On Data Fusion for Wireless Localization

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***Abstract*—This paper presents a data fusion framework for wireless localization via the weighted least square estimator (WLSE). Three types of fusion schemes are presented: measurement fusion, estimate fusion and mixed fusion. Theoretical performance comparison among these schemes in terms of the estimation error covariance matrix is conducted. We show that, if the raw measurement vectors are correlated, then measurement fusion achieves the best performance, followed by mixed fusion and estimate fusion is the worst. If the raw measurement vectors are uncorrelated, then they can achieve the same performance. The benefits that can be earned from data fusion are also investigated and numerical case studies are presented to validate our theoretical analysis.**

***Index Terms*—Wireless localization, data fusion, weighted least square estimator (WLSE), Cramer-Rao lower bound (CRLB).**

I. Introduction

Wireless localization has attracted great attention over the past decade [1]–[3]. Currently used localization systems are mainly in four categories: satellite-based, cellular network (CN)-based, wireless local area network (WLAN)-based and wireless sensor network (WSN)-based. Satellite-based localization system, e.g., the Global Positioning System (GPS) [4], utilizes pseudo time-of-arrival (TOA), code phase or Doppler shift measurements to estimate the mobile station (MS) location. The location-pertaining measurements the other three kinds of systems utilized are TOA, time-difference-of-arrival (TDOA), round trip time (RTT), time advance (TA), angle-of-arrival (AOA), received signal strength (RSS), received signal level (RXLEV, actually quantized RSS) and so forth. The localization problem is to estimate the MS location based on location-pertaining measurements with respect to a set of reference stations (RSs). For analytical purposes, we consider only the parametric model based localization techniques in this paper.

Localization techniques based on different types of measurements have their own advantages and disadvantages. In certain scenarios, the techniques based on single type of measurement may not be satisfactory. Hybrid methods (data fusion) which seek performance gain by fusing data from different types of measurements and/or multiple localization systems are thus resorted to. We classify data fusion methods as three types: measurement fusion, estimate fusion and mixed fusion. In measurement fusion, only the raw location-pertaining measurements are fused. In estimate fusion, only the local estimates (already an estimate of the MS location, e.g., a GPS location fix) are fused. In mixed fusion, combinations of raw measurements and local estimates are fused. Here, we use "mixed" fusion to distinguish from so called "hybrid" methods used in the existing literature concerning data fusion in wireless localization. Most of the hybrid methods proposed are actually measurement fusion since they fuse only different types of raw measurements, while our defined mixed fusion fuses both raw measurements and local estimates.

Concerning measurement fusion, [5] proposes an AOA assisted TOA positioning system and [6] propose hybrid TOA/AOA techniques for non-line-of-sight (NLOS) [7] mitigation. A hybrid TDOA/AOA technique is proposed in [8] for wideband code division multiple access (WCDMA) cellular systems. [9] explores the combination of TDOA and RSS for cellular network positioning. [10] resorts to a triplet hybrid scheme using TOA, time sum of arrival (TSOA) and TDOA to reduce the NLOS errors. [11] proposes hybrid pseudo TOA/TA/RXLEV methods between GPS and a CN-based localization system to achieve enhanced accuracy. Besides measurement fusion, estimate fusion is used in [12] to fuse two location estimates from TOA and TDOA estimators. Recently, [13] uses mixed fusion to improve the initial GPS location fix accuracy by making use of terrestrial TOA and AOA measurements.

However, there does not exist a general framework for all these types of fusion schemes and the performance comparison among these fusion schemes is also unaddressed. Therefore, in this paper, we propose a unified data fusion framework based on the weighted least square estimator (WLSE) [15] for wireless localization where the MS location is treated as a deterministic unknown vector. The proposed data fusion framework can effectively fuse all the available raw measurements and/or local estimates. We also analyze and compare the theoretical performance of these fusion schemes in terms of the estimation error covariance matrix. Assuming that all the estimates (local estimates and data fusion) are done via the WLSE, we show that

1) *Property 1*: If the raw measurement vectors are uncorrelated, then the three fusion schemes achieve the same performance.
2) *Property 2*: If the raw measurement vectors are correlated, then measurement fusion achieves the best performance, followed by mixed fusion and lastly estimate fusion.

