Dynamic Spectrum Management for Green DSL

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***Abstract*—Dynamic spectrum management (DSM) has been recognized as a key technology for tackling multi-user crosstalk interference in DSL broadband access. Up to now, DSM design has mainly been focusing on maximization of data rates. However, recently, reducing the total power has become a main target, as IT power consumption has been identified as a significant contributor to global warming. In this paper we extend traditional DSM design towards a much wider power-efficient scope and show how to tackle the corresponding optimization problems. This leads to a unifying mathematical representation of more general DSM problems which will be referred to as 'green DSL'. Using this 'green DSL' approach, we quantify the power-rate trade-off for different practical settings with some surprising numerical results. We demonstrate that for some VDSL scenarios power savings of up to 50% can be achieved while still guaranteeing 95% of full-power data rate performance. This somewhat surprising trade-off result is due to not just the logarithmic dependence of rate on SINR but also the characteristics of crosstalk in DSL systems. Finally, the analysis is extended towards suboptimal DSM algorithms, leading to a power-rate-complexity trade-off.**

I. Introduction

Digital subscriber line (DSL) technology refers to a family of technologies that provides digital broadband access over the local telephone network. It is still the dominating broadband access technology with 66% of all broadband access subscribers worldwide using DSL to access the Internet [1]. The main reason of its popularity is its low deployment cost as DSL reuses the twisted pairs of the existing telephone network infrastructure for connecting the subscribers to the Internet backbone.

In order to cope with the increasing demands of the users (end-users as well as service providers) and to stay competitive with other broadband access technologies, e.g., cable connection, wireless, satellite, etc, DSL technology is continuously extended to counter new technological issues. One of the major challenges is to overcome the electromagnetic interference, also called *crosstalk*, generated among different lines operating in the same cable bundle. Different lines (i.e. users) indeed interfere with each other, leading to a very challenging interference environment where proper management of the resources is required to prevent a huge performance degradation.

Dynamic spectrum management (DSM) is recognized as a key technology for tackling this crosstalk problem [2]. The main idea of DSM is to prevent and/or remove crosstalk by using spectrum [3]–[13] and/or signal coordination [14]–[18] among different users. Note that in this work the focus will be on spectrum level coordination, also referred to as spectrum management, spectrum balancing or multi-carrier power control. More specifically, spectrum management comes down to allocating the transmit spectrum, i.e. transmit powers over all frequencies, to the different users so as to achieve some design objective. When the objective is to maximize the data rates, transmit powers should be balanced carefully, as increasing one user's transmit power increases its data rate but also causes crosstalk degradation for the other users. Next to data rates, transmit spectrum can also be allocated so as to achieve different objectives. The exact choice of the design objectives and/or constraints will be referred to as DSM design.

The major research efforts in DSM[1] algorithm design, have been focusing on maximizing the users' data rates (i.e. rate-adaptive DSM) without any power minimizing design objective, except for the power constraints defined by DSL standards. However, recently, power consumption has started to gain a lot of importance (ITU-T Study Group 15, European Code of Conduct for broadband equipment). Information and Communication Technologies (ICTs) have been identified as significant contributors to global warming [19]. As the ICT industry is growing faster than the rest of the economy, it is likely that this share will increase over time. Broadband equipment contributes to the electricity consumption and depending on the penetration level, the specifications of the equipment and the requirements of the service provider, a total European consumption of up to 50 TWh per year can be estimated for the year 2015 [20]. Therefore the European Code of Conduct for Broadband Equipment takes initiative in setting up general principles and actions, and targets to limit the (maximum) electricity consumption to 25 TWh per year which is equivalent to 5.5 Million ton of oil equivalent (TOE) and to a total saving of about € 7.5 Billion per year.

DSL, as the most deployed broadband technology today, plays an important role in this trend [21]. The DSL Forum encourages international standards bodies to develop techniques for power reduction within the scope of their activities and to maximize power savings while preserving and enhancing quality of service [21]. One of the technologies that fits well in this framework is DSM.

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[1] In the sequel, DSM refers to spectrum coordination.

In this paper, we therefore revisit DSM and extend its design with power related objectives. This approach improves upon the traditional pure data rate maximizing approach in two ways [22]: (i) Adding objectives and/or constraints for limiting transmit power reduces overall power consumption by DSL systems, as power consumed by DSL modems is indeed often dominated by circuits used for transmitting power, and (ii) smaller transmit powers encourage a 'polite' behaviour as less crosstalk is radiated into other DSL modems. These benefits of making the copper 'greener' in an evolving and increasingly energy-efficient world, were also recently pointed out in [22].

The main contributions of this paper are as follows:

(i) We extend the traditional rate-adaptive DSM design to a wider 'green' setting, incorporating power limiting objectives and/or constraints. This leads to a unifying mathematical representation of more general DSM problems, which will be referred to as 'green DSL'.

(ii) We provide a general systematic procedure for tackling the corresponding extended set of (non-convex) DSM problems by introducing extended Lagrange multipliers and modifying existing DSM algorithms. These insights are furthermore used to analyze the underlying mechanisms of the different DSM designs.

(iii) We quantify the power-rate trade-off for different practical settings with some surprising numerical results. We demonstrate that for some VDSL scenarios power savings of up to 50% can be achieved while still guaranteeing 95% of full-power data rate performance. We explain that this somewhat surprising tradeoff result is due to not just the logarithmic dependence of data rate on the signal-to-interference-plus-noise ratio (SINR) but also the characteristics of crosstalk in DSL systems. Finally, the analysis is extended towards suboptimal DSM algorithms, leading to a power-rate-complexity trade-off.

The rest of the paper is organized as follows. In Section II, the system model is introduced together with the concept of 'DSM design' through spectral coordination. In Section III, traditional data rate driven DSM design is extended towards a unifying 'green DSL' formulation. It is shown that this corresponds to an extended set of DSM designs that allow a much more power-efficient DSL configuration. In Section IV, a general systematic procedure is proposed for green DSL, the relations among the various DSM designs are analyzed, and the infeasibility issue is discussed. Finally in Section V, the engineering impact of green DSL is investigated for different realistic DSL scenarios.

II. System Model and DSM Design

A. System Model

We consider a system consisting of $\mathcal{N} = \{1, \ldots, N\}$ interfering DSL users (i.e., lines, modems) with standard synchronous discrete multi-tone (DMT) modulation with $K = \{1, \ldots, K\}$ tones (i.e., frequency carriers). The transmission can be modeled independently on each tone k as follows:

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{z}_k.$$

The vector $\mathbf{x}_k = [x_k^1, \ldots, x_k^N]^T$ contains the transmitted signals on tone k, where x_k^n refers to the signal transmitted by user n on tone k. Vectors $\mathbf{z}_k$ and $\mathbf{y}_k$ have similar structures; $\mathbf{z}_k$ refers to the additive noise on tone k, containing thermal noise, alien crosstalk, radio frequency interference (RFI), etc, and $\mathbf{y}_k$ refers to the received signals on tone k. $\mathbf{H}_k$ is an $N \times N$ matrix with $[\mathbf{H}_k]_{n,m} = h_k^{n,m}$ referring to the channel gains from transmitter m to receiver n on tone k. The diagonal elements are the direct channels and the off-diagonal elements are the crosstalk channels.

The transmit power of user n on tone k, also referred to as transmit power spectral density, is denoted as $s_k^n \triangleq \Delta_f E\{|x_k^n|^2\}$, where Δ_f refers to the tone spacing. The vector $\mathbf{s}_k \triangleq \{s_k^n, n \in \mathcal{N}\}$ denotes the transmit powers of all users on tone k. The vector $\mathbf{s}^n \triangleq \{s_k^n, k \in \mathcal{K}\}$ denotes the transmit powers of user n on all tones. The received noise power by user n on tone k, also referred to as noise spectral density, is denoted as $\sigma_k^n \triangleq \Delta_f E\{|z_k^n|^2\}$.

Note that we assume no signal coordination at the transmitters and at the receivers, and that the interference is treated as additive white Gaussian noise. Under this assumption the bit loading for user n on tone k, given the transmit spectra $\mathbf{s}_k$ of all users on tone k, is

$$b_k^n \triangleq \log_2\left(1 + \frac{1}{\Gamma}\frac{|h_k^{n,n}|^2 s_k^n}{\sum_{m\neq n}|h_k^{n,m}|^2 s_k^m + \sigma_k^n}\right) \text{ bits/Hz}, \quad (1)$$

where Γ denotes the SNR-gap to capacity, which is a function of the desired BER, the coding gain and noise margin [23]. The DMT symbol rate is denoted as f_s. The achievable total bit rate for user n and the total power used by user n are equal to, respectively:

$$\begin{array}{l} R^n \triangleq f_s \sum_{k\in\mathcal{K}} b_k^n, \\ P^n \triangleq \sum_{k\in\mathcal{K}} s_k^n. \end{array} \quad (2)$$

B. DSM Design

The basic goal of DSM through spectrum level coordination is to allocate the transmit powers dynamically in response to physical channel conditions (channel gains and noise) so as to pursue certain design objectives and/or satisfy certain constraints. A number of design constraints are imposed by DSL standards. More specifically, there is an upper bound on the total power that each user is allowed to allocate over its tones. There are also constraints on the transmit power allocated into one tone, which are referred to as spectral mask constraints. This set of constraints can be summarized by the following set of allowed power allocations:

$$\mathcal{S} = \left\{(s_k^n : n \in \mathcal{N}, k \in \mathcal{K}) : \textstyle\sum_{k\in K} s_k^n \leq P^{n,\text{tot}}, 0 \leq s_k^n \leq s_k^{n,\text{mask}}\right\}, \quad (3)$$

where $P^{n,\text{tot}}$ refers to the total available power budget for user n and $s_k^{n,\text{mask}}$ refers to the spectral mask constraint for user n on tone k.

A typical design objective [3]–[13], is to achieve some Pareto optimal allocation of the data rates R^n. The set of all possible data rate allocations, which satisfy the constraints of set $\mathcal{S}$ in (3), can be characterized by the achievable rate region $\mathcal{R}$:

$$\mathcal{R} = \left\{(R^n : n \in N) | R^n = f_s \sum_{k\in K} b_k^n(\mathbf{s}_k), \{\mathbf{s}^n, n \in \mathcal{N}\} \in \mathcal{S}\right\}.$$

However, other design objectives are also possible, considering other performance measures like transmit power, fairness, etc. In general we will refer to a *DSM design* as an optimization problem where the transmit powers correspond to the optimization variables, and where the objectives and/or constraints are functions of these transmit powers that reflect the quality of service (QoS) requirements for the corresponding DSL network. Note that the constraints corresponding to set $\mathcal{S}$ in (3) are needed in all DSM designs in order to be compliant with DSL standards.

III. Green DSL

Research efforts in DSM up to now have mainly been focusing on DSM designs involving pure data rate maximizing objectives [3]–[13]. However, as mentioned in Section I, power consumption is becoming an increasingly important issue. In this section we will revisit traditional DSM designs and then extend these to a much wider 'green' setting, incorporating relevant power limiting objectives and/or constraints. This will lead to a unifying DSM design framework, which will be referred to as *'green DSL'* in this paper.

