, G_1)$)* – such that:

(1) $\forall g\, \big(I(g,\underline{G_1}) \leftrightarrow \exists G_2\, (\varphi(g,G_2)\ \&\ {\sim}I(g,G_2))\big)$
($\underline{G_1}$ includes a groupable g iff, for some G_2 that does not include g, $\varphi(g,G_2)$.)

Surjectiveness-of-$\varphi(g,G_1)$ entails that, for some groupable $\underline{g}$:

(2) $\varphi(\underline{g},\underline{G_1})$

We can classically prove $\mathbf{I(\underline{g},\underline{G_1})\&{\sim}I(\underline{g},\underline{G_1})}$. Either $I(\underline{g},\underline{G_1})$ or ${\sim}I(\underline{g},\underline{G_1})$. Suppose $I(\underline{g},\underline{G_1})$. By (1), $\varphi(\underline{g},\underline{G_2})$ and ${\sim}I(\underline{g},\underline{G_2})$ for some $\underline{G_2}$. By (2) and Functionality-of-$\varphi(g,G_1)$, $\underline{G_1}=\underline{G_2}$. So, ${\sim}I(\underline{g},\underline{G_1})$. Hence, $I(\underline{g},\underline{G_1})\&{\sim}I(\underline{g},\underline{G_1})$. Next, suppose ${\sim}I(\underline{g},\underline{G_1})$. By (1), there is no G_2 such that $\varphi(\underline{g},G_2)$ and ${\sim}I(\underline{g},G_2)$. Given (2), $I(\underline{g},\underline{G_1})$ follows. Hence, $I(\underline{g},\underline{G_1})\&{\sim}I(\underline{g},\underline{G_1})$.

RMP. RMP comes in different versions. Each version uses Cantor to show that structurism is classically inconsistent with two prima facie attractive ontological principles. The *first* principle is Proposition-Plenitude.

(Proposition-Plenitude) $\quad \exists p\, (p = A)$, $\boldsymbol{p}$ not free in A
(There is such a proposition as $\langle A \rangle$.)

The *second* is a plenitude principle for groups of propositions of the form displayed by Group-Plenitude. The paradox shows that structurists have to reject either one of these principles or classical logic.

Different versions of RMP involve plenitude principles for different kinds of groups of propositions. I will consider four versions, called RMP_{set}, RMP_{plu}, RMP_{con} and RMP_{pty}, since they feature plenitude principles for sets, pluralities, conjunctions and properties of propositions, respectively. I will discuss the first three versions in this section and $\mathrm{RMP}_{\mathrm{pty}}$ in §3.

RMP$_{set}$. (Cp. Russell 1996: 527) Structurism is classically inconsistent with Proposition-Plenitude and Set-Plenitude. ($\boldsymbol{s_1}, \boldsymbol{s_2}$ range over sets, $\in$ is a membership predicate whose first and second places take a propositional and an individual term, respectively.)

(Set-Plenitude)	$\exists s_1\ \forall p\ (p \in s_1 \leftrightarrow \mathrm{A})$, $\boldsymbol{s_1}$ not free in A (Some set contains exactly those propositions p such that A.)

Predication-Structure entails Functionality-of-$\boldsymbol{p}$=($\boldsymbol{s_1}$=$\boldsymbol{s_1}$),[3] Proposition-Plenitude entails Surjectiveness-of-$\boldsymbol{p}$=($\boldsymbol{s_1}$=$\boldsymbol{s_1}$).

(Functionality-of-$\boldsymbol{p} = (\boldsymbol{s_1} = \boldsymbol{s_1})$)	$\forall p\ \forall s_1\ \forall s_2\ (((p = (s_1 = s_1))\ \&\ (p = (s_2 = s_2))) \rightarrow s_1 = s_2)$ (For any p, there is at most one s_1 such that $p = \langle s_1 = s_1 \rangle$.)
(Surjectiveness-of-$\boldsymbol{p} = (\boldsymbol{s_1} = \boldsymbol{s_1})$)	$\forall s_1\ \exists p\ (p = (s_1 = s_1))$ (For any s_1, there is some p such that $p = \langle s_1 = s_1 \rangle$.)