Property 1 tells us if the raw measurement vectors are uncorrelated, then there is no performance degradation in the raw measurements → local estimates → estimate fusion or mixed fusion processes compared with measurement fusion which directly fuse all the raw measurements. Therefore, providing only local estimates is equivalent to providing all the raw measurements. While the former offers privacy since the information about the RSs used does not need to be released. *Property 2* tells us if some raw measurement vectors are correlated, to obtain good performance, it is better to fuse them together to generate a local estimate, rather than produce separate local estimates based on individual raw measurement vectors and then fuse these local estimates together. For Gaussian measurement errors, the WLSE is equivalent to the maximum likelihood estimator (MLE) and the estimation error covariance matrix then attains the corresponding Cramer-Rao lower bound (CRLB), which sets a lower bound on the covariance matrix of any unbiased estimator for deterministic unknowns. In consequence, the corresponding estimation mean square error (MSE) then becomes the minimum MSE.

Benefits from data fusion and the pros and cons of the three fusion schemes are also investigated. For example, data fusion can bring lowered estimation error, improved resilience to bad geometric layouts, enhanced localization availability, improved data usage, easy cooperation among different localization systems with maintained privacy, distributed implementation with retained performance and so forth.

The remainder of this paper is organized as follows. Section II introduces the WLSE and the data models used in wireless location. Section III presents the data fusion framework, derives the estimation error covariance matrices and discusses the relationship among the three fusion schemes. Section IV investigates the benefits from data fusion and the pros and cons of respective fusion methods. Section V provides numerical case studies in comparing the performance among measurement fusion, estimate fusion and mixed fusion. Finally, Section VI concludes the paper.

II. Preliminaries

In this section, we first introduce the definition and properties of the WLSE. Then, we present the data models for four kinds of commonly used raw measurements, followed by the data model for a local estimate. Throughout this paper, we assume that all the covariance matrices are invertible, and thus positive definite [14]. We denote a positive definite matrix $\mathbf{A}$ as $\mathbf{A} \succ \mathbf{0}$ and a positive semidefinite matrix $\mathbf{B}$ as $\mathbf{B} \succeq \mathbf{0}$.

A. WLSE

Given data that modeled as $\mathbf{z} = \mathbf{s}(\boldsymbol{\theta}) + \mathbf{n}$ ($\mathbf{s}(\boldsymbol{\theta})$ is a known function of $\boldsymbol{\theta}$; $\mathbf{n}$ has zero mean and known covariance matrix $\mathbf{C}_\mathsf{n}$; $\mathbf{z}$, $\mathbf{s}(\boldsymbol{\theta})$ and $\mathbf{n}$ are all $N \times 1$ vectors), to estimate the unknown parameter $\boldsymbol{\theta}$ ($\boldsymbol{\theta}$ is treated as a deterministic $p \times 1$ vector), the WLSE [15] is to minimize the following penalty function

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} (\mathbf{z} - \mathbf{s}(\boldsymbol{\theta}))^T \mathbf{C}_\mathsf{n}^{-1} (\mathbf{z} - \mathbf{s}(\boldsymbol{\theta})). \tag{1}$$

1) Case I: $\mathbf{s}(\boldsymbol{\theta})$ is linear in $\boldsymbol{\theta}$, i.e., $\mathbf{s}(\boldsymbol{\theta}) = \mathbf{H}\boldsymbol{\theta}$ ($\mathbf{H}$ is a known $N \times p$ matrix). The data model becomes $\mathbf{z} = \mathbf{H}\boldsymbol{\theta} + \mathbf{n}$.

In this case, $\hat{\boldsymbol{\theta}}$ has closed-form expression

$$\hat{\boldsymbol{\theta}} = (\mathbf{H}^T \mathbf{C}_\mathsf{n}^{-1} \mathbf{H})^{-1} \mathbf{H}^T \mathbf{C}_\mathsf{n}^{-1} \mathbf{z}. \tag{2}$$

with covariance matrix

$$\mathbf{C} = (\mathbf{H}^T \mathbf{C}_\mathsf{n}^{-1} \mathbf{H})^{-1}. \tag{3}$$

WLSE is then identical to the best linear unbiased estimator (BLUE) which has the minimum MSE among all the linear unbiased estimators [15].

