# ELE539A: Optimization of Communication Systems Lecture 7: Scheduling

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## **Lecture Outline**

- Models
- Problem space
- Key algorithms

This is a new lecture, most of the materials will be given Yung Yi and I on blackboard during the lecture

## **Model I**

Resource minimization with infinite backlog (coloring)

• Constrained queuing model

1. Switching: bipartite graph

- Constraint: matching

2. (Link-level) Scheduling: general graph

- Constraint: independent set (via a graph transformation)

## **Model II**

## Constraint for scheduling:

- Graph-based model
- 1. 1-hop: Bluetooth or FH-CDMA network
- 2. 2-hop: IEEE 802.11 network
- 3. K-hop: general form
- SIR-based model
- 1. fixed power model
- 2. power control model

## **Problem Space I**

- Traffic characteristics: mean or variance of stochastic arrivals
- Channel characteristics
- Connectivity graph
- Constraint set
- Performance objective: throughput and delay
- Complexity: on-line/off-line, centralized/distributed

Tradeoff of the above

# **Problem Space II**

## Performance:

- Throughput-optimality, stability region: focus of this lecture
- Delay
- Utility

## Complexity:

- Time
- Space
- Message size
- Local computation

## **Throughput-optimality**

- Definitions
- 1. Stability
- 2. A feasible load
- 3. Throughput region or Stability region
- 4. Maximum stability region (converse theorem)
- 5. Throughput-optimality of a scheduling algorithm (achievability theorem)
- What is a throughput-optimal algorithm which is **unaware** of the statistics of arrivals?

Max-Rate Rule: Choose a schedule S[t], such that

$$S[t] \in \arg\max_{S \in \mathcal{S}} \sum_{l \in L} Q_l(t) \times S_l.$$

Intuition: "minimize the maximum queue length"

# **Algorithms**

- Collision-free
- Collision-based
- Fully distributed
- Distributed with message passing
- Centralized

# **Collision-Free Algorithms**

#### Centralized:

- Max-Rate Rule: Maximum weight independent set: NP-hard!
- transformation of the original graph into its conflict graph

#### Distributed:

- Maximal scheduling and maximal weight independent set (approximation algorithm)
- Random mixing, pick and compare (randomized algorithm)

## **Random Mixing**

Take a randomized algorithm  $\pi$  which satisfies the following:

**P1:** Prob $(R[t] == S^*) \geq \delta$ .

**P2:** Choose  $S[t] = \max[S[t-1], R[t]].$ 

Then,  $\pi$  is throughput-optimal.

- Implications: complexity reduction
- Why does this random-mixing works?

## **Collison-Based Algorithms**

#### Distributed:

- •: RTS-CTS
- Distributed scheduling based on random access
- exchange of queue length info among interfering links
- use it for determining a smart "attempt probability"

## Fully distributed:

- Aloha
- CSMA/CD, CSMA/CA

In general, 2 phases: collision avoidance and collision resolution (exponential backoff)

# **Key Scheduling Mechanisms**

- LCQ policy for time-varying rate regions (i.e., schedule sets) and variations
- Backpressure
- $\bullet~\mu c~{
  m rule}$
- Max Projection and generalizations
- Utility maximization

## **Proof Techniques**

- Graph theory: coloring, matching
- Approximation and randomized algorithm
- Lyapunov function, Foster criterion
- Markov chain
- Fluid model
- Utility maximization

## Readings

## **Key readings:**

- L. Tassiulas, A. Ephremides, "Stability properties of constrained queueing systems and scheduling policies for maximum throughput in multihop radio networks" IEEE Transactions on Automatic Control, Vol. 37, No. 12, pp. 1936-1949, December 1992
- J.G. Dai, B. Prabhakar, "The throughput of data switches with and without speedup", Proceedings of the IEEE INFOCOM, 2:556-564, Tel Aviv, Israel, March 2000

Prasanna Chaporkar, Koushik Kar, Saswati Sarkar, "Throughput Guarantees Through Maximal Scheduling in Wireless Networks," Proceedings of 43rd Annual Allerton Conference on Communication, Control and Computing, Allerton, Monticello, Illinois, September 28-30, 2005

L. Tassiulas, "Linear complexity algorithms for maximum throughput in radio networks and input queuing switches", Proceedings of INFOCOM98, San Francisco, California, 1998.

Xiaojun Lin and Shahzada Rasool, "Constant-Time Distributed

Scheduling Policies for Ad Hoc Wireless Networks," in Proceedings of IEEE Conference on Decision and Control, San Diego, December 2006

## Recommended readings:

- X. Wu and R. Srikant. Bounds on the Capacity Region of Multi-hop Wireless Networks under Distributed Greedy Scheduling. In IEEE INFOCOM 2006
- E. Modiano, D. Shah, and G. Zussman, Maximizing Throughput in Wireless Networks via Gossiping, Proc. ACM SIGMETRICS / IFIP Performance'06, June 2006
- Changhee Joo and Ness B. Shroff, "Performance of Random Access Scheduling Schemes in Multi-hop Wireless Networks", to appear in IEEE Infocom 2007
- S. Sanghavi, L. Bui, and R. Srikant, Distributed Link Scheduling with Constant Overhead. http://arxiv.org/abs/cs.NI/0611064
- L. Tassiulas, A. Ephremides, "Dynamic server allocation to parallel queues with randomly varying connectivity," IEEE Transactions on Information Theor y, Vol. 39, No. 2, pp. 466-478, March 1993.