Cantor entails that Set-Plenitude, Functionality-of-$\boldsymbol{p} = (\boldsymbol{s_1} = \boldsymbol{s_1})$ and Surjectiveness-of- $\boldsymbol{p} = (\boldsymbol{s_1} = \boldsymbol{s_1})$ are classically inconsistent. The proof is like the above proof of Cantor. By Set-Plenitude, there is a *diagonal set* relative to $\boldsymbol{p} = (\boldsymbol{s_1} = \boldsymbol{s_1})$, i.e. a set $\underline{s}_1$ such that:

$$\forall p\ (p \in \underline{s}_1 \leftrightarrow \exists s_2\ ((p = (s_2 = s_2))\ \&\ (p \notin s_2)))$$

($\underline{s_1}$ contains exactly those p such that, for some s_2, p=$\langle s_2$=$s_2 \rangle$ and $p \notin s_2$.)

Surjectiveness-of-$\boldsymbol{p}$=($\boldsymbol{s_1}$=$\boldsymbol{s_1}$) entails that, for some $\underline{p}$, $\underline{p}$=$\langle \underline{s_1}$=$\underline{s_1} \rangle$. Using Functionality-of-$\boldsymbol{p}$=($\boldsymbol{s_1}$=$\boldsymbol{s_1}$), we can classically prove $(\underline{\boldsymbol{p}} \in \underline{\boldsymbol{s_1}})\ \&\ (\underline{\boldsymbol{p}} \notin \underline{\boldsymbol{s_1}})$.[4]

The following results can be proven by analogous reasoning.

RMP$_{plu}$. (McGee and Rayo 2000, Uzquiano 2015) Structurism is classically inconsistent with Proposition-Plenitude and Plurality-Plenitude. (Read $\boldsymbol{p \prec pp}$ as 'p is among pp'.)

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[3] Functionality-of-$\boldsymbol{p}$=($\boldsymbol{s_1}$=$\boldsymbol{s_1}$) follows from $\forall \boldsymbol{s_1}\ \forall \boldsymbol{s_2}\ (((\boldsymbol{s_1} = \boldsymbol{s_1}) = (\boldsymbol{s_2} = \boldsymbol{s_2})) \rightarrow (\boldsymbol{s_1} = \boldsymbol{s_2}))$, which follows from an instance of Predication-Structure.

[4] I used $\boldsymbol{p}$=($\boldsymbol{s_1}$=$\boldsymbol{s_1}$) to replace $\varphi(g, \mathrm{G}_1)$ in Functionality-of-$\varphi(g, \mathrm{G}_1)$ and Surjectiveness-of-$\varphi(g, \mathrm{G}_1)$. I could instead have used $\boldsymbol{p}$=$\mathbf{A}(\boldsymbol{s_1})$ for any other formula $\mathrm{A}(s_1)$ containing free occurrences of $\boldsymbol{s_1}$. However, had I used $\boldsymbol{p}$=$\mathbf{A}(\boldsymbol{s_1})$ for some *complex* formula $\mathrm{A}(s_1)$, I would have needed further structurist principles in addition to Predication-Structure to prove Functionality-of-$\boldsymbol{p}$=$\mathbf{A}(\boldsymbol{s_1})$. (For example, had I used $\boldsymbol{p}$=$\exists \boldsymbol{s_2}(\boldsymbol{s_2}$=$\boldsymbol{s_1})$, I would have needed Existential-Structure.) The proof would otherwise have been the same.

(Plurality-Plenitude) $\exists pp\, \forall p\, (p \prec pp \leftrightarrow \mathrm{A})$, $\boldsymbol{pp}$ not free in A[5]

RMP_{con}. (Cp. Russell 1996: 527) Structurism is classically inconsistent with Proposition-Plenitude and Conjunction-Plenitude. ($\boldsymbol{c}$ ranges over conjunctive propositions, $\boldsymbol{Cpc}$ (pronounced 'p is a conjunct of c') abbreviates $\exists \boldsymbol{pp}\, ((\boldsymbol{c} = \boldsymbol{\bigwedge pp})\ \&\ (\boldsymbol{p} \prec \boldsymbol{pp}))$.)

