ELE539A: Optimization of Communication Systems
Lecture 1: Motivation and Introduction

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Communication Systems

How to send information from one point to another (and for other source-destination configurations) over a medium?
Questions

• How to meet the requirements from the applications of the information (like accuracy, throughput, latency, jittering, mobility support...)?

• How to represent and use the information?

• How to utilize the communication medium?

• How to connect users?

• How to reach one point from another?

• How to coordinate among the transmitters and receivers?

• How to regulate competition among users?

• How to make the system robust to failures, attacks, variations, growth across space and over time?
Layered Architecture

Divide and conquer: break the overall big problem into smaller ones with standardized interfaces

Each layer provides a service to upper layers and utilizes the services provided by lower layers

Performance may not be ‘optimal’, but makes the architecture simple and flexible
Point-to-Point Communication Channel

Compress analog signals into digital data
Add redundancy to protect against channel impairments
Map digital data onto physical waveforms suitable for the medium
Questions

• How to describe the channel and estimate its characteristics (twisted pair, coaxial cable, optic fiber, radio, acoustic, storage)?
• How fast can data be sent reliably?
• How to compress signals?
• How to mitigate noise (thermal noise, impulse noise ...) and manage interference (from other users, from reflections, among symbols) ...
• How to use the communication resources (time, frequency, engineering design parameters) efficiently?
• What happens when multiple transmitters send data to multiple receivers?
Communication Networks

Not necessarily a direct link, but a networked communication system

Questions on last slide remain, plus more questions (and opportunities)
Questions

• Fixed or dynamic topology? Who are transceivers and who are relays?
• Direct link or switched architecture? Circuit switch or packet switch or something else?
• How to divide into (possibly different types of) subnetworks?
• End-to-end control or hop-by-hop control?
• How to get on the communication medium?
• How to get from one point to another?
• How to monitor and adjust overall state of the network?
• How to ensure accurate, secure, dependable, timely, and usable transfer of information across space among competing users?
Model, Analysis and Design

Empirical data from field trials and deployments
Test-bed operations
Computer simulations

Analytic tools
- Information theory, coding theory, communication theory
- Queuing theory and other probabilistic tools
- Systems control theory, graph theory, game theory, economics modelling, physics/biology modelling...
- Optimization theory and distributed algorithms
Optimization

\[
\begin{align*}
\text{minimize} & \quad f(x) \\
\text{subject to} & \quad x \in C
\end{align*}
\]

Optimization variables: \(x\). Constant parameters describe objective function \(f\) and constraint set \(C\).
Questions

- How to describe the constraint set?
- Can the problem be solved globally and uniquely?
- What kind of properties does it have? How does it relate to another optimization problem?
- Can we numerically solve it in an efficient, robust, and distributed way?
- Can we optimize multiple objectives simultaneously?
- Can we optimize over a sequence of time instances?
- Can we find the problem for a given solution?
Applications

Theory and algorithms of optimization are extremely powerful:

- Communication systems

- Other information science areas: signal/image/video processing, systems control, algorithms, graphics, data analysis, theoretical computer science ...

- Other engineering disciplines: aerospace, mechanical, chemical, civil, transportation, computer architecture, analog circuit design ...

- Physics, chemistry, biology ...

- Economics, finance, management ...

- Analysis, probability, statistics, differential equations ...

Three Application-Appreciation Cycles since 1940s
Methodologies

Widely known: linear programming is powerful and easy to solve

Modified view: watershed between easy and hard optimization problems is not linearity, but convexity

- Local optimality is also global optimality
- Lagrange duality theory well developed
- Know a lot about the problem and solution structures
- Efficiently compute the solutions numerically

Need to know how to recognize and formulate convex optimization, and use the recently developed tools to solve your problem (an objective of this course)

Active research area with many exciting recent and ongoing developments, and other challenges (discrete optimization, nonconvex problems, robust and distributed algorithms...)