A. Traditional Data Rate Driven DSM Designs

The 'traditional' approach to DSM is based on a 'data rate maximizing' point of view. This is sometimes also referred to as rate-adaptive DSM [24]. A typical DSM design [4] [9] [25] is formulated as follows:

$$\begin{array}{l} \max_{\{\mathbf{s}^n, n\in\mathcal{N}\}} R^1 \\ \text{s.t. } R^n \geq R^{n,\text{target}}, \quad \forall n > 1 \\ \text{s.t. } P^n \leq P^{n,\text{tot}}, \quad \forall n \\ \text{s.t. } 0 \leq s_k^n \leq s_k^{n,\text{mask}}, \quad \forall n, \forall k, \end{array} \tag{4}$$

where $R^{n,\text{target}}$ denotes the minimum data rate constraint for user n.

It is shown in [4] [9] that the optimal solution of (4) can also be obtained by solving the following optimization problem for some w_n, which can be seen as weights that specify the importance of each user n:

$$\begin{array}{l} \max_{\{\mathbf{s}^n, n\in\mathcal{N}\}} \sum_{n\in\mathcal{N}} w_n R^n \\ \text{s.t. } P^n \leq P^{n,\text{tot}}, \quad \forall n, \\ \text{s.t. } 0 \leq s_k^n \leq s_k^{n,\text{mask}}, \quad \forall n, \forall k. \end{array} \tag{5}$$

Furthermore data rate constraints can be added to formulation (5):

$$\begin{array}{l} \max_{\{\mathbf{s}^n, n\in\mathcal{N}\}} \sum_{n\in\mathcal{N}} w_n R^n \\ \text{s.t. } R^n \geq R^{n,\text{target}}, \quad \forall n, \\ \text{s.t. } P^n \leq P^{n,\text{tot}}, \quad \forall n, \\ \text{s.t. } 0 \leq s_k^n \leq s_k^{n,\text{mask}}, \quad \forall n, \forall k, \end{array} \tag{6}$$

which can be viewed as a generalization of (4), i.e. (4) is obtained by fixing the constants in (6) to $w_1 = 1, R^{1,\text{target}} = 0, \text{and } \forall n \neq 1 : w_n = 0$.

An alternative DSM design, introduced in [26], is as follows:

$$\begin{array}{l} \max_{\{R, \mathbf{s}^n, n\in\mathcal{N}\}} R \\ \text{s.t. } R^n \geq \beta_n R, \quad \forall n \\ \text{s.t. } P^n \leq P^{n,\text{tot}}, \quad \forall n \\ \text{s.t. } 0 \leq s_k^n \leq s_k^{n,\text{mask}}, \quad \forall n, \forall k, \end{array} \tag{7}$$

where β_n is a constant that specifies the minimum required proportion in data rate with respect to a base rate R for user n.

For a two-user DSL scenario, the solutions of these DSM designs can be visualized in the rate region as in Figure 1. The green dot indicates the solution to (4). The blue curve indicates the set of possible solutions to DSM design (5). The red curve

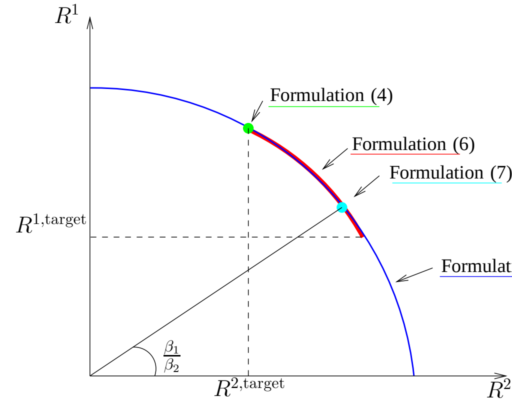


Fig. 1. Characterization of working points for data driven DSM designs in a two-user achievable rate region. The green dot indicates the solution to DSM design (4). The blue curve indicates the set of possible solutions to DSM design (5). The red curve indicates the set of possible solutions to DSM design (6). The exact solutions in these sets are determined by the chosen weights $\{w_n : n \in \mathcal{N}\}$. The cyan dot corresponds to the solution of (7) for given values β_1 and β_2.

indicates the set of possible solutions to (6). The exact solutions in these sets are determined by the chosen weights $\{w_n : n \in \mathcal{N}\}$. By chosing the appropriate weights, these DSM designs can all lead to the same solution indicated by the green dot. The cyan dot corresponds to the solution of (7) for given values β_1 and β_2.

DSM research has so far mainly been focusing on DSM designs (4)-(7). Note that these data rate driven DSM designs can only characterize one particular point on the border of the rate region, not a working point inside the rate region. These border points mostly correspond to transmit power allocations where all users fully use their available transmit power. Obviously, this is not the best strategy in terms of power consumption, which calls for new DSM designs that also enable more power-efficient working points.

B. Green DSL: Power-Efficient DSM designs

Traditional data-rate driven DSM designs can be modified or extended in several ways towards a more power-efficient design, by changing the objective and/or adding constraints that involve transmit power.

A first relevant extension would be to impose a constraint on the sum of all allocated transmit powers so as to reduce the total consumed power by factor α, i.e.,

$$\begin{array}{l} \max_{\{\mathbf{s}^n, n\in\mathcal{N}\}} \sum_{n\in\mathcal{N}} w_n R^n \\ \text{s.t. } R^n \geq R^{n,\text{target}}, \quad \forall n \\ \text{s.t. } P^n \leq P^{n,\text{tot}}, \quad \forall n \\ \text{s.t. } \sum_{n\in\mathcal{N}} P^n \leq \alpha \sum_{n\in\mathcal{N}} P^{n,\text{tot}} \\ \text{s.t. } 0 \leq s_k^n \leq s_k^{n,\text{mask}}, \quad \forall n, \forall k, \end{array} \tag{8}$$

where $0 < \alpha \leq 1$ is a chosen constant, and denotes the required power reduction with respect to full power usage. This formulation can be relevant when service providers aim to reduce their total power consumption by a factor α. In [20], a 50% decrease in power consumption was targeted for the year 2015. For the above DSM design, this would correspond to setting α equal to 0.5.

A special case of (8) is where only the data rate of one user is maximized subject to minimum data rate constraints for the other users. This corresponds to the following DSM design, which is (8) with $w_1 = 1$ and $w_n = 0$ for $n > 1$:

$$\begin{array}{l} \max_{\{\mathbf{s}^n, n\in\mathcal{N}\}} R^1 \\ \text{s.t. } R^n \geq R^{n,\text{target}}, \quad \forall n \\ \text{s.t. } P^n \leq P^{n,\text{tot}}, \quad \forall n \\ \text{s.t. } \sum_{n\in\mathcal{N}} P^n \leq \alpha \sum_{n\in\mathcal{N}} P^{n,\text{tot}} \\ \text{s.t. } 0 \leq s_k^n \leq s_k^{n,\text{mask}}, \quad \forall n, \forall k. \end{array} \tag{9}$$

Another relevant DSM design would be to drive the full objective towards power minimization subject to minimum data rate constraints as follows:

$$\begin{array}{l} \min_{\{\mathbf{s}^n, n\in\mathcal{N}\}} \sum_{n\in\mathcal{N}} P^n \\ \text{s.t. } R^n \geq R^{n,\text{target}}, \quad \forall n \\ \text{s.t. } P^n \leq P^{n,\text{tot}}, \quad \forall n \\ \text{s.t. } 0 \leq s_k^n \leq s_k^{n,\text{mask}}, \quad \forall n, \forall k, \end{array} \tag{10}$$

The solutions of these DSM designs (8)-(10) are visualized in Figure 2 for a two-user scenario. The green dot indicates the solution to (10). The red curve indicates the set of possible solutions for (8). The exact solutions in this set are determined by the chosen weights $\{w_n : n \in \mathcal{N}\}$. The blue dot indicates the solution to (9).

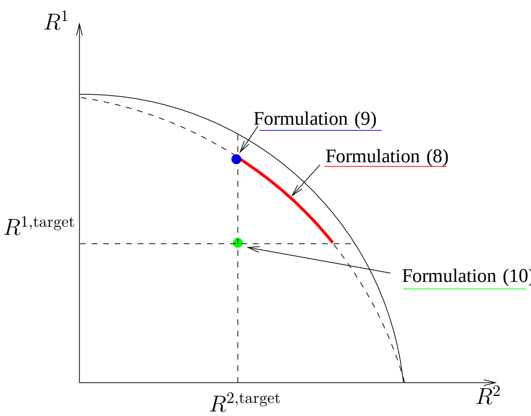


Fig. 2. Characterization working points for power-efficient DSM designs in a two-user achievable rate region. The blue dot indicates the solution to DSM design (9). The red curve indicates the set of possible solutions to DSM design (8). The exact solutions in this set are determined by the chosen weights $\{w_n : n \in \mathcal{N}\}$. The green dot indicates the solution to DSM design (10).

A more general formulation of (10) would be to add a weighting t_n for the different users as follows:

$$\begin{array}{l} \min_{\{\mathbf{s}^n, n\in\mathcal{N}\}} \sum_{n\in\mathcal{N}} t_n P^n \\ \text{s.t. } R^n \geq R^{n,\text{target}}, \quad \forall n \\ \text{s.t. } P^n \leq P^{n,\text{tot}}, \quad \forall n \\ \text{s.t. } 0 \leq s_k^n \leq s_k^{n,\text{mask}}, \quad \forall n, \forall k, \end{array} \tag{11}$$

The relevance of this formulation becomes more clear by introducing the notion of the ‘power region’. This can be viewed as the counterpart of the rate region. Note that the border of the (achievable) rate region is defined as the optimal trade-off in data rates for fixed maximum power budgets $P^{n,\text{tot}}$ for the users. Similarly the border of the ‘power region’ can be defined as the optimal trade-off in powers for fixed minimum data rate constraints $R^{n,\text{target}}$. These two regions are shown in Figure 3 for a two-user scenario.

By changing the weights t_n, different operating points can be obtained on the power region border. The notion of power region also leads to other relevant DSM designs. For instance, we can pursue some proportion in terms of the total powers consumed by the individual users, leading to following DSM design:

$$\begin{array}{l} \min_{\{P, \mathbf{s}^n, n\in\mathcal{N}\}} P \\ \text{s.t. } P^n \leq \gamma_n P, \quad \forall n \\ \text{s.t. } R^n \geq R^{n,\text{target}}, \quad \forall n \\ \text{s.t. } P^n \leq P^{n,\text{tot}}, \quad \forall n \\ \text{s.t. } 0 \leq s_k^n \leq s_k^{n,\text{mask}}, \quad \forall n, \forall k, \end{array} \tag{12}$$

where the $\{\gamma_n : n \in \mathcal{N}\}$ indicate the proportions with respect to a base power P. One interesting point would for instance be to choose the point on the power region border with the objective of minimizing the maximum of all powers P^n, which corresponds to (12) where $\{n \in \mathcal{N} : \gamma_n = 1\}$.

The objective function can also be made ‘symmetric’ in data rates and powers, i.e. with weighted data rate and weighted power optimization under per-user (maximum) total power constraints and (minimum) data rate constraints, as follows:

$$\begin{array}{l} \max_{\{\mathbf{s}^n, n\in\mathcal{N}\}} \sum_{n\in\mathcal{N}} w_n R^n - \sum_{n\in\mathcal{N}} t_n P^n \\ \text{s.t. } R^n \geq R^{n,\text{target}}, \quad \forall n \\ \text{s.t. } P^n \leq P^{n,\text{tot}}, \quad \forall n \\ \text{s.t. } 0 \leq s_k^n \leq s_k^{n,\text{mask}}, \quad \forall n, \forall k, \end{array} \tag{13}$$

The solution to this DSM design corresponds to a working point somewhere inside the rate region $\mathcal{R}$ and power region $\mathcal{P}$, where the exact position is determined by the chosen weights w_n, t_n.