(Conjunction-Plenitude) $\exists c\, \forall p\, (Cpc \leftrightarrow \mathrm{A})$, $\boldsymbol{c}$ not free in A
(Some conjunction conjoins exactly those p such that A.)

This presents a problem for structurists, since they have reasons to endorse Conjunction-Plenitude. For, Conjunction-Plenitude follows from Conjunction-Structure and the attractive principles Plurality-Plenitude and (3).[6]

(3) $\forall pp\, \exists c\, (c = \bigwedge pp)$
(Any propositions have a conjunction.)

One solution to RMP is to reject structurism. For example, denying Predication-Structure, we can say that there are distinct sets of propositions s_1, s_2 such that $\langle s_1 = s_1\rangle = \langle s_2 = s_2\rangle$. Consequently, *Functionality-of-$\boldsymbol{p=(s_1=s_2)}$* fails and $\mathrm{RMP_{set}}$ collapses. $\mathrm{RMP_{plu}}$ and $\mathrm{RMP_{con}}$ have analogous solutions. Call this the *anti-structurist solution* to RMP.

What are the structurist's options? Russell (1908) responded to RMP by developing the ramified theory of types. (Also see Church 1974, 1976, Whittle 2017, Hodes 2015.) Structurists disinclined towards ramification and unwilling to accept a contradiction need to reject at least one assumption of each version of RMP – either a principle of plenitude or a logical principle. They

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[5] $\exists \boldsymbol{pp}\, \forall \boldsymbol{p}\, (\boldsymbol{p} \prec \boldsymbol{pp} \leftrightarrow (\boldsymbol{p} \neq \boldsymbol{p}))$ instantiates Plurality-Plenitude, which therefore entails the existence of a plurality of zero propositions. Those who reject this conclusion might prefer (11) to Plurality-Plenitude.

(11) $(\exists p\, \mathrm{A}) \rightarrow (\exists pp\, \forall p\, (p \prec pp \leftrightarrow \mathrm{A}))$, $\boldsymbol{pp}$ not free in A

(11) suffices to generate the paradox. For, Plurality-Plenitude is only needed in $\mathrm{RMP_{plu}}$ to prove that there is a diagonal plurality, i.e. that (12) holds.

(12) $\exists pp\, \forall p\, \big(p \prec pp \leftrightarrow \exists qq\, ((p = (qq = qq))\ \&\ {\sim}(p \prec qq))\big)$

But (12) also follows from (11) and (13).

(13) $\exists p\, \exists qq\, ((p = (qq = qq))\ \&\ {\sim}(p \prec qq))$

Moreover, (13) is provable. Given any complex sentence B, Proposition-Plenitude entails $\exists \boldsymbol{p}\, (\boldsymbol{p} = \mathbf{B})$. By $\exists \boldsymbol{p}\, (\boldsymbol{p} = \mathbf{B})$ and (11), some $\underline{pp}$ include only $\langle \mathrm{B}\rangle$. By Proposition-Plenitude, there is a proposition $\langle \underline{pp} = \underline{pp}\rangle$. $\langle \underline{pp} = \underline{pp}\rangle$ is atomic. By Atomic-Complex-Structure, $\langle \mathrm{B}\rangle \neq \langle \underline{pp} = \underline{pp}\rangle$. Therefore, $\langle \underline{pp} = \underline{pp}\rangle \nprec \underline{pp}$. (13) follows.

[6] By Plurality-Plenitude, some $\underline{pp}$ include exactly those p such that A. By (3), $\underline{pp}$ have a conjunction, $\underline{c}$. By Conjunction-Structure, $\underline{c}$ is not the conjunction of any plurality except $\underline{pp}$. Hence, a proposition p satisfies $\exists \boldsymbol{pp}\, ((\underline{c} = \boldsymbol{\bigwedge pp})\ \&\ (\boldsymbol{p} \prec \boldsymbol{pp}))$ iff $\boldsymbol{p} \prec \underline{\boldsymbol{pp}}$ holds, i.e. iff p is such that A. Therefore, $\forall \boldsymbol{p}\, (\mathbf{C}\boldsymbol{p}\underline{\boldsymbol{c}} \leftrightarrow \mathbf{A})$ is true. This shows that Conjunction-Plenitude holds.

should (i) give reasons for rejecting these principles that are independent of RMP (to avoid charges of ad hocness) and (ii) offer independently motivated substitutes. I will argue that natural assumptions about grounding allow structurists to discharge these obligations in a unified way for RMP_{plu}, RMP_{set} and RMP_{con}.