2) Case II: $\mathbf{s}(\boldsymbol{\theta})$ is nonlinear in $\boldsymbol{\theta}$. Generally there is no closed-form expression for $\hat{\boldsymbol{\theta}}$. The optimization of (1) can be solved by iterative algorithms given an initial point, e.g., the Levenberg-Marquardt method [16].

The linearization of $\mathbf{s}(\boldsymbol{\theta})$ at the true value of $\boldsymbol{\theta}$ ($\boldsymbol{\theta}_0$) yields (retaining only the first two terms of the Taylor series)

$$\mathbf{s}(\boldsymbol{\theta}) = \mathbf{s}(\boldsymbol{\theta}_0) + \mathbf{P}(\boldsymbol{\theta} - \boldsymbol{\theta}_0) \tag{4}$$

where

$$[\mathbf{P}]_{i,j} = \left.\frac{\partial [\mathbf{s}(\boldsymbol{\theta})]_i}{\partial [\boldsymbol{\theta}]_j}\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0}, \tag{5}$$

$[\mathbf{P}]_{i,j}$ denotes the (i,j)th element of matrix $\mathbf{P}$ and $[\mathbf{s}(\boldsymbol{\theta})]_i$ represents the ith element of vector $\mathbf{s}(\boldsymbol{\theta})$.

The data model then becomes

$$\begin{aligned} & \mathbf{z} = \mathbf{s}(\boldsymbol{\theta}_0) + \mathbf{P}(\boldsymbol{\theta} - \boldsymbol{\theta}_0) + \mathbf{n} \\ \Rightarrow\quad & \mathbf{z} - \mathbf{s}(\boldsymbol{\theta}_0) + \mathbf{P}\boldsymbol{\theta}_0 = \mathbf{P}\boldsymbol{\theta} + \mathbf{n}. \end{aligned}$$

Referring to *Case I*, we have

$$\hat{\boldsymbol{\theta}} = (\mathbf{P}^T \mathbf{C}_\mathsf{n}^{-1} \mathbf{P})^{-1} \mathbf{P}^T \mathbf{C}_\mathsf{n}^{-1} (\mathbf{z} - \mathbf{s}(\boldsymbol{\theta}_0) + \mathbf{P}\boldsymbol{\theta}_0) \tag{6}$$

with covariance matrix

$$\mathbf{C} = (\mathbf{P}^T \mathbf{C}_\mathsf{n}^{-1} \mathbf{P})^{-1}. \tag{7}$$

In either case, if $\mathbf{n}$ is Gaussian, i.e., $\mathbf{n} \sim \mathcal{N}(\mathbf{0}, \mathbf{C}_\mathsf{n})$, the WLSE is then equivalent to the MLE, and the estimation covariance matrix $\mathbf{C}$ attains the corresponding CRLB for large enough data records. The CRLB sets the lower bound on the covariance matrix of any unbiased estimator for deterministic unknowns.

It is easy to verify that by the WLSE, $\hat{\boldsymbol{\theta}}$ is unbiased, i.e. $\mathbb{E}(\hat{\boldsymbol{\theta}}) = \boldsymbol{\theta}$ (for the nonlinear case, this holds for large enough N), and thus, the estimation MSE is given by

$$\mathbb{E}\|\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}\|^2 = \mathbb{E}\mathbf{tr}\big((\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})^T\big) = \mathbf{tr}(\mathbf{C}). \tag{8}$$

In addition, since $\boldsymbol{\theta}$ is treated as a deterministic unknown parameter vector, $\mathbf{C}$ is also the covariance matrix for the estimation error.

B. Data Models for Wireless Localization and Asymptotic Estimation Covariance Matrices

We consider 2-D scenarios and assume there are altogether N RSs. The true but unknown MS location is denoted as $\boldsymbol{\theta} = [x\ y]^T$ and the known location of the ith RS is denoted as $\boldsymbol{\zeta}_i = [a_i\ b_i]^T$.