Finally we can construct a general DSM design that includes all previous DSM designs as special cases as follows:

$$\begin{array}{l} \max_{\{R, P, \mathbf{s}^n, n\in\mathcal{N}\}} R - P + \sum_{n\in\mathcal{N}} w_n R^n - \sum_{n\in\mathcal{N}} t_n P^n \\ \text{s.t. } R^n \geq \beta_n R, \quad \forall n \\ \text{s.t. } P^n \leq \gamma_n P, \quad \forall n \\ \text{s.t. } R^n \geq R^{n,\text{target}}, \quad \forall n \\ \text{s.t. } P^n \leq P^{n,\text{tot}}, \quad \forall n \\ \text{s.t. } \sum_{n\in\mathcal{N}} P^n \leq \alpha \sum_{n\in\mathcal{N}} P^{n,\text{tot}} \\ \text{s.t. } 0 \leq s_k^n \leq s_k^{n,\text{mask}}, \quad \forall n, \forall k. \end{array} \tag{14}$$

The relevance of this DSM design is that it combines data rate objectives and constraints with power objectives and constraints, leading to a unifying mathematical representation of more general DSM problems. This results in a high flexibility in characterizing different working points inside the rate region as well as inside the power region. In this way DSL operation can be configured towards a more ‘green’ power-efficient setting. This unifying DSM design will be referred to as *‘green DSL’* in this paper. All the DSM designs in Section III can be derived from the ‘green DSL’ formulation (14), e.g., (10) can be obtained from (14) by removing the proportional data rate and power constraints and setting the weights w_n to zero and the weights t_n to one, and setting α to one.

Generally, this leads to a suite of DSM designs that can be obtained by removing different constraints and/or parts of the objective function from the general DSM design (14), leading to the tree structure in Figure 4 where each node corresponds to a DSM design and the arrows indicate a removal of a constraint or a part of the objective function. Each node is indicated by a sequence of letters where each letter corresponds to the presence of a constraint or part of the objective function. The specific

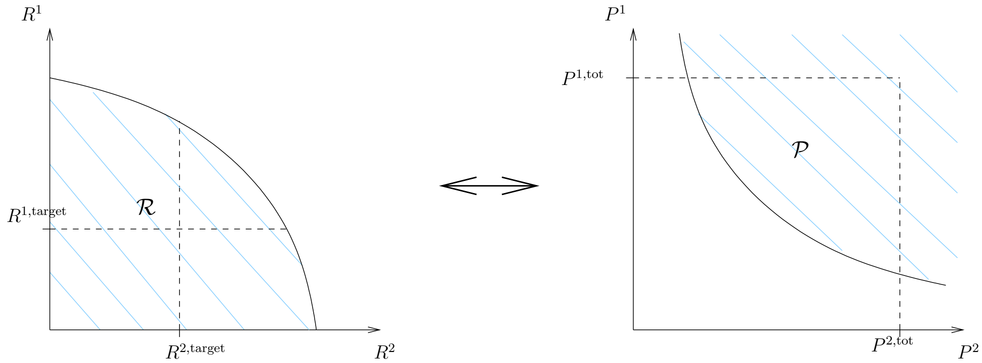


Fig. 3. Left: two-user rate region $\mathcal{R}$ = set of all achievable data rates for per-user power constraints $P^n \leq P^{n,\text{tot}}$, Right: two-user power region $\mathcal{P}$ = set of all possible powers for per-user data rate constraints $R^n \geq R^{n,\text{target}}$.

meaning of the letters is explained in the caption of Figure 4. The top node corresponds to the unifying green DSL DSM design as specified in (14). The other nodes correspond to other (reduced) DSM designs. As an example, DSM designs (5), (6) and (7) correspond to nodes **a**, **ae** and **c** respectively. Note that either **a**, **c** or **e** has to be present for each node because otherwise the solution is trivial, i.e. zero transmit powers. Also note that, as mentioned in Section II-B, the constraints corresponding to set $\mathcal{S}$ in (3) are present in all DSM designs in order to be compliant with DSL standards. The bold nodes in the tree correspond to DSM designs that have been addressed in literature, where DSM algorithms have been designed to solve the corresponding optimization problems. All other nodes have not been addressed so far.

The DSM designs from the tree in Figure 4 can be interesting for different practical scenarios and this leads to an increased potential in DSM configuration. The most appropriate DSM design can be selected so as to satisfy specific QoS requirements. In Section V, the performance impact of the proposed extended set of DSM designs will be demonstrated for realistic DSL scenarios, with some surprising numerical results, proving their practical relevance.

IV. DSM Algorithm Design for Green DSL

In this section we will propose a general systematic procedure for tackling the extended set of DSM designs, as proposed in the previous section. To this end, we will start from the unifying green DSL DSM design (14) and derive a general solution procedure. Furthermore we show that all other DSM designs can be solved using this procedure by redefining the dual Lagrange multipliers. This redefinition also allows to reuse existing DSM algorithms up to minor modifications, where each of the algorithms then leads to a trade-off in terms of complexity and performance. Moreover these insights allow an analysis of the relation between the different DSM designs. Lastly the infeasibility issue for the DSM designs is addressed by including an online detection and adaptation mechanism and a concrete practical implementation is given, highlighting some practical aspects.

A. General Systematic Procedure

The DSM designs proposed in the previous section are non-convex optimization problems, because of the non-convex relation between the data rates R^n and the transmit powers s_k^n, as can be seen from (1). Even for moderately sized problems (with 5-20 users and 200-4000 tones), finding the globally optimal solution is computationally prohibitive. In [27] [4], a dual decomposition approach was proposed to tackle this complexity issue for the rate-adaptive DSM designs of Section III-A. This dual approach is justified by the following observations: (i) the duality gap goes to zero as the number of tones goes to infinity [27] [28], (ii) the Lagrangian function can be decomposed into independent, be it still non-convex, per-tone subproblems of much smaller dimensions [27] [4], which are much more manageable. The number of tones in practice is not infinitely large but still very large, e.g. up to 4000 for some VDSL bandplans. The duality gap is thus assumed to be approximately zero. Consequently the optimal solution to the primal problem can be found by solving its decomposable dual problem.

1) Dual Decomposition: We first present a dual decomposition approach for the general green DSL DSM design (14). More specifically, the procedure consists of solving two main problems, corresponding to a master problem (15) and a slave problem (16) or (17), namely:

$$\begin{array}{l} \min_{\theta} L(\theta) \\ \text{s.t. } \theta \geq 0 \end{array} \qquad (15)$$

with $\theta = [\mu_1, \ldots, \mu_N, \nu_1, \ldots, \nu_N, \kappa_1, \ldots, \kappa_N, \lambda_1, \ldots, \lambda_N, \lambda_a]^T$ and '$\geq$' denoting a component-wise inequality and with the Lagrangian function $L(\theta)$ defining the slave optimization problem:

$$L(\theta) = \begin{cases} \max_{\{R,P,\mathbf{s}^n,n\in\mathcal{N}\}} & R - P + \sum_n \mu_n(R^n - \beta_n R) + \sum_n w_n R \\ & - \sum_n \kappa_n(P^n - \gamma_n P) - \sum_n \lambda_n(P^n - P \\ \text{s.t. } 0 \leq s_k^n \leq s_k^{n,\text{mask}}, & \forall k, \forall n. \end{cases} \qquad (16)$$

In order to obtain a non-trivial solution of (16), i.e., $\{R \neq \infty$ and/or $P \neq -\infty\}$, the Lagrange multipliers μ_n and κ_n have to satisfy equalities $1 = \sum_n \beta_n \mu_n$ and $1 = \sum_n \gamma_n \kappa_n$, which can always be obtained by a simple scaling, as explained in [26]. By using these equalities, problem (16) can be transformed into an alternative slave optimization problem, which consists of K

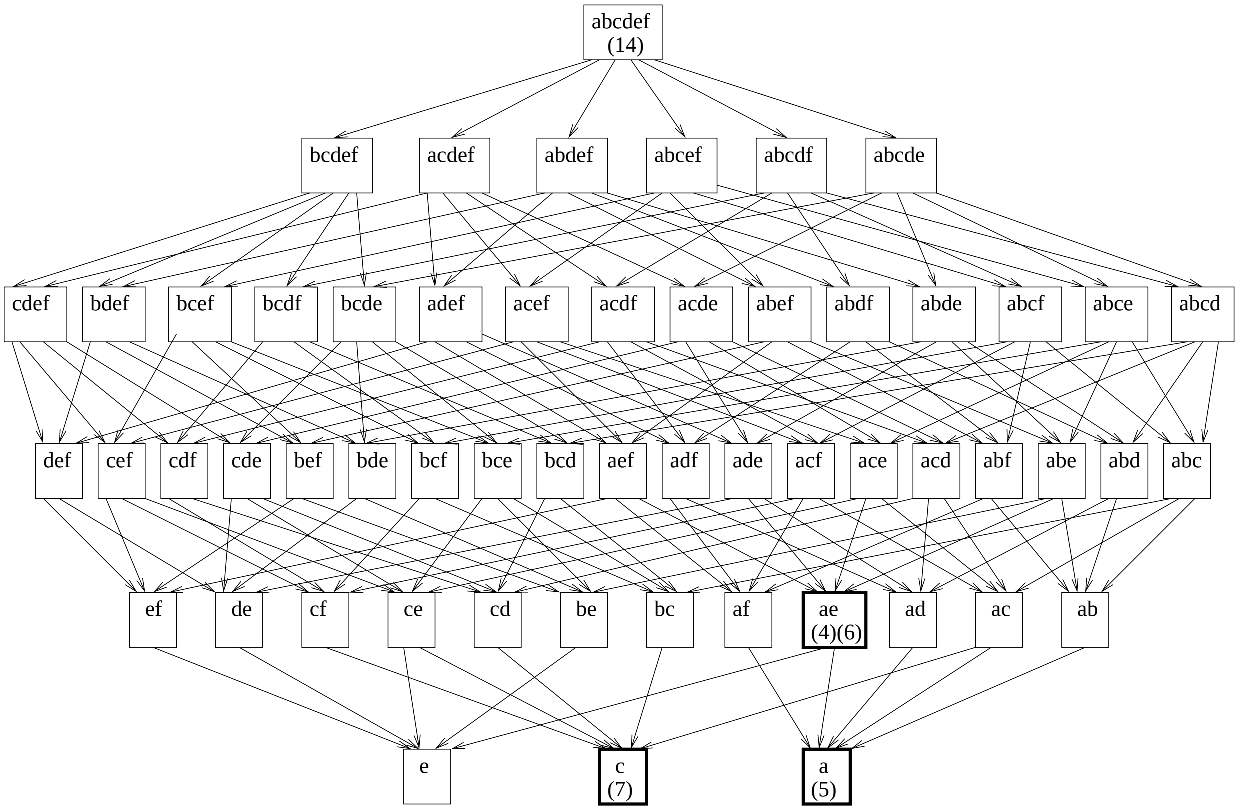


Fig. 4. DSM design taxonomy represented in a tree. Each node corresponds to a DSM design containing objectives and/or constraints indicated by letters. These letters range from a to f and are explained as follows: $\mathbf{a}$ indicates the presence of a weighted data rate sum in the objective $\sum_n w_n R^n$. $\mathbf{b}$ indicates the presence of a weighted power sum in the objective $\sum_n t_n P^n$. $\mathbf{c}$ indicates the presence of proportional data rate constraints $R^n \geq \beta_n R, \forall n \in \mathcal{N}$ and an extra data rate variable R in the objective as in problem (7). $\mathbf{d}$ indicates the presence of proportional power constraints $P^n \leq \gamma_n P, \forall n \in \mathcal{N}$ and an extra power variable P in the objective as in problem (12). $\mathbf{e}$ indicates the presence of target data rate constraints $R^n \geq R^{n,\text{target}}, \forall n \in \mathcal{N}$. $\mathbf{f}$ indicates the presence of a total sum power constraint $\sum_n \sum_k s_k^n \leq \alpha \sum_n P^{n,\text{tot}}$. Also note that, as mentioned in Section II-B, the constraints corresponding to set $\mathcal{S}$ in (3) are present in all DSM designs in order to be compliant with DSL standards.