2. Grounding and RMP

Grounding (Schaffer 2009, Rosen 2010, Koslicki 2012, Fine 2012) is commonly understood as a non-causal explanatory relation between metaphysically non-fundamental facts and the more fundamental facts that give rise to them.[7,8] Some notation, terminology and assumptions. Where S is a true sentence or name for a true sentence, ⌜[S]⌝ will designate the fact stated by that sentence. Facts *ff partially ground* (*ground$_p$*) fact *g* iff *g* is grounded in facts that include *ff*. I will remain neutral on whether grounding (or grounding$_p$) is transitive and use 'grounds*' ('grounds$_p$*') to express the ancestral relation of grounding (grounding$_p$).[9] Grounding* (grounding$_p$*) is transitive by definition.

While not completely uncontroversial, Non-Circularity is accepted by many grounders. I will assume its truth.

(Non-Circularity) No fact grounds$_p$* itself.[10]

I will also make the following very natural assumptions.

(Plurality-Grounding) A plurality's existence is grounded$_p$* in the existence of each entity it includes.[11]

(Set-Grounding) A set's existence is grounded$_p$* in the existence of each entity it contains.[12]

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[7] However, Schaffer (2009) argues that the relata of grounding include many entities besides facts.

[8] For grounding skepticism, see Hofweber 2009, Sider 2011:ch.8, Wilson 2014.

[9] Rosen (2010: 116), Schaffer (2009: 376), Audi (2012), Raven (2013) accept transitivity. Schaffer (2012: §2) opposes it; for replies, see Javier-Castellanos 2014, Litland 2013, Makin 2019, Raven 2013.

[10] Non-Circularity follows from the common assumptions that grounding$_p$ is transitive (footnote 9) and irreflexive (Audi 2012, Rosen 2010: 115, Schaffer 2009: 376, Raven 2013; for arguments against unrestricted irreflexivity, see Correia 2014: 54-5, Woods 2018; cp. Jenkins 2011). However, Non-Circularity does not require transitivity.

[11] Plurality-Grounding assumes, somewhat controversially, that pluralities exist in addition to the singular entities they include. For discussion, see Rayo 2007, Florio and Linnebo 2016.

[12] Plurality-Grounding and Set-Grounding leave open whether the existence of the entities included in a plurality or set K *grounds** or merely *grounds$_p$** K's existence. In the former case, the existence of ∅ and of the empty plurality (the plurality of zero entities) is grounded* in the empty plurality of facts (Fine 2012: 47).

(Proposition-Grounding) A Russellian proposition's existence is grounded$_p$* in the existence of each of its constituents.

The constituents of a Russellian proposition p include (among other things) the entities and pluralities that p is about and p's constituent propositions (if any). For example, a's existence grounds$_p$* $\langle Fa\rangle$'s existence, and $\langle Fa\rangle$'s existence grounds$_p$* $\langle Fa\&Gb\rangle$'s existence.

Each of RMP_{set}, RMP_{plu} and RMP_{con} uses instances of two plenitude principles. The instances, and consequently the principles, are jointly classically inconsistent with the foregoing grounding principles.

Consider RMP_{set}. The above grounding principles classically entail:

(4) For every proposition p and set s_2, if $p=\langle s_2=s_2\rangle$, then $p\notin s_2$.