Denote the true distance from the MS to the ith RS as $d_i = \|\boldsymbol{\theta} - \boldsymbol{\zeta}_i\|$ (the norm is the Euclidean norm), and the true angle between the positive x-axis and the line segment connecting the MS and the i-th RS as $\phi_i = \arctan \frac{y-b_i}{x-a_i}$. Therefore,

$$\cos\phi_i = \frac{a_i - x}{d_i},\ \sin\phi_i = \frac{b_i - y}{d_i}. \tag{9}$$

First, we consider the data models used in line-of-sight (LOS) scenarios for four kinds of commonly used raw measurements: TOA, TDOA, AOA and RSS.

In all the formulations below, we use $n_{\mathsf{x},i}$ to denote the measurement noise with x being a subscript indicating the type of the corresponding measurement - t for TOA, d for TDOA, a for AOA and r for RSS. The measurement noise vector $\mathbf{n}_\mathsf{x}$ is constructed as $\mathbf{n}_\mathsf{x} = [n_{\mathsf{x},1}\ n_{\mathsf{x},2}\ \dots]^T$. $\mathbf{P}_\mathsf{x}$ denotes the differential matrix given by (5) for measurement type x.

1) TOA: The scalar data model for TOA is

$$t_i = \frac{1}{c}\|\boldsymbol{\theta} - \boldsymbol{\zeta}_i\| + n_{\mathsf{t},i},\ i = 1, \dots, N \tag{10}$$

where t_i denotes the ith measured TOA and c is the speed of light. TOA can be measured by either one-way ranging or two-way ranging [17]. The measurement noise is modeled as $\mathbf{n}_\mathsf{t} \sim \mathcal{N}(\mathbf{0}, \mathbf{C}_\mathsf{t})$.

In reference to Section II-A, obviously, for TOA, $[\mathbf{s}(\boldsymbol{\theta})]_i = \frac{1}{c}\|\boldsymbol{\theta} - \boldsymbol{\zeta}_i\|$, and we have

$$\mathbf{P}_\mathsf{t}^T = \frac{1}{c}\begin{bmatrix}\cos\phi_1 & \dots & \cos\phi_N \\ \sin\phi_1 & \dots & \sin\phi_N\end{bmatrix}. \tag{11}$$

The corresponding estimation covariance matrix by the WLSE (for $\hat{\boldsymbol{\theta}}$ as well as the estimation error; the same below) is $\mathbf{C} = (\mathbf{P}_\mathsf{t}^T\mathbf{C}_\mathsf{t}^{-1}\mathbf{P}_\mathsf{t})^{-1}$.

2) TDOA: TDOA uses differences in TOA measurements. Assuming that the N-th RS is used as the reference, then the scalar data model for TDOA is given by

$$\tau_i = \frac{1}{c}\left(\|\boldsymbol{\theta} - \boldsymbol{\zeta}_i\| - \|\boldsymbol{\theta} - \boldsymbol{\zeta}_N\|\right) + n_{\mathsf{d},i},\ i = 1, \dots, N-1 \tag{12}$$

where τ_i denotes the i-th TDOA measurement and $n_{\mathsf{d},i} = n_{\mathsf{t},i} - n_{\mathsf{t},N}$.

$\mathbf{n}_\mathsf{d}$ relates to $\mathbf{n}_\mathsf{t}$ through $\mathbf{n}_\mathsf{d} = [\mathbf{I}_{N-1}\ \ -\mathbf{1}_{N-1}]\mathbf{n}_\mathsf{t} \triangleq \mathbf{D}\mathbf{n}_\mathsf{t}$. Therefore, $\mathbf{n}_\mathsf{d} \sim \mathcal{N}(\mathbf{0}, \mathbf{C}_\mathsf{d})$ with $\mathbf{C}_\mathsf{d} = \mathbf{D}\mathbf{C}_\mathsf{t}\mathbf{D}^T$.