independent subproblems as follows:

$$L(\theta) = \textstyle\sum_k L_k(\theta)$$
$$\text{with } L_k(\theta) = \begin{cases} \max_{\mathbf{s}_k} \sum_n \tilde{w}_n f_s b_k^n - \sum_n \tilde{\lambda}_n s_k^n \\ \text{s.t. } 0 \leq s_k^n \leq s_k^{n,\text{mask}}, \quad \forall n, \end{cases} \tag{17}$$

where

$$\begin{aligned} \tilde{w}_n &= w_n + \mu_n + \nu_n \\ \tilde{\lambda}_n &= t_n + \kappa_n + \lambda_n + \lambda_\alpha, \end{aligned} \tag{18}$$

and where Lagrange multipliers μ_n, $\nu_n, \kappa_n, \lambda_n, \lambda_a$ correspond to constraints $R^n \geq \beta_n R$, $R^n \geq R^{n,\text{target}}$, $P^n \leq \gamma_n P$, $P^n \leq P^{n,\text{tot}}$, and $\sum_n P^n \leq \alpha \sum_n P^{n,\text{tot}}$, respectively. We will call $\tilde{w}_n$, $\tilde{\lambda}_n$ (18) the *extended Lagrange multipliers*.

It remains to be shown how master problem (15) and slave problem (17) can be solved.

2) Master Problem: Master problem (15) consists of a convex but non-differentiable objective function. Thus, we use the standard subgradient update approach to finding the optimal dual variables $\mu_n, \kappa_n, \nu_n, \lambda_n, \lambda_a, n \in \mathcal{N}$, as follows:

$$\mu_n = [\mu_n + \iota(\beta_n R - R^n)]^+ \quad \text{with } R = \min_n R^n/\beta_n, \tag{19}$$
$$\kappa_n = [\kappa_n + \zeta(P^n - \gamma_n P)]^+, \tag{20}$$
$$\nu_n = [\nu_n + \epsilon(R^{n,\text{target}} - R^n)]^+, \tag{21}$$
$$\lambda_n = [\lambda_n + \delta(P^n - P^{n,\text{tot}})]^+, \tag{22}$$
$$\lambda_\alpha = [\lambda_\alpha + \eta(\sum_n P^n - \alpha \sum_n P^{n,\text{tot}})]^+, \tag{23}$$

until the corresponding complementarity conditions are satisfied:

$$\mu_n(\beta_n R - R^n) = 0, \qquad \forall n, \tag{24}$$
$$\kappa_n(P^n - \gamma_n P) = 0, \qquad \forall n, \tag{25}$$
$$\nu_n(R^{n,\text{target}} - R^n) = 0, \qquad \forall n, \tag{26}$$
$$\lambda_n(P^n - P^{n,\text{tot}})) = 0, \qquad \forall n, \tag{27}$$
$$\lambda_\alpha(\sum_n P^n - \alpha \sum_n P^{n,\text{tot}}) = 0, \tag{28}$$

where $\iota, \zeta, \epsilon, \delta, \eta$ are stepsizes that can be chosen using different approaches [27] [8], and where $[x]^+ = \max(0, x)$.

3) Slave Problem: The slave problem (17) consists of K independent non-convex optimization problems for given Lagrange multipliers θ. The main observation here is that these independent slave subproblems have a similar structure as the slave subproblems of traditional data rate driven formulations (4)(5)(6). The main difference is the use of extended Lagrange multipliers $\tilde{w}_n$, $\tilde{\lambda}_n$ instead of the usual Lagrange multipliers w_n, λ_n. As a consequence, existing DSM algorithms (IW [3], ASB(2) [9] [13], SCALE [11], MIW [12], DSB [13], MS-DSB [13], ISB [5] [6], OSB [4], PBB [7], BB-OSB [8]) can be modified to tackle the general green DSL formulation by redefining the Lagrange multipliers using the set of extended Lagrange multipliers. For example, the iterative transmit power updates so as to solve slave subproblems (17) are modified to the following formulas for IW, ASB2, SCALE and MIW/DSB, respectively:

$$s_k^n = \left[\frac{\tilde{w}_n f_s}{\log(2)(\tilde{\lambda}_n)} - \frac{\Gamma(\sum_{m \neq n} |h_k^{n,m}|^2 s_k^m + \sigma_k^n)}{|h_k^{n,n}|^2} \right]_0^{s_k^{n,\mathrm{mask}}} \quad \text{(IW)} \tag{29}$$

$$s_k^n = \left[\frac{\tilde{w}_n f_s / \log(2)}{\tilde{\lambda}_n + P_k^{\mathrm{ASB-2},n}} - \frac{\sum_{m \neq n} \Gamma |h_k^{n,m}|^2 s_k^m + \Gamma \sigma_k^n}{|h_k^{n,n}|^2} \right]_0^{s_k^{n,\mathrm{mask}}} \quad \text{(ASB2)} \tag{30}$$

$$s_k^n = \left[\frac{\tilde{w}_n f_s \alpha_k^n}{\log(2)(\tilde{\lambda}_n) + \sum_{m \neq n} \frac{\tilde{w}_m f_s \alpha_k^m \Gamma |h_k^{m,n}|^2}{\sum_{p \neq m} \Gamma |h_k^{m,p}|^2 s_k^p + \Gamma \sigma_k^m}} \right]_0^{s_k^{n,\mathrm{mask}}} \quad \text{(SCALE)} \tag{31}$$

$$s_k^n = \left[\frac{\tilde{w}_n f_s / \log(2)}{\tilde{\lambda}_n + \sum_{m \neq n} \frac{\tilde{w}_m f_s \Gamma |h_k^{m,n}|^2}{\log(2)} \left(\frac{1}{\mathrm{int}_k^m} - \frac{1}{\mathrm{rec}_k^m}\right)} - \frac{\mathrm{int}_k^n}{|h_k^{n,n}|^2} \right]_0^{s_k^{n,\mathrm{mask}}} \quad \text{(MIW/DSB)}$$

with

$$\mathrm{int}_k^n = \sum_{m \neq n} \Gamma |h_k^{n,m}|^2 s_k^m + \Gamma \sigma_k^n$$
$$\mathrm{rec}_k^n = \sum_m |h_k^{n,m}|^2 s_k^m + \Gamma \sigma_k^n, \tag{32}$$

where $P_k^{\mathrm{ASB-2},n}$ and α_k^n are defined as in [13] and $[x]_0^y$ means $\max(\min(x,y),0)$. Note that extended Lagrange multipliers $\tilde{w}_n$, $\tilde{\lambda}_n$ are used. For ASB [9], the powers can similarly be obtained by replacing the usual Lagrange multiplier λ_n, corresponding to the total power constraint for user n, with $\tilde{\lambda}_n$ and solving the corresponding cubic equation, as explained in [9].

Remark: The above existing DSM algorithms with modified Lagrange multipliers do not necessarily find the globally optimal solution to the slave problems (17). They can be divided into three types of algorithms: (i) globally optimal DSM algorithms, e.g. OSB, BB-OSB, PBB, (ii) locally optimal DSM algorithms, e.g. SCALE, MIW, DSB, MS-DSB, and (iii) heuristic DSM algorithms, e.g. IW, ASB. The globally optimal algorithms have an exponential complexity. The other type of algorithms have only polynomial complexity. Each of the algorithms computes a solution with a particular performance and has a particular complexity, leading to a trade-off between performance and complexity [13]. This trade-off will be highlighted in the Section V-B.

4) General Procedure for Green DSL: The subgradient update strategy for the master problem and solution strategy for the slave subproblems can be combined to obtain a general procedure for solving (14). This procedure is visualized in Figure 5. The master part consists of two parts. The first part consists of a set of subgradient updates of the Lagrange multipliers θ, where, for (14), the set $\mathcal{D}$ is defined as $\mathcal{D} = \{(19),(20),(21),(22),(23)\}$. The second part is the calculation of the extended Lagrange multipliers $\tilde{w}_n$, $\tilde{\lambda}_n$ (18) based on the obtained Lagrange multipliers θ.

For given extended Lagrange multipliers, the slave problem consists of solving problem (17) for each tone k. For this, one can use one of the modified DSM algorithms as explained in Section IV-A3. This has to be repeated until the corresponding complementarity conditions (24)-(28) are satisfied, meaning that the constraints are satisfied.

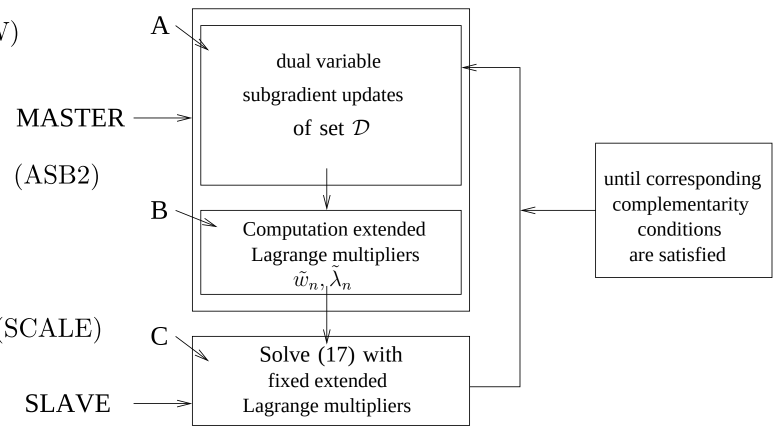


Fig. 5. General systematic procedure for green DSL DSM design. Two main parts: master and slave. Master part consists of two parts: A and B. Set $\mathcal{D}$ and extended Lagrange multipliers $\tilde{w}_n, \tilde{\lambda}_n$ are determined by Table I and particular DSM design

5) Implementation for specific DSM designs: The procedure for the unifying green DSL formulation (14) corresponding to node $\mathbf{abcdef}$ in Figure 4, can also be used to tackle all the other DSM designs of the tree in Figure 4, by simply redefining the extended Lagrange multipliers and the set of subgradient update formulas $\mathcal{D}$. In Table I we have summarized these definitions for the DSM designs that have been discussed in Section III. The first column of this table indicates the DSM designs. The second column and third column contain the extended Lagrange multipliers $\tilde{w}_n$ and $\tilde{\lambda}_n$, respectively. The last column indicates which subgradient update formulas to use in order to find the corresponding optimal dual variables. To obtain specific procedures for the different DSM designs, Part A and B of the general procedure of Figure 5 should use column 2,3,4 for defining the extended Lagrange multipliers $\tilde{w}_n$ and $\tilde{\lambda}_n$ and the specific set $\mathcal{D}$ of subgradient update formulas. Table I contains only the definitions for the DSM designs discussed in Section III, but similar definitions can be constructed for all the other DSM designs of the tree in Figure 4.