(*Proof.* Suppose $p=\langle s_2=s_2\rangle$ and $p\in s_2$. Given that s_2 is a constituent of p, Proposition-Grounding entails that [s_2 exists] grounds$_p$* [p exists]. By Set-Grounding, [p exists] grounds$_p$* [s_2 exists]. Hence, [s_2 exists] grounds$_p$* itself, contrary to Non-Circularity.) The instances of Set-Plenitude and Proposition-Plenitude used in RMP_{set} say that, for some $\underline{s}_1$ and $\underline{q}$:

(5) $\underline{s}_1=\{p\colon \exists s_2\,((p=\langle s_2=s_2\rangle)\ \&\ (p\notin s_2))\}$

(6) $\underline{q}=\langle \underline{s}_1=\underline{s}_1\rangle$

(4) and (5) entail that $\underline{s}_1=\{p\colon \exists s_2\,(p=\langle s_2=s_2\rangle)\}$. Given (6), it follows that $\underline{q}\in\underline{s}_1$. That contradicts (4).

Analogous reasoning (employing Plurality-Grounding and Proposition-Grounding instead of Set-Grounding) applies to RMP_{plu} and RMP_{con}. Structurist grounders who accept the above grounding principles thus have reasons independent of RMP to deny the pairs of plenitude principles underlying the three versions of RMP.

They should tell us which principle in each pair to reject and offer replacements. An additional grounding principle makes these tasks easier. Let the term 'SPP-item' cover sets, propositions and pluralities. Call SPP-item K *existentially dependent on* SPP-item K* iff [K* exists] grounds$_p$* [K exists]. The additional grounding principle runs thus:

(Wellfoundedness) There is no infinite sequence of SPP-items $K_1,K_2,\ldots$ such that K_i existentially depends on K_{i+1} for $i=1,2,\ldots$.

Wellfoundedness allows grounders to say that all entities and pluralities form an iterative hierarchy. Level 0 includes all singular entities except propositions and sets, but no pluralities. Level $\alpha+1$ includes all entities and pluralities existing at level α and all SPP-items that existentially depend only on entities and pluralities existing at level α (i.e., sets and pluralities of level-α-entities; negations, conjunctions, etc. of level-α-propositions; propositions about entities and pluralities existing at level α; etc.). A limit level includes all entities and pluralities existing at levels below it. Every entity and plurality exists at some level. On this account:

(7) There is a set (plurality, conjunction) of all propositions satisfying condition C only if, for some level α, all propositions satisfying C exist at levels below α.

The simplest version of this view incorporates the following principle.

(8) There is no highest level. New sets, pluralities and propositions come to exist at every level.

(7) and (8) entail that there is no set, plurality or conjunction of all propositions. That invalidates Set-Plenitude, Plurality-Plenitude and Conjunction-Plenitude.[13] However, provided the Level-0 entities form a set, we can adopt the following replacements.

(Set- (Plurality-, Conjunction-) Plenitude*). For any level α and definable condition C, some set (plurality, conjunction) contains (includes, conjoins) exactly those propositions at levels below α that satisfy C.

Proposition-Plenitude remains valid.

Set-Plenitude* does not entail the existence of the paradox-generating diagonal set $\underline{s}=\{p{:}\exists s_1((p=(s_1=s_1))\ \&\ p\notin s_1)\}$. For, there is no level α such that all propositions that satisfy $\boldsymbol{\exists s_1\,((p=(s_1=s_1))\ \&\ p\notin s_1)}$ exist at levels below α.[14] However, for every level α, there is a *level-relative* diagonal set $\underline{s}_\alpha =_{\text{def}} \{p{:}\ \exists s_1\,((p=(s_1=s_1))\ \&\ p\notin s_1)$ and p exists below Level $\alpha\}$. $\underline{s}_\alpha$ does not engender paradox. Since $\langle \underline{s}_\alpha=\underline{s}_\alpha\rangle$ comes to exist at Level $\alpha+1$, $\underline{s}_\alpha$'s restriction to propositions existing below Level α guarantees that $\langle \underline{s}_\alpha=\underline{s}_\alpha\rangle\notin\underline{s}_\alpha$. There is no way to prove the contradiction $\boldsymbol{\langle \underline{s}_\alpha=\underline{s}_\alpha\rangle\in\underline{s}_\alpha\ \&\ \langle \underline{s}_\alpha=\underline{s}_\alpha\rangle\notin\underline{s}_\alpha}$. By analogous reasoning, Plurality-Plenitude* (Conjunction-Plenitude*) does not entail the existence of a paradox-generating diagonal plurality (conjunction), but only of harmless level-relative diagonal pluralities (conjunctions).