Obviously, for TDOA, $[\mathbf{s}(\boldsymbol{\theta})]_i = \frac{1}{c}(\|\boldsymbol{\theta} - \boldsymbol{\zeta}_i\| - \|\boldsymbol{\theta} - \boldsymbol{\zeta}_N\|)$, and we have

$$\mathbf{P}_\mathsf{d}^T = \frac{1}{c}\begin{bmatrix}\cos\phi_1 - \cos\phi_N & \dots & \cos\phi_{N-1} - \cos\phi_N \\ \sin\phi_1 - \sin\phi_N & \dots & \sin\phi_{N-1} - \sin\phi_N\end{bmatrix}. \tag{13}$$

$\mathbf{P}_\mathsf{d}$ relates to $\mathbf{P}_\mathsf{t}$ through $\mathbf{P}_\mathsf{d} = \mathbf{D}\mathbf{P}_\mathsf{t}$.

The corresponding estimation covariance matrix is $\mathbf{C} = (\mathbf{P}_\mathsf{d}^T\mathbf{C}_\mathsf{d}^{-1}\mathbf{P}_\mathsf{d})^{-1}$.

3) AOA: The scalar data model for AOA is given by

$$\alpha_i = \arctan\frac{y - b_i}{x - a_i} + n_{\mathsf{a},i},\ i = 1, \dots, N \tag{14}$$

where α_i denotes the i-th measured AOA. The measurement noise is modeled as $\mathbf{n}_\mathsf{a} \sim \mathcal{N}(\mathbf{0}, \mathbf{C}_\mathsf{a})$.

Obviously, for AOA, $[\mathbf{s}(\boldsymbol{\theta})]_i = \arctan\frac{y-b_i}{x-a_i}$, and we have

$$\mathbf{P}_\mathsf{a}^T = \begin{bmatrix}-\frac{\sin\phi_1}{d_1} & \dots & -\frac{\sin\phi_N}{d_N} \\ \frac{\cos\phi_1}{d_1} & \dots & \frac{\cos\phi_N}{d_N}\end{bmatrix}. \tag{15}$$

The corresponding estimation covariance matrix is $\mathbf{C} = (\mathbf{P}_\mathsf{a}^T\mathbf{C}_\mathsf{a}^{-1}\mathbf{P}_\mathsf{a})^{-1}$.

4) RSS: The scalar data model for RSS is given by

$$L_i = L_0 + 10\gamma\log_{10}\frac{\|\boldsymbol{\theta} - \boldsymbol{\zeta}_i\|}{d_0} + n_{\mathsf{r},i},\ i = 1, \dots, N \tag{16}$$

where L_0 is the path loss (in dB) at the reference distance d_0, L_i denotes the measured path loss for a distance greater than d_0, γ is the path loss exponent and $n_{\mathsf{r},i}$ is a Gaussian random variable representing the log-normal shadow fading effects. The values of L_0, d_0 and γ are assumed known a priori from system calibration or on-line survey. $\mathbf{n}_\mathsf{r}$ is modeled as $\mathbf{n}_\mathsf{r} \sim \mathcal{N}(\mathbf{0}, \mathbf{C}_\mathsf{r})$.

Obviously, for RSS, $[\mathbf{s}(\boldsymbol{\theta})]_i = L_0 + 10\gamma\log_{10}\frac{\|\boldsymbol{\theta}-\boldsymbol{\zeta}_i\|}{d_0}$, and we have

$$\mathbf{P}_\mathsf{r}^T = \frac{10\gamma}{\ln 10}\begin{bmatrix}\frac{\cos\phi_1}{d_1} & \dots & \frac{\cos\phi_N}{d_N} \\ \frac{\sin\phi_1}{d_1} & \dots & \frac{\sin\phi_N}{d_N}\end{bmatrix}. \tag{17}$$

The corresponding estimation covariance matrix is $\mathbf{C} = (\mathbf{P}_\mathsf{r}^T\mathbf{C}_\mathsf{r}^{-1}\mathbf{P}_\mathsf{r})^{-1}$.

As has been proved in [18], if the statistics of NLOS induced errors (for single type of measurement) are not available, the CRLB for the MS location depends only on the LOS signals, i.e., all the NLOS signals should be rejected. Therefore, in NLOS scenarios, a selection procedure should be done first and the problem that remains is the same as localization in LOS scenarios. That is to say, the LOS data models presented above are also enough for NLOS scenarios without the statistics of NLOS induced errors. Since the statistics of NLOS induced errors are quite environmental dependent and do not have a unified model, we restrict our attention to the LOS models presented above.