B. Analysis of DSM Designs

The various DSM designs differ in the particular choice of extended Lagrange multipliers. This insight allows a comparison of the different designs and their mechanisms to obtain certain working points in the rate and power region. An interesting comparison can be made between the pure data rate driven DSM designs (4)(5)(6) on the one hand and the power-efficient DSM designs on the other hand, where the former are only able to obtain working points on the border of the rate region, while the latter are also able to obtain working points inside the rate region.

To facilitate further study, we introduce a normalization of the extended Lagrange multipliers $\tilde{w}_n$, using $\tilde{W} = \sum_n \tilde{w}_n$. This normalization transforms the extended Lagrange multipliers $\tilde{w}_n, \tilde{\lambda}_n$ of Table I into the normalized extended Lagrange multipliers $\hat{w}_n = \tilde{w}_n / \tilde{W}, \hat{\lambda}_n = \tilde{\lambda}_n / \tilde{W}$ of Table II (with

TABLE I
EXTENDED LAGRANGE MULTIPLIERS AND SET OF SUBGRADIENT UPDATES FOR DIFFERENT DSM DESIGNS. FIRST COLUMN INDICATES CONCERNING DSM DESIGN. SECOND AND THIRD COLUMNS INDICATES VALUE FOR $\tilde{w}_n$ AND $\tilde{\lambda}_n$ RESPECTIVELY. FOURTH COLUMN INDICATES SET OF SUBGRADIENT UPDATES.

DSM design	$\tilde{w}_n$	$\tilde{\lambda}_n$	$\mathcal{D}$
(4)	$n = 1 : 1 + \nu_1$, $n > 1 : \nu_n$	λ_n	(21)(22)
(5)	w_n	λ_n	(22)
(6)	$w_n + \nu_n$	λ_n	(21)(22)
(7)	μ_n	λ_n	(21)(22)
(8)	$w_n + \nu_n$	$\lambda_n + \lambda_\alpha$	(21)(22)(23)
(9)	$n = 1 : 1 + \nu_1$, $n > 1 : \nu_n$	$\lambda_n + \lambda_\alpha$	(21)(22)(23)
(10)	ν_n	$\lambda_n + 1$	(21)(22)
(11)	ν_n	$\lambda_n + \kappa_n$	(21)(22)
(12)	ν_n	$\lambda_n + \kappa_n$	(21)(20)(22)
(13)	$w_n + \nu_n$	$\lambda_n + t_n + \lambda_\alpha$	(21)(22)(23)
(14)	$w_n + \mu_n + \nu_n$	$\lambda_n + t_n + \kappa_n + \lambda_\alpha$	(21)(22)(23)(19) (20)

TABLE II
NORMALIZED EXTENDED LAGRANGE MULTIPLIERS AND SET OF SUBGRADIENT UPDATES FOR DIFFERENT DSM DESIGNS. FIRST COLUMN INDICATES CONCERNING DSM DESIGN. SECOND AND THIRD COLUMNS INDICATES VALUE FOR $\hat{w}_n$ AND $\hat{\lambda}_n$ RESPECTIVELY. FOURTH COLUMN INDICATES SET OF SUBGRADIENT UPDATES.

DSM design	$\hat{w}_n$	$\hat{\lambda}_n$	$\mathcal{D}$
(4)	$n = 1 : \frac{1+\nu_1}{\tilde{W}}$, $n > 1 : \frac{\nu_n}{\tilde{W}}$	$\frac{\lambda_n}{\tilde{W}}$	(21)(22)
(5)	$\frac{w_n}{\tilde{W}}$	$\frac{\lambda_n}{\tilde{W}}$	(22)
(6)	$\frac{w_n+\nu_n}{\tilde{W}}$	$\frac{\lambda_n}{\tilde{W}}$	(21)(22)
(7)	$\frac{\mu_n}{\tilde{W}}$	$\frac{\lambda_n}{\tilde{W}}$	(21)(22)
(8)	$\frac{w_n+\nu_n}{\tilde{W}}$	$\frac{\lambda_n+\lambda_\alpha}{\tilde{W}}$	(21)(22)(23)
(9)	$n = 1 : \frac{1+\nu_1}{\tilde{W}}$, $n > 1 : \frac{\nu_n}{\tilde{W}}$	$\frac{\lambda_n+\lambda_\alpha}{\tilde{W}}$	(21)(22)(23)
(10)	$\frac{\nu_n}{\tilde{W}}$	$\frac{\lambda_n+1}{\tilde{W}}$	(21)(22)
(11)	$\frac{\nu_n}{\tilde{W}}$	$\frac{\lambda_n+\kappa_n}{\tilde{W}}$	(21)(22)
(12)	$\frac{\nu_n}{\tilde{W}}$	$\frac{\lambda_n+\kappa_n}{\tilde{W}}$	(21)(20)(22)
(13)	$\frac{w_n+\nu_n}{\tilde{W}}$	$\frac{\lambda_n+t_n+\lambda_\alpha}{\tilde{W}}$	(21)(22)(23)
(14)	$\frac{w_n+\mu_n+\nu_n}{\tilde{W}}$	$\frac{\lambda_n+t_n+\kappa_n+\lambda_\alpha}{\tilde{W}}$	(21)(22)(23)(19) (20)

$\sum_n \hat{w}_n = 1$).

The normalized extended Lagrange multipliers $\hat{\lambda}_n$ can be viewed as prices for allocating transmit power. Their lower bound determines the minimum price that can be imposed on the allocated transmit power. The smaller this lower bound, the smaller we can make the penalty for the transmit powers and the less the allocated transmit powers can be penalized, which means that enough power can be allocated to tightly satisfy the power constraints $P^n \leq P^{n,\text{tot}}$. If the lower bound increases, we are forced to penalize the transmit powers with at least that amount, and so above a certain threshold value for the lower bound the corresponding total power constraints can not be tightly satisfied anymore. This increase of the lower bound creates a mechanism for obtaining a point inside the rate region.

For the pure data rate driven DSM designs (4)-(7), the extended Lagrange multipliers $\tilde{\lambda}_n$ are simply the Lagrange multipliers λ_n corresponding to the per-user total power constraints, which can range from lower bound 0 up to ∞. So $\hat{\lambda}_n$ can also range from 0 up to ∞. Thus there is no mechanism for increasing the lower bound on these Lagrange multipliers and so reducing the power. Consequently one will always obtain a point on the border of the rate region.

For the power minimimizing DSM design (10) the lower bound of $\hat{\lambda}_n$ is $1/\tilde{W}$, meaning that the lower bound is larger than zero. In fact, as long as the data rates are larger than the target data rates, and thus the constraints are not tightly satisfied, $\tilde{W}$ will decrease and so the lower bound will increase, leading to a working point inside the rate region. The process will converge to the point inside the rate region corresponding to the target data rates $R^{n,\text{target}}, \forall n,$.

For DSM design (8), the lower bound corresponds to $\lambda_a/\tilde{W}$. Contrary to (10) where the normalisation $\tilde{W}$ can decrease up to 0, the normalisation for (8) cannot decrease to less than $\sum_{n \in \mathcal{N}} w_n$. However there is a second parameter that defines the lower bound, namely λ_a. The larger this Lagrange multiplier, the larger the lower bound. So both (8) and (10) are seen to have a mechanism to obtain a point inside the rate region, be it that these mechanisms are different. This explains the relation between DSM designs (8) and (10) in terms of their extended Lagrange multipliers.

C. Infeasibility

The unifying green DSL DSM design (14) can be seen as a parameterized optimization problem, where the parameters reflect the QoS requirements. More specifically, the parameters correspond to $\psi \triangleq \{w_1, \ldots, w_N, t_1, \ldots, t_N, \beta_n, \ldots, \beta_N, \gamma_1, \ldots, \gamma_N, R^{1,\text{target}}, \ldots, R^{N,\text{target}}, \alpha\}$, where $\psi \in \Psi = \mathbf{R}_+^N \times \mathbf{R}_+^N \times \mathbf{R}_+^N \times \mathbf{R}_+^N \times \mathbf{R}_+^N \times \mathbf{R}_+$. Note that not all values for ψ correspond to a feasible DSM design. Some of the DSM designs may be infeasible, i.e. there may not exist a feasible solution satisfying all constraints. For example, the target data rates $R^{n,\text{target}}$ may correspond to a working point which is outside the achievable rate region for given channel, noise and power resources $\mathcal{S}$. Note that the achievable rate region is difficult to characterize (this amounts to solving (5) for all possible w_n's) and generally one does not know this region in advance. Now, Ψ can be divided into two disjoint subsets Ψ_1 and Ψ_2, where $\Psi_1 \subseteq \Psi$ consists of parameter values that correspond to a feasible DSM design, and where $\Psi_2 \subseteq \Psi$ consists of parameter values that correspond to infeasible DSM designs.

The tree in Figure 4 can be divided into two disjoint sets of nodes as follows:

$$\begin{aligned}
\mathcal{E} &= \{\text{all nodes}\} = \mathcal{E}_1 \cup \mathcal{E}_2 \\
\mathcal{E}_1 &= \{\text{abcdf, bcdf, acdf, abdf, abcf, abcd, cdf, bcf, bcd, adf, acf, acd, abf, abd, abc, cf, cd, bc, af, ad, ac, ab, c, a}\}, \\
\mathcal{E}_2 &= \{\text{abcdef, bcdef, acdef, abdef, abcef, abcde, cdef, bdef, bcef, bcde, adef, acef, acde, abde, abef, abce, def,} \\
&\qquad \text{cef, cde, bef, bde, bce, aef, ade, ace, abe, ef, de, ce, be, ae, e}\} \\
&= \mathcal{E} \setminus \mathcal{E}_1
\end{aligned} \tag{33}$$

All DSM designs in $\mathcal{E}_1$ are always feasible, i.e. all possible parameter values for $\mathcal{E}_1$ are within set Ψ_1. This is due to the fact that there are no lower bounds on the data rates and so there always exists at least one feasible solution for the set of constraints, namely the zero transmit power solution.

However, the problem formulations in $\mathcal{E}_2$ do not guarantee feasibility. In order to cope with some concrete instances of this set of problem formulations, we provide a number of sufficient conditions that detect infeasibility, i.e. if these conditions occur during execution of the DSM algorithm, then we know that the corresponding DSM design for the chosen parameters is infeasible.

Infeasibility certificate 1:
For all DSM designs of set $\mathcal{E}_2$, when a transmit power allocation occurs where all total powers $P^n, \forall n$, are tightly satisfied, i.e. $P^n = P^{n,\text{tot}}, \forall n$, and all corresponding data rates R^n violate the target data rate constraints, i.e. $R^n < R^{n,\text{target}}, \forall n$, then the corresponding DSM design is infeasible.

Proof: An (optimal) transmit power allocation $(P^{1,*}, \ldots, P^{N,*})$ where all total power constraints are tightly satisfied, i.e. $P^{n,*} = P^{n,\text{tot}}, \forall n$, means, by definition of optimality, that no other better transmit power allocation exists satisfying stricter constraints, i.e. $P^n < P^{n,\text{tot}}$. So no transmit power allocation with total powers $P^n, \forall n$, within region A of Figure 6 has a better objective function value. As the weighting of the data rates in the objective $\tilde{w}_n, \forall n$, can never be smaller than 0, i.e. $\tilde{w}_n \geq 0, \forall n$, this means that no transmit power allocation in region A corresponds to data rates where $\sum_n \tilde{w}_n R^n > \sum_n \tilde{w}_n R^{n,*}$. So all data rate combinations in set B of Figure 6 can never be attained and correspond to infeasible data rate constraints. If the target data rates $R^{n,\text{target}}$ are within this set B, they are infeasible and hence the DSM design is infeasible.