Optimization of Communication Systems

Three meanings of ‘optimization of communication systems’:

- Formulate the problem as an optimization problem
- Interpret a given solution as an optimizer/algorith for an optimization problem
- Extend the underlying theory by optimization theoretic techniques

A remarkably powerful, versatile, widely applicable and not yet fully recognized viewpoint

Applications in communication systems also stimulate new developments in optimization theory and algorithms
Three Meanings

- Problems
- Solutions
- Theories
Two Views

- Network to be optimized
 Optimization is a tool

- Network as an optimizer
 Optimization is a mentality and language
Lecture Outline

- Communication systems
- Optimization: theory, algorithm, mentality
- Overview
- Why take this course
- Course arrangements
What This Course Is About

How problems in communication systems can be formulated and solved as optimization problems

- Classic results (starting with 1940s)
- Current research (papers being published as we speak)

Applications topics:
Information theory problems, transmitter and receiver design, channel decoding, detection and estimation, multiple antenna beamforming, network resource allocation and utility maximization, optical network topology design, wireless power control and medium access, network flow problems, IP routing, TCP congestion control, cross layer design

Methodology topics:
Linear programming, convex optimization, quadratic programming, geometric programming, integer programming, robust optimization, Pareto optimization, dynamic programming, nonconvex optimization, Lagrange duality, KKT optimality conditions, gradient methods, interior point methods, distributed algorithms
What This Course Is Not About

- Not a math course on convex analysis (not many rigorous proofs)
- Not an OR course on nonlinear optimization (only basic optimization/algorithm topics)
- Not an EE course on digital communication (cover only selected topics)
- Not an EE/CS course on networking (cover only selected topics)
- Not a CS course on algorithms (little computational complexity analysis)

‘Just enough’ background materials presented ‘just-in-time’
Lecture 2: Convex Sets and Convex Functions

\[ f(y) = f(x) + \nabla f(x)^T (y - x) \]
Lecture 3: Convex Optimization and Lagrange Duality
Lecture 4: LP and QP
Lecture 5: NFP

[Diagram of network flow problems]
Lecture 6: GP
Lecture 7: GP Applications

Diagram:

Source → Source encoder → Channel encoder → Channel → Channel decoder → Source decoder → Destination

Compression \( X \) → Encoder → Rate \( R \) → Decoder → \( \hat{X} \)

Transmission \( W \) → Sender → Channel \( p(y|x) \) → Receiver → \( \hat{W} \)
Lecture 8: Distributed Algorithms
Lecture 9: NUM and TCP

Average utilization

- Linux TCP: txq=100, 19%
- Linux TCP: txq=10000, 95%
- FAST: txq=100, 16%
- Linux TCP: txq=10000, 48%
- Linux TCP: txq=10000, 92%
Lecture 10: Heterogeneous Congestion Control Protocol
Lectures 12 and 13: Layering As Optimization Decomposition
Lecture 14: Wireless Power Control

Distributed control convergence

User rate (bps/Hz/user)

- Optimal utility
- Distributed control

Iteration
Lecture 16: DSL Spectrum Management

Optimal Spectrum Balancing
Iterative Spectrum Balancing
Autonomous Spectrum Balancing
Iterative Waterfilling

RT1 @ 2 Mbps, RT2 @ 2 Mbps
Lecture 17: Alternative Decompositions

- Master Problem
- Secondary Master Problem
- Subproblem
- Subproblem 1
- Subproblem 2
- Subproblem N
Lecture 19: Stochastic NUM

Normalized Source Rates

Normalized Congestion Prices

Flow AF
Flow BE
Lecture 20: SDP and Applications
Lecture 21: Nonconvex Optimization

![Graph of U(x)](image)

The graph shows the function $U(x)$ plotted against $x$. The x-axis represents $x$ values from 0 to 12, and the y-axis represents $U(x)$ values from 0 to 3. The function appears to be nonconvex, with multiple local maxima and minima.
Lecture 22: Unconstrained Optimization Algorithm
Lecture 23: Interior Point Algorithm

![Graphs showing duality gap and NT. Ite. vs. iterations and m]

- The left graph displays the duality gap on a logarithmic scale against NT. Ite., with several iterations marked as \( t_1, t_2, t_3 \).
- The right graph illustrates NT. Ite. on a logarithmic scale against m.