There have been previous attempts to resolve versions of RMP by abandoning their underlying ontological assumptions (e.g., Deutsch 2014, Walsh 2016, Yu 2017), but they were not motivated ground-theoretically. The account superficially most similar to mine is Yu's solution to RMP_{plu}, which employs an iterative hierarchy of propositions in which the propositions at each level are about entities and pluralities existing at lower levels. But even this view differs significantly from mine. Yu does not use the notion of grounding to motivate his account. While all propositions in

[13] Proof: $\boldsymbol{\exists s\,\forall p\,(p\in s \leftrightarrow (p=p))}$, $\boldsymbol{\exists pp\,\forall p\,(p\prec pp \leftrightarrow (p=p))}$, $\boldsymbol{\exists c\forall p(Cpc\leftrightarrow(p=p))}$ instantiate Set-Plenitude, Plurality-Plenitude, Conjunction-Plenitude, respectively.

[14] Proof. Let α be any level. By (8), some set of propositions $\underline{s}_2$ comes to exist at α. $\langle \underline{s}_2=\underline{s}_2\rangle$ comes to exist at $\alpha+1$. Hence, $\langle \underline{s}_2=\underline{s}_2\rangle\notin\underline{s}_2$. Therefore, $\langle \underline{s}_2=\underline{s}_2\rangle$ satisfies $\boldsymbol{\exists s_1\,((p=(s_1=s_1))\ \&\ p\notin s_1)}$. So, not all propositions satisfying this formula exist below α.

my hierarchy actually exist, Yu's hierarchy consists of *possible* propositions some of which do not exist. Compound propositions sometimes appear at the same level of his hierarchy as their constituent propositions, so that his levels do not reflect the existential dependence of complex propositions on simpler ones. The account consequently has no resources to address RMP_{con}.

3. Another form of RMP

What makes the unified ground-theoretic treatment of RMP_{set}, RMP_{plu} and RMP_{con} possible is an important commonality between these versions of RMP: they involve groups (sets, pluralities, conjunctions) whose existence is grounded in the existence of the propositions they include. Other versions with this feature possess analogous solutions. However, it is not straightforward to extend this approach to variants that lack this feature, such as RMP_{pty}.

> *RMP_{pty}*. (Dorr 2016, Goodman 2017) Let **X**, **Y** be monadic predicate variables ranging over monadic properties of propositions. Cantor can be used to show that Property-Plenitude, Proposition-Plenitude and Predication-Structure are jointly classically inconsistent.

(Property-Plenitude) $\exists X \, \forall p \, (Xp \leftrightarrow A)$, $\boldsymbol{X}$ not free in A

To solve RMP_{pty} in the same way as RMP_{set}, RMP_{plu} and RMP_{con}, we would need Property-Grounding.

(Property-Grounding) The existence of a property is $\text{grounded}_{\text{p}}$* in the existence of each entity or plurality instantiating it.

However, unlike Plurality-Grounding, Set-Grounding and Proposition-Grounding, Property-Grounding is highly implausible: Socrates instantiates humanity, but [Socrates exists] does not ground_{p}* [Humanity exists]. Extending my framework to versions of RMP like RMP_{pty} remains a task for future work.

Objection 1. My goal is to sketch the beginning of a structurist solution to RMP that is no worse than the anti-structurist's solution. However, by requiring different solutions for different versions of RMP, my proposal is less unified than the anti-structurist's.

Reply. Evaluating this objection requires us to look beyond RMP. RMP belongs to a large family of paradoxes that rest on instances of Cantor. (Call such paradoxes 'Cantorian.') Here is another well-known example.

> *Russell's paradox*. The following variant of naïve set comprehension is prima facie attractive (***s*** ranges over sets, $\boldsymbol{h_1}$,$\boldsymbol{h_2}$ over sets of sets).