Infeasibility certificate 2:
For all DSM designs of set $\mathcal{E}_2$, when a transmit power allocation occurs where all data rates $R^n, \forall n$, are tightly satisfied, i.e. $R^n = R^{n,\text{target}}, \forall n$, and all corresponding total powers P^n violate the total power constraints, i.e. $P^n > P^{n,\text{tot}}, \forall n$, then the corresponding DSM design is infeasible.

The proof of this sufficient condition is very similar to the proof of infeasibility certificate 1 and is omitted.

Note that these are only sufficient conditions and not necessary conditions, meaning that there exist infeasible problems that may not satisfy these conditions. In the case that infeasibity is detected, we may stop the algorithm and return the infeasibility result, or we can opt to provide a mechanism steering towards feasibility. Generally this mechanism would have to relax the data rate constraints and/or power constraints. As the power constraints are implied by DSL standards we propose to relax the data rate constraints. This can be done in different ways but one possible mechanism is to proportionally decrease all target data rates by multiplying these with a factor smaller than one as follows,

$$\forall n : R^{n,\text{target}} = \rho R^{n,\text{target}}, \tag{34}$$

with $\rho < 1$. In the following section, a concrete example is shown that demonstrates how to include this infeasibility detection mechanism in a DSM algorithm.

D. Practical Implementation for DSM Design (8)

Although the general procedure has already been explained in Section IV-A, we will explain a specific implementation for DSM design (8) in more detail, and highlight some practical aspects that can improve convergence speed. We also include the infeasibility check and the mechanism for steering the DSM design towards feasibility. As shown in Table I, the solution for (8) requires three different subgradient updates. These can be performed jointly, but practical simulation results show that it may be wiser to enforce a particular hierarchy in these updates so as to improve convergence speed. This can be viewed as giving different time scales to these subgradient updates. In Algorithm 1 we propose the following hierarchy 'time scale λ_a' $>$ 'time scale ν_n' $>$ 'time scale λ_n', as simulations show that this is a good order. Note that other orders may also converge well, i.e. much better than joint updates. To improve a specific implementation, it is important to check different orders so as to identify which orders improve the convergence speed for the given DSM design.

The Lagrange multipliers $\lambda_n, \forall n$, appear in the inner loop and so have the smallest time scale. For every new update of λ_a and/or ν_n, the optimal Lagrange multipliers λ_n are searched so that the corresponding complementarity constraints are satisfied. Line 9 corresponds to the solution of the K slave subproblems. Remember that the slave subproblems can be tackled using slave subproblem procedures of existing DSM algorithms with modified Lagrange multipliers, i.e. the extended Lagrange multipliers, as discussed in Section IV-A3. The infeasibility check is included on line 10 and on line 11 where the data rate constraints are relaxed by a factor $\rho = 0.95$ in case of infeasibility, to steer the DSM design towards feasibility.

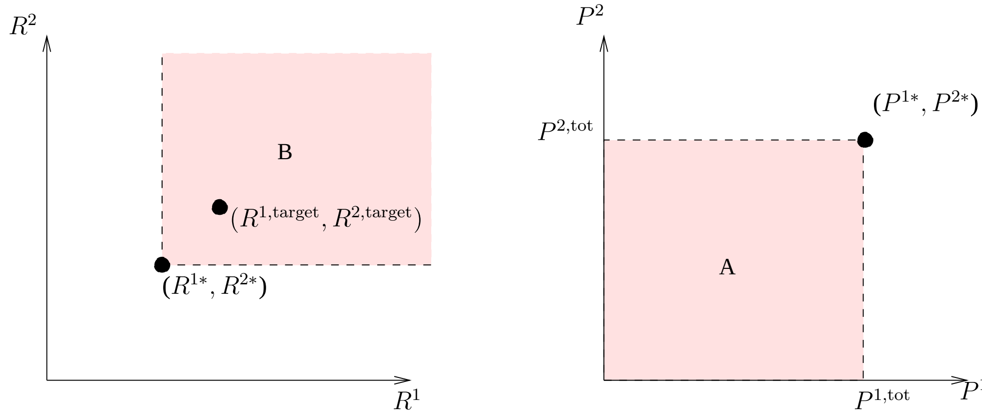


Fig. 6. Two-user rate region left, two-user power region right. $(P^{1,*}, P^{2,*})$ and $(R^{1,*}, R^{2,*})$ correspond to optimal powers and data rates respectively.

Algorithm 1 Specific implementation for DSM design (8)

```
ρ = 0.95;
while complementarity condition (28) not satisfied do
   Update dual variable λ_α using (23) {different possible
   strategies for stepsizes: [27] [8] or bisection}
   while complementarity condition (26) not satisfied do
      Update dual variables ν_n, ∀n using (21) {different
      possible strategies for stepsizes: [27] [8]}
      while complementarity condition (27) not satisfied do
         Update dual variables λ_n, ∀n using (22) {different
         possible strategies for stepsizes: [27] [8]}
         Compute extended Lagrange multipliers w̃_n and λ̃_n
         using Table I for problem formulation (8)
         ∀k : Apply slave subproblem procedure of existing
         DSM algorithm with extended Lagrange multipliers
         if infeasibility certificates 1 and/or 2 satisfied then
            ∀n : R^{n,target} = ρR^{n,target}
         end if
      end while
   end while
end while
```

V. Power-Rate Trade-off in Green DSL

In the earlier sections, traditional data rate maximizing DSM design has been extended towards a unifying power-efficient DSM design, which was referred to as 'green DSL'. Concrete procedures have been proposed to tackle the corresponding optimization problems. In this section we will make use of this extra design freedom to evaluate the possible impact of a power-efficient approach in practice. As the main motivation of this paper is reduction of power consumption, our main interests lie in the trade-off between power saving and data rate performance (power-rate trade-off). More specifically, we will analyze these trade-offs for different realistic scenarios and for different parameters: signal-to-noise ratio (SNR), network size, xDSL technology. Furthermore we will provide lower bounds on the data rate performance versus power usage. Finally we will assess the same trade-off for different DSM algorithms as proposed in Section IV-A3. This leads to an interesting trade-off between data rate, power usage and computational complexity.

A. Trade-off: Data Rates vs Power Usage

The achievable data rates strongly depend on the available power resources. By saving power, i.e. reducing the power usage, the achievable data rates will obviously decrease. This trade-off between data rates and power usage is of great importance and will give us a clear picture of how much power savings are possible without sacrificing too much of data rate performance in the green DSL context.

Simulation Setup

The following parameter settings are assumed for the DSL scenarios. The twisted pair lines have a diameter of 0.5 mm (24 AWG). The maximum transmit power is 20.4 dBm for the ADSL scenarios and 11.5 dBm for the VDSL scenarios. The SNR gap Γ is 12.9 dB, corresponding to a coding gain of 3 dB, a noise margin of 6 dB and a target symbol error probability of 10^{-7}. The tone spacing Δ_f is 4.3125 kHz. The DMT symbol rate f_s is 4 kHz. Note that we used the general procedure from Section IV-A4 in combination with the BB-OSB algorithm [8] with extended Lagrange multipliers as in Table I to solve the non-convex slave subproblems (17) for all DSM designs in this section. This combination with the branch and bound (BB) algorithm succeeds in finding the globally optimal solution up to a certain accuracy, which was taken very high in these examples, i.e. very small granularity.

Performance Measures

To quantify the trade-off between data rate performance and power resources, we define the notion of *power usage* as follows:

Definition V.1 (Power usage)**.** *The power usage c for the unifying green DSL DSM design (14) is defined as the ratio of the sum of all allocated transmit powers and the sum of all available power budgets defined by the DSL standard, i.e.,*

$$c(\psi) \triangleq \frac{\sum_{n\in N} P^{n,*}(\psi)}{\sum_{n\in N} P^{n,\text{tot}}} \leq \alpha \leq 1, \tag{35}$$

where $P^{n,*}(\psi)$ refers to the optimal allocated power of user n for (14) with given parameters
$\psi \triangleq \{w_1, \ldots, w_N, t_1, \ldots, t_N, \beta_n, \ldots, \beta_N, \gamma_1, \ldots, \gamma_N, R^{1,\text{target}}, \ldots, R^{N,\text{t}}$

1) Impact of Signal-to-Noise Ratio: A first DSL scenario is shown in Figure 7(a). This is a so-called near-far scenario which is known to be challenging, where DSM can make a substantial difference. Its corresponding rate region is shown in Figure 7(c) where the blue curve is the rate region at full power, obtained by solving the traditional DSM design (5), and the green curve is the rate region at half power, obtained by solving DSM design (8) with $\alpha = 0.5$. In Figure 7(d) the blue curve shows the percental data rate performance as a function of the power usage c for this near-far scenario. This curve is obtained by solving the proposed DSM design (10) for the different target rates $R^{n,\text{target}}, \forall n$, indicated in Figure 7(c) with red circles, corresponding to 40%, 50%, 60%, 70%, 80%, 90%, 95% and 100% of the achievable data rate performance. More formally, this curve corresponds to the percental data rate performance β as a function of $c(0, 0, 1, 1, 0, 0, \infty, \infty, \beta R^{1,o}, \beta R^{2,o}, 1)$, where $(R^{1,o}, R^{2,o})$ is indicated in Figure 7(c) and β ranges from 0 to 1. One can observe that saving 50% of total power leads to only a small decrease in achievable rate region (green curve in Figure 7(c)). More specifically, it corresponds to 85% of data rate performance as can be seen in Figure 7(d) (blue curve).

To understand this phenomenon more clearly, the evolution of the bit loading is shown in Figure 7(e) for a linearly increasing power budget ranging from 20% to 100% (i.e. full power) in steps of 10%. One can observe *a law of diminishing returns*, i.e. a linear increase of power leads to a diminishing data rate increase.

In Figure 7(b) a symmetric DSL scenario is shown and its corresponding trade-off between data rate performance and power saving is shown as the red curve in Figure 7(d). One can observe a data rate performance of 72% for a 50% power saving. Furthermore the evolution of the bit loading is shown in Figure 7(f) for a linearly increasing power budget ranging from 10% to 100% in steps of 10%. One can observe here that the effect of diminishing returns is less obvious than for Figure 7(e). The data rate performance for given power usage thus depends on the type of scenario.

It is challenging to explicitly and generally characterize the data rate performance decrease in relation to power savings. However, as Theorem V.1 states, we are able to study the worst case decrease of optimal data rate performance as a function of power savings, irrespective of the scenario:

Theorem V.1. *In the worst case the optimal data rate performance decreases linearly as a function of a decreasing power usage c. Furthermore, for a minimum bit loading of 1 bit after power reduction, the lower bound on the optimal data rate performance $g_{1\text{bit}}$ as a function of the power usage c is given by:*

$$g_{1\text{bit}}(c) = \log_2(1 + 1/c)^{-1}. \tag{36}$$

Proof: A first observation is that the worst case corresponds to a zero-crosstalk case. This is easy to understand if we know that the more crosstalk is present, the more power we need to increase the data rate and so the less effective that power becomes, leading to a smaller slope of the data rate performance as a function of the power usage. This will also be demonstrated in Section V-A2. For this zero-crosstalk worst case scenario the percental data rate performance in function of the power usage c on a tone for a user can be expressed as

$$f(\text{SNR}, c) = \frac{\log_2(1 + SNR \times c)}{\log_2(1 + SNR)}, \tag{37}$$

where SNR denotes the signal-to-noise ratio. Note that although this focuses on only one tone and user, the global effect can be seen as an average of these effects on all tones and users. The function f is increasing in SNR. Using l'Hôpital's rule, one can verify the following:

$$\begin{array}{l} \lim_{\text{SNR}\to 0} f(\text{SNR}, c) = c \\ \lim_{\text{SNR}\to \infty} f(\text{SNR}, c) = 1 \end{array} \tag{38}$$

This illustrates two points. As the SNR becomes very large, the data rate performance loss will vanish. Secondly, the percental data rate performance is minimized when SNR is zero and this leads to a *linear* absolute lower bound on the optimal data rate performance in function of the power usage c.