The graphs likely represent the performance of an Interior Point Algorithm, showing how the duality gap decreases and NT. Ite. increases as the algorithm progresses.
Guest Lectures

These are really fun lectures:

- **Students**: Some of last two years’ projects
- **Professor Jennifer Rexford** (Princeton CS): Internet IP routing
- **Dr. Daniel Palomar** (Princeton EE): Alternative Decompositions
- **Steve Sposato and Raj Savoor** (AT&T Labs): Optimizing real networks
Acknowledgements

The first course devoted to systematic treatment of the subject

Course materials drawn from a variety of sources (many textbooks, a number of recent journal/conference papers, ongoing research projects...) and distilled into a common framework

Jointly developed with Professor Steven Low at Caltech (netlab.caltech.edu)

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- Professor Stephen Boyd (Stanford)
- Professor Tom Luo (U. Minnesota)
- Professor Wei Yu (U. Toronto)
Prerequisite

• Willing to learn
• Interested in communication systems
• Comfortable with math
• Know advanced calculus, elementary linear algebra, basic probability, fundamentals of digital communications
• Basic working knowledge of Matlab

Not required but would be helpful:
• Linear programming
• Communications networking
Schedule and Lectures

Comprehensive schedule in Information Sheet

Need exceptions: please inform me early

Date: M. W. F. lectures (11am-12:20pm for MW, different times for F)
Place: J323, E Quad
Office Hours: B328, W afternoon, or by appointment
Website: www.princeton.edu/~chiangm/class.html (Outdated lecture notes so far)
Email: chiangm@princeton.edu
Homework and Midterm

Four Homework assignments (20%):
- Each 5% of grade, graded on scale of 1-4
- Some graded by yourselves, others by me
- Not emphasized in grading, but important for learning

Take-home Midterm exam (25%):
- Open books, open lecture notes, open library, open Internet, open mind
- Get exam from my office, return solutions after 24 hours
- 8 hours should be enough
Projects and Presentation

This is the fun part (and serious part)!
What’s the point of learning the materials if you can’t use it?
- Individual projects. 2 projects can collectively tackle a large problem
- Either original research or comparative study of existing solutions
- Cycle through initial and final project proposal
- You’ve got a whole month to do the project (and nothing else on this course) April 12 – May 22
- Expect a lot of interaction with me (and help from me if necessary)
- 15-20 page Report (40%) graded on both content and presentation
- 12-15 minute Presentation (15%) May 15 Pizza and Snack afternoon
Reading List

Required (all free):

- M. Chiang, *ELE539 Lecture Notes 2004* Uploaded to website and distributed in class. Does not contain all the information, and complemented by a lot of discussion and graphs in class
- Drafts of course reader from M. Chiang and S. Low
- About 15 recent journal/conference papers. Distributed in class

Other references will be mentioned in lectures and notes
Why Take This Course

• Learn the tools and mentality of optimization (surprisingly useful for other study you may engage in later on)

• Learn classic and recent results on optimization of communication systems (over a surprisingly wide range of problems)

• Train the ability to do original research in academia or industry

• Turn the final project into journal/conference publications, part of Ph.D. dissertations, M.S. theses, or undergraduate senior theses
To Do and Not To Do

TO DO:
- Learn A LOT
- To be an optimizer, think like an optimizer
- Have fun

NOT TO DO:
- Worry about grades
- Be lazy
Lecture Summary

- General overview of communication systems
- The Optimization view
- Overview of course materials
- Get ready for a lot of fun
- Next two lectures we’ll start with some math but will soon see its power in applications