5) Local Estimate: Finally, we construct the data model for a location estimate. Assume we obtain a location estimate $\hat{\boldsymbol{\theta}}_i$ with covariance matrix $\mathbf{C}_{\mathsf{m},i}$ from a system (e.g., a GPS location fix). In data fusion, we term it a local estimate since it is not the final location estimate. If we consider a local estimate $\hat{\boldsymbol{\theta}}_i$ as a processed measurement, we can then construct a model relating $\hat{\boldsymbol{\theta}}_i$ to $\boldsymbol{\theta}$ as

$$\hat{\boldsymbol{\theta}}_i = \boldsymbol{\theta} + \mathbf{m}_i \tag{18}$$

where $\mathbf{m}_i$ denotes the estimation error vector ($\hat{\boldsymbol{\theta}}_i$, $\boldsymbol{\theta}$ and $\mathbf{m}_i$ are all $p \times 1$ vectors). $\hat{\boldsymbol{\theta}}_i$ is often modeled as $\hat{\boldsymbol{\theta}}_i \sim \mathcal{N}(\boldsymbol{\theta}, \mathbf{C}_{\mathsf{m},i})$. By applying the WLSE, we have

$$\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}_i,\ \mathbf{C} = \mathbf{C}_{\mathsf{m},i}.$$

In other words, we cannot perform further processing based on a single local estimate. However, it contains useful information and can be combined with other raw measurements and/or local estimates to bring about performance enhancement.

III. Data Fusion for Wireless Localization

This section presents a data fusion framework for wireless localization based on the WLSE. The accuracy metric is defined in terms of the localization error covariance matrix (also the location estimation covariance matrix). Throughout this paper, mea is short for measurement, est is short for estimate and mix is short for mixed.

Assume that in the geometric area of interest there are M different sources that can perform wireless localization with respect to the same unknown MS location $\boldsymbol{\theta}$. Each source obtains a measurement vector $\mathbf{z}_i$ containing N_i raw measurements (i.e., $\mathbf{z}_i \in \mathbb{R}^{N_i \times 1}$) from a set of RSs (may be different for different sources). The ith source can either perform a local estimate $\hat{\boldsymbol{\theta}}_i$ based on $\mathbf{z}_i$ or just forward $\mathbf{z}_i$ and other necessary information to another entity. There also exists a fusion center (can be dedicated or one of the M sources)

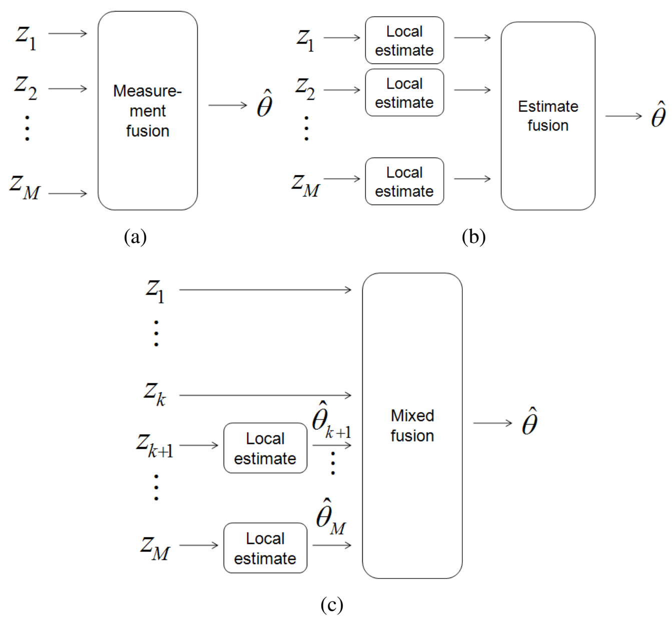


Fig. 1. (a) Measurement fusion. (b) Estimate fusion. (c) Mixed fusion.

which can perform data fusion and produce an overall estimate $\hat{\boldsymbol{\theta}}$ based on all the available data from the M sources.