However zero SNR has no practical meaning and therefore it is better to put a certain lower bound on the SNR. One practical intuitive way of lower bounding is by enforcing a minimum bit loading after power reduction of one bit so that at least one bit can be transmitted on the line. This corresponds to the constraint $\text{SNR} \times c = 1$, which leads to the following relation:

$$g_{1\text{bit}}(c) = f(1/c, c) = \frac{\log_2(1+1)}{\log_2(1+1/c)} = \log_2(1 + 1/c)^{-1}. \tag{39}$$

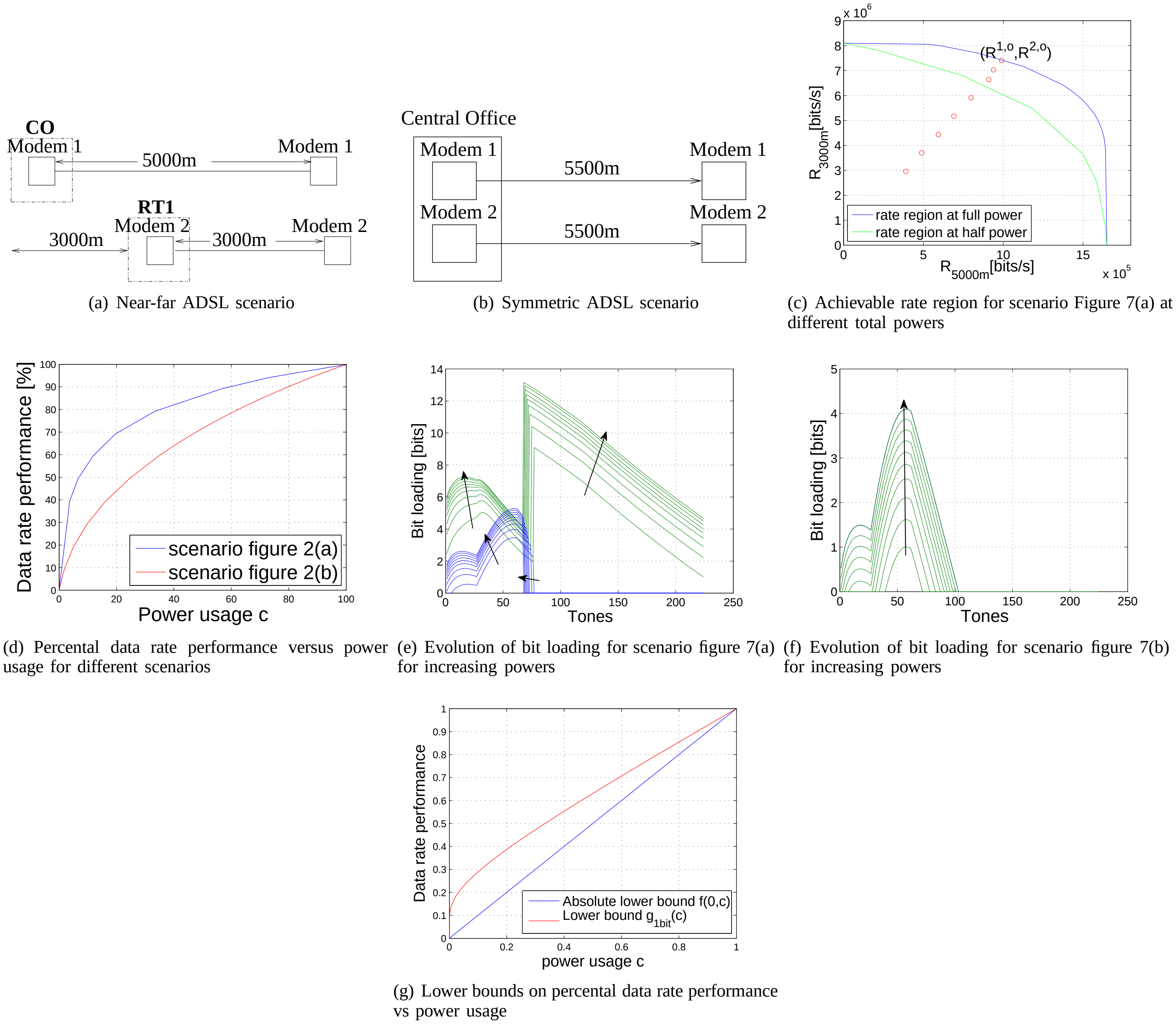
■

Note that (36) corresponds to a lower bound on data rate performance of 63% for 50% power saving. The lower bounds of Theorem V.1 are summarized in Figure 7(g)

Note that the law of diminishing returns is not just stating the concavity of the logarithmic relation between bits and powers as in (1), it is more than that. Adding one extra bit requires at least a doubling of the signal-to-noise ratio (SNR), and leads to an increase of the bit rate by a factor $(m + 1)/m$, where m is the bit rate before doubling the power. This logarithmic relation in itself is bad news. Then for the 'green DSL case' this is good news in a sense, as it means that from a given starting point, larger power savings can be made with small impact on the data rate.

The larger the bit rate, the less effective power becomes. In Figure 7(e), the bitloadings range from 0 to 13 bits, with an average of 6 bits. This large average bit loading leads to poor power efficiency. This means that the last added bits require a large amount of extra power and so by reducing power to 50% of the full power budget, only a small decrease in data rate is incurred, i.e. 85%. In Figure 7(f), the bitloadings range from 0 to 4 bits, with an average of 2 bits. This small average leads to better power efficiency and so also a smaller data rate performance for 50% power usage, i.e. 72%.

Power efficiency depends on the SNR, which in turn depends on the line attenuation and hence on the length of the lines. Thus in scenarios with long line lengths, i.e. large line attenuation and small SNRs, larger decreases in data rate are observed when the total power is reduced. In the limit when SNR goes to zero (very large attenuation or very large noise), the logarithm behaves as a linear function and decreasing power by a factor 2 leads to a decrease in rate by a factor 2. However, we strongly emphasize that, in practice, typical scenarios have an average SNR which is much larger than zero, leading to much smaller data rate performance losses in function of power savings.



(a) Near-far ADSL scenario

(b) Symmetric ADSL scenario

(c) Achievable rate region for scenario Figure 7(a) at different total powers

(d) Percental data rate performance versus power usage for different scenarios

(e) Evolution of bit loading for scenario figure 7(a) for increasing powers

(f) Evolution of bit loading for scenario figure 7(b) for increasing powers

(g) Lower bounds on percental data rate performance vs power usage

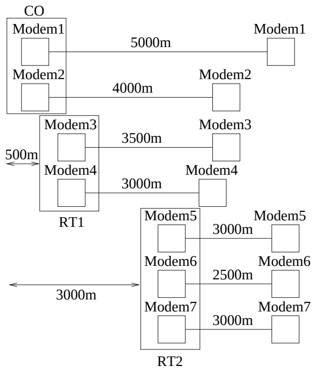
Fig. 7. Simulation Scenarios and Results

2) Impact of Network Size: The size of the DSL network, measured by the number of interfering users, may have a significant impact on the trade-off between data rates and power usage. An increasing number of users generally leads to an increasing amount of crosstalk. As mentioned in the previous section, the more crosstalk the smaller the slope of the data rate performance as a function of the power usage. To demonstrate this, in Figure 8 we show the trade-off between data rate performance and power usage for a symmetric scenario where the crosstalk is simply scaled with factors ranging from 0 (i.e. no crosstalk) to 1/4,1/2,1,2,4,8 and 16. As the crosstalk increases, one can observe that the percental data rate performance increases for a given power usage.

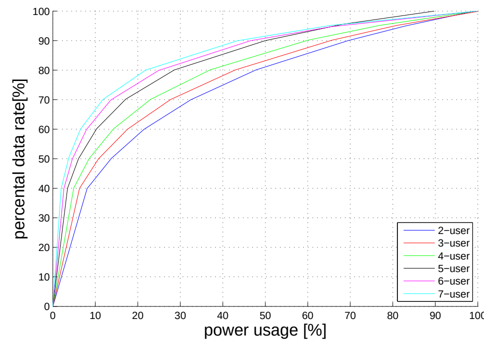
So as the crosstalk increases, the law of diminishing returns hits us even harder which is bad news. But at the same time, somewhat surprisingly, this gives us scenarios where green DSL can save a lot of power without too much impact on the data rates.

In Figure 9(a), a multiuser ADSL scenario is shown. For this scenario, we simulated the trade-off for the two-user case up to the seven-user case. The four-user case, for example, consists of active modems 1, 2, 3 and 4 where modems 5, 6 and 7 are inactive. The results are shown in Figure 9(b). It can be seen that as the size of the DSL network increases, the degradation in data rate performance is smaller under power savings. For the seven-user case, the data rate performance is 91% for 50% power usage. This result is quite promising for practice, where typically 20-100 DSL lines are binded into one cable bundle.

3) Impact of DSL technology: Different DSL technologies (ADSL(2)+, VDSL(2)) use different settings in terms of number of tones, band plan, total available power resources, etc. This has an impact on the studied trade-off. In Figure 10(a) a four-user upstream VDSL scenario is shown. The corresponding



(a) N-user ADSL downstream scenario



(b) Trade-off percental data rate performance vs power usage for two- up to seven-user ADSL downstream scenario of Figure 9(a)

Fig. 9.

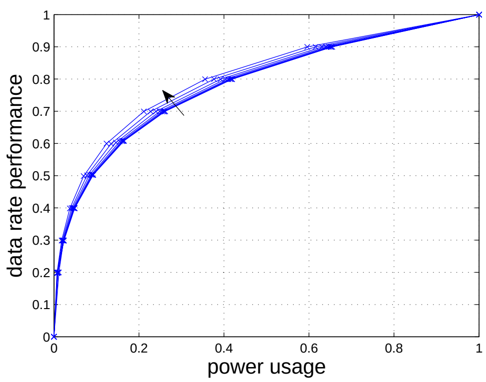


Fig. 8. Percental data rate performance in function of power usage for symmetric DSL scenario for increasing level of interference: [-∞dB -6dB -3dB 0dB 3dB 6dB 9dB 12dB]

trade-off is shown in Figure 10(b). One can observe that the results are even more pronounced compared to those for the ADSL scenarios. For 50% power usage, the data rate performance is still 95% of full power data rate performance. Even stronger, for 20% power usage, one can still transmit at 80% of the full power data rate performance.

It is not so easy to make a general statement of which technology (ADSL(2) versus VDSL(2)) is more favourable in this 'green' setting, i.e. which technology has smaller data rate performance losses for a certain power usage. The reason is that this depends on many factors: used transmit tones, band plan, line lengths, average SNRs, crosstalk interference level, etc. However simulations show that typically VDSL scenarios are more favourable. The main reason is shorter line lengths and so higher average SNRs, which results in smaller data rate performance losses under power savings as explained in Section V-A1.