(Set-of-Sets-Plenitude) $\exists h_1 \, \forall s \, (s \in h_1 \leftrightarrow A)$, $\boldsymbol{h_1}$ not free in A

Functionality-of-$\boldsymbol{s=h_1}$ follows from the transitivity of identity, Surjectiveness-of-$\boldsymbol{s=h_1}$ is trivial.

(Functionality-of-$\boldsymbol{s=h_1}$) $\quad \forall s\, \forall h_1\, \forall h_2\, ((s=h_1 \,\&\, s=h_2) \rightarrow h_1=h_2)$

(Surjectiveness-of-$\boldsymbol{s=h_1}$) $\quad \forall h_1\, \exists s\, (s=h_1)$

(Every set of sets is a set.)

By Set-of-Sets-Plenitude, there is a diagonal set $\underline{h_1}$ relative to $\boldsymbol{s=h_1}$. $\underline{h_1} = \{s\!: \exists h_1\, (s=h_1 \,\&\, s\notin h_1)\}$, i.e. $\underline{h_1}$ contains exactly the non-self-containing sets of sets. We can classically prove $\boldsymbol{\underline{h_1}\in\underline{h_1}\ \&\ \underline{h_1}\notin\underline{h_1}}$.

Anti-structurism provides a unified solution to RMP, my account does not. However, I will argue that (9) holds.

(9) (i) My account yields an attractive unified solution to $\mathrm{RMP}_{\mathrm{set}}$, $\mathrm{RMP}_{\mathrm{plu}}$, $\mathrm{RMP}_{\mathrm{con}}$ and Russell's Paradox.

(ii) Anti-structurism does not.

In light of (9), the anti-structurists's overall view no longer looks more unified. It merely differs in what it unifies with what.

Argument for (9)(i). The grounding principles of §2 provide reasons independent of Russell's Paradox for rejecting (the paradox-generating instance of) Set-of-Sets-Plenitude. By Set-Grounding and Non-Circularity:

(10) No set contains itself.

Hence, if there were a diagonal set $\underline{h_1}$ of all non-self-containing sets of sets, $\underline{h_1}$ would contain *all* sets of sets. But then $\underline{h_1}$ would contain itself, contrary to (10). So, no diagonal set exist. Moreover, the iterative view of §2 provides a workable replacement for Set-of-Sets-Plenitude:

(Set-of-Sets-Plenitude*) For any level α and definable condition C, some set contains exactly those sets at levels below α that satisfy C.

(Set-of-Sets-Plenitude* is essentially the plenitude principle of the familiar iterative view of sets (Boolos 1971).)

Argument for (9)(ii). The anti-structurist solution to RMP rejects the paradox's functionality assumptions (Functionality-of-$\boldsymbol{p=(s_1=s_1)}$, *Functionality-of-*$\boldsymbol{p=(pp=pp)}$, etc.). To give a *unified* treatment of Russell's Paradox and any given version of RMP, anti-structurists would have to solve Russell's Paradox in the analogous way, by denying Functionality-of-$\boldsymbol{s=h_1}$. But that would require rejecting the transitivity of identity – a highly unattractive move.

Objection 2. The versions of RMP form a more unified class than $\mathrm{RMP}_{\mathrm{set}}$, $\mathrm{RMP}_{\mathrm{plu}}$, $\mathrm{RMP}_{\mathrm{con}}$ and Russell's Paradox. It is therefore more important to provide a unified solution to RMP than to the latter paradoxes. That consideration favors the anti-structurist solution.

Reply. Cantorian paradoxes differ (among other things) in the entities that play the roles of groupables in the corresponding instances of Cantor and in the entities playing the role of groups. What unifies the versions of RMP is that they involve the same entities (propositions) as groupables. RMP_{set} and Russell's Paradox are unified in a different way: they involve entities of the same kind (sets) as groups. We could say that paradoxes unified in the first way should receive unified treatment. My account violates this constraint. But we could equally reasonably insist that paradoxes unified in the second way should have a unified solution. The anti-structurist solution to RMP forces us to violate this second constraint, by ruling out a unified treatment of RMP_{set} and Russell's Paradox. The first violation is not obviously worse than the second.[15]

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