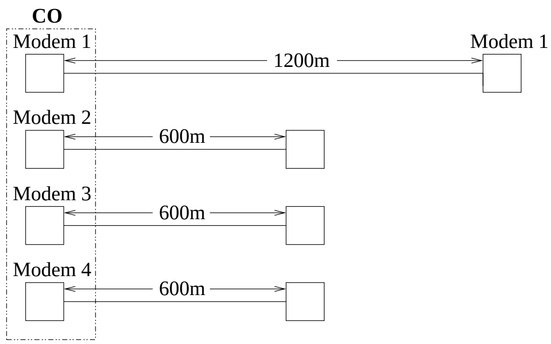
B. *Trade-off: Data Rates vs Power Usage vs Complexity*

In Section V-A, we analyzed the trade-off between 'optimal' data rate performance and power usage. This basically comes down to solving the green DSL DSM design (14) for some specific range of parameters. Note that this requires an optimal solution of the corresponding optimization problems, i.e. non-convex problem (17) needs to be solved with global optimality. Possible algorithms that can be used for this are OSB, BB-OSB and PBB with extended Lagrange multipliers as explained in Section IV-A3. These procedures deterministically find the globally optimal solutions, however they require a large computational complexity, i.e. an *exponential complexity* in the number of users. Therefore it is also interesting to focus on *polynomial complexity* algorithms[2] such as IW, ASB(2), SCALE, MIW, DSB, MS-DSB as the underlying procedure for green DSL. This leads to a much lower hence 'manageable' computational complexity. However the obtained solution is not necessarily globally optimal and this results in a sacrifice in the trade-off between data rate performance and power usage. The main goal of this section is to assess this sacrifice for different DSL scenarios so as to display the general trade-off between data rate performance, power usage and computational complexity.

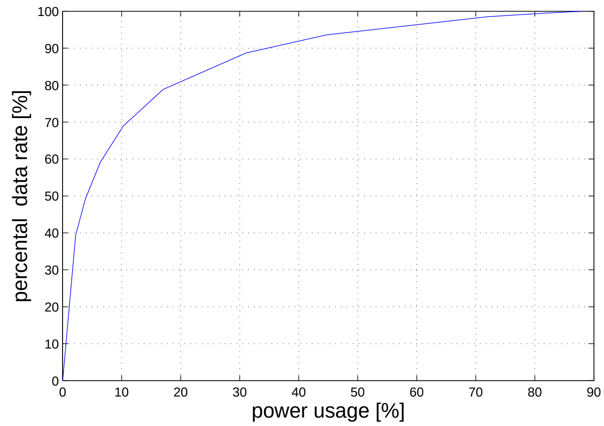
1) Near-far ADSL Downstream Scenario: In Figure 11 the trade-offs are shown for the near-far scenario of Figure 7(a) for different DSM algorithms IW, ASB-2, SCALE, MIW/DSB and MS-DSB with extended Lagrange multipliers, i.e. using transmit power updates (29)-(32). Note that the parameters of the reference line for the ASB-2 algorithm are chosen based on the guidelines provided in [12]. For instance, the CO line is chosen as the reference line. One can observe that the performance of MS-DSB is globally optimal, i.e. the same performance is obtained as BB-OSB. ASB-2 performs near-optimally. SCALE and DSB perform slightly worse but still quite good. IW performs suboptimally. Note that the trade-off for IW stops at 60% power usage. This can be explained as follows. Remember that the trade-off is generated by plotting the percental data rate performance β as a function of the power usage $c(0,0,1,1,0,0,\infty,\infty,\beta R^{1,o},\beta R^{2,o},1)$, where $(R^{1,o},R^{2,o})$ corresponds to the target data rate on the border of the rate region. The crossing of the achievable rate region of IW and the straight line $(\beta R^{1,o},\beta R^{2,o})$ for varying β, corresponds to a total of $0.6\sum_n P^{n,\mathrm{tot}}$ power consumption for IW.

The order of increasing complexity is as follows: 'IW' $<$ 'ASB-2' $<$ 'SCALE/DSB/MIW' $<$ 'MS-DSB' [3]. For this DSL scenario, ASB-2 achieves a very good trade-off in terms of data rate performance, power usage and complexity.

[2]See [13] for an extensive comparison of DSM algorithms.
[3]See [13] for a discussion on the complexities.



(a) VDSL upstream scenario



(b) Percental data rate performance versus power usage for 4-user VDSL upstream scenario of Figure 10(a)

Fig. 10.

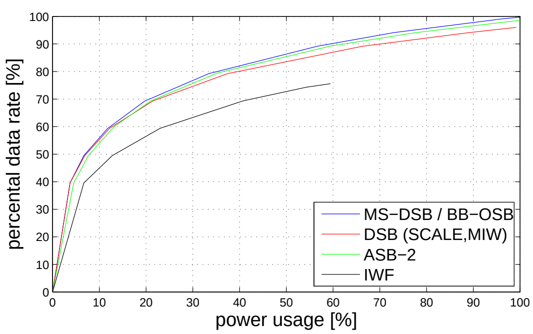


Fig. 11. Trade-off of percental data rate performance versus power usage for near-far scenario of Figure 7(a) for different underlying DSM algorithms

2) N-user ADSL downstream scenario: In Figures (12(a)), (12(b)), (12(c)), (12(d)), (12(e)) and (12(f)) the trade-offs are shown for the N-user ADSL scenario of Figure 9(a) for the two-user case up to the seven-user case, respectively.

The MS-DSB algorithm performs globally optimal for all scenarios. Furthermore the suboptimal algorithms (ASB-2, SCALE, DSB, MIW) have a small sacrifice with respect to the optimal performance, especially for the 5-user case. More specifically, for 50% power usage we can see a degradation from 90% to 86% in data rate performance. IW performs worse for the 5-user case up to the 7-user case because of the presence of large crosstalk. Again ASB-2 achieves a very good performance in the trade-off. The main reason is that the reference line is chosen to protect the 5000m line which appears to be a good strategy.

3) VDSL upstream scenario: In Figure 13 the trade-off is shown for the four-user scenario of Figure 10(a). MS-DSB performs globally optimal. SCALE, MIW and DSB perform near-optimally. ASB-2 and IW perform suboptimally. The reason why ASB-2 performs worse is the fact that the reference line is chosen as the long line (1200m) which appears not to be a very good strategy. The locally optimal algorithms SCALE, MIW and DSB achieve a very good trade-off in terms of data rate, power usage and complexity.

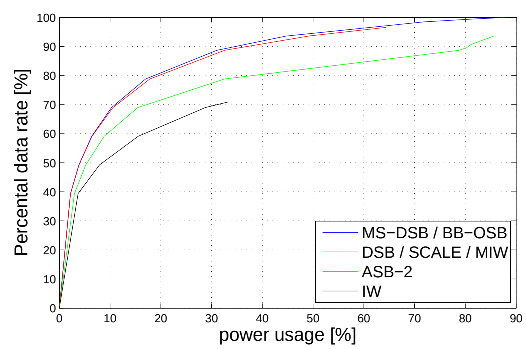


Fig. 13. Trade-off of percental data rate performance versus power usage for vdsl scenario of Figure 10(a) for different underlying DSM algorithms

The polynomial complexity algorithms seem to be very good candidate algorithms for green DSL. Generally MS-DSB performs globally optimally for most DSL scenarios. The ASB-2 algorithm performs very well with a very low complexity where however a good choice of the reference line is important. Furthermore SCALE, MIW and DSB perform near-optimally. Their performance depends on the initial points, as also explained in [13].

VI. Conclusion

Power saving is becoming an important goal in broadband access design. In this paper we have provided a first unifying framework for a "green DSL" formulation as well as a systematic procedure for tackling the corresponding optimization problems. Using this "green DSL" approach, we have quantified the power-rate trade-off for different practical settings with some surprising numerical results. We have demonstrated that for typical DSL scenarios large power savings can be achieved with only small impact on data rates. This somewhat surprising tradeoff result is due to not just the logarithmic dependence of rate on SINR but also the characteristics of crosstalk in DSL systems.

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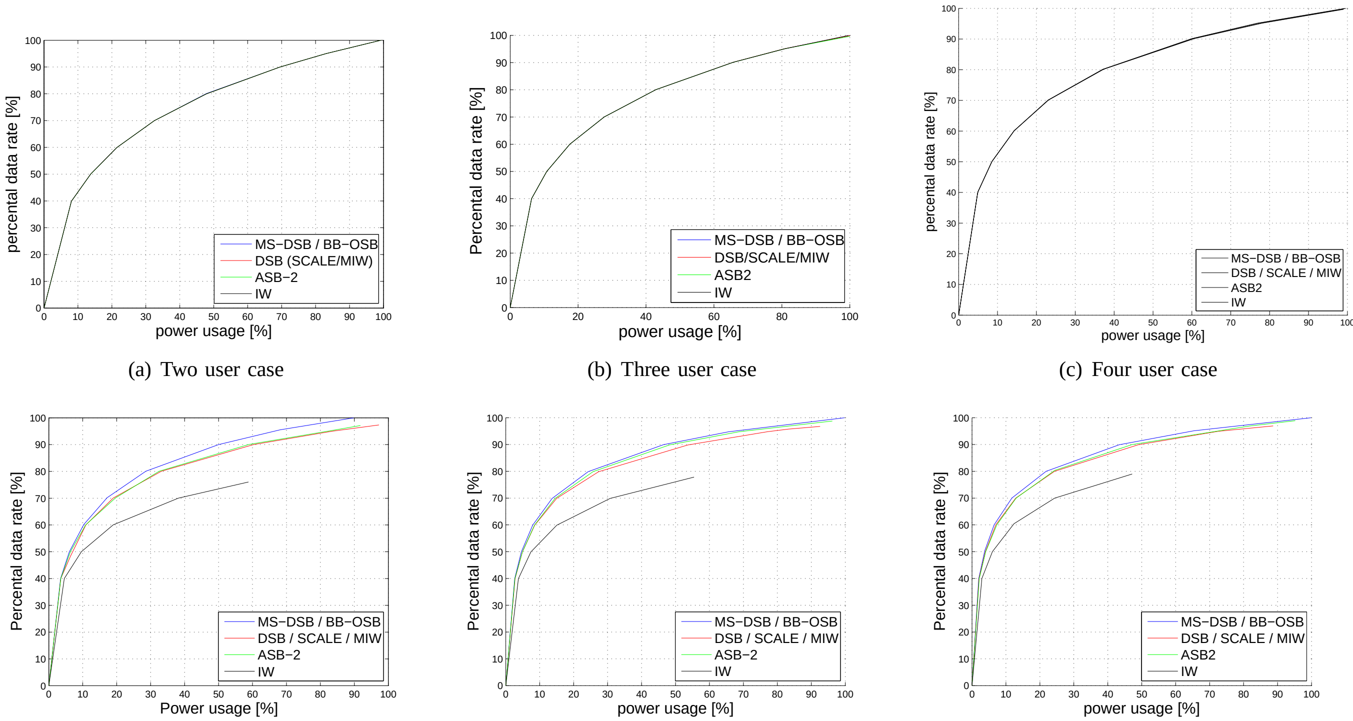

(a) Two user case (b) Three user case (c) Four user case

(d) Five user case (e) Six user case (f) Seven user case

Fig. 12. Trade-off of percental data rate performance versus power usage for N-user ADSL scenario of Figure 9(a) for different underlying DSM algorithms and for different number of users.

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