The unified model for the raw measurement vector is simply $\mathbf{z}_i = \mathbf{s}_i(\boldsymbol{\theta}) + \mathbf{n}_i$, where $\mathbf{s}_i(\boldsymbol{\theta})$ is a known N_i-dim function of $\boldsymbol{\theta}$, and the noise vector $\mathbf{n}_i$ has zero mean and known covariance matrix $\mathbf{C}_{\mathsf{n},i}$. The unified model for a local estimate is given by (18). The covariance matrix associated with each $\hat{\boldsymbol{\theta}}_i$ (i.e., $\mathbf{C}_{\mathsf{m},i}$) is also assumed known.

Concerning data fusion, we define three types of fusion schemes: measurement fusion, estimate fusion and mixed fusion. In measurement fusion, the fusion center fuses only the raw measurements $\mathbf{z}_i$'s to generate $\hat{\boldsymbol{\theta}}$. In estimate fusion, the fusion center fuses only the local estimates $\hat{\boldsymbol{\theta}}_i$'s. In mixed fusion, the fusion center processes combinations of raw measurements and local estimates. The three types of fusion schemes are shown in Fig. 1.

A. Mixed Fusion

Without loss of generality, we assume that the first 1 to k sources just forward raw measurements $\mathbf{z}_i$ to the fusion center and the remaining $k+1$ to M sources send their respective local estimates $\hat{\boldsymbol{\theta}}_i$ to the fusion center.

To apply the WLSE, we need to construct an overall data vector as

$$\mathbf{z} = \begin{bmatrix} \mathbf{z}_1 \\ \vdots \\ \mathbf{z}_k \\ \hat{\boldsymbol{\theta}}_{k+1} \\ \vdots \\ \hat{\boldsymbol{\theta}}_M \end{bmatrix} = \begin{bmatrix} \mathbf{s}_1(\boldsymbol{\theta}) \\ \vdots \\ \mathbf{s}_k(\boldsymbol{\theta}) \\ \boldsymbol{\theta} \\ \vdots \\ \boldsymbol{\theta} \end{bmatrix} + \begin{bmatrix} \mathbf{n}_1 \\ \vdots \\ \mathbf{n}_k \\ \mathbf{m}_{k+1} \\ \vdots \\ \mathbf{m}_M \end{bmatrix} \triangleq \mathbf{s}(\boldsymbol{\theta}) + \mathbf{n}. \tag{19}$$

The covariance matrix of $\mathbf{n}$ is

$$\mathbf{C}_{\mathsf{n}} = \begin{bmatrix} \mathbf{C}_{11} & \mathbf{C}_{12} & \dots & \mathbf{C}_{1M} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{C}_{M1} & \mathbf{C}_{M2} & \dots & \mathbf{C}_{MM} \end{bmatrix}, \tag{20}$$

where $\mathbf{C}_{ij}$'s are all block matrices. Since $\mathbf{n}_i$'s and $\mathbf{m}_i$'s are assumed zero mean, $\mathbf{n}$ is also zero mean and thus $\mathbf{C}_{ij} = \mathbb{E}([\mathbf{n}]_i[\mathbf{n}]_j^T)$ ($[\mathbf{n}]_i$ denotes the ith element of vector $\mathbf{n}$).

It is ready to apply the WLSE (1) on model (19) to generate $\hat{\boldsymbol{\theta}}$, and the covariance matrix of $\hat{\boldsymbol{\theta}}$ can be determined by (7).

If the data from different sources are uncorrelated, i.e., $\mathbb{E}([\mathbf{n}]_i[\mathbf{n}]_j^T) = \mathbf{0}$ for $i \neq j$, by the WLSE, we have

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \sum_{i=1}^{k} (\mathbf{z}_i - \mathbf{s}_i(\boldsymbol{\theta}))^T \mathbf{C}_{\mathsf{n},i}^{-1} (\mathbf{z}_i - \mathbf{s}_i(\boldsymbol{\theta})) + \sum_{i=k+1}^{M} (\hat{\boldsymbol{\theta}}_i - \boldsymbol{\theta})^T \mathbf{C}_{\mathsf{m},i}^{-1} (\hat{\boldsymbol{\theta}}_i - \boldsymbol{\theta}). \tag{21}$$

with covariance matrix

$$\mathbf{C}_{\text{mix}} = \left(\sum_{i=1}^{k} \mathbf{P}_i^T \mathbf{C}_{\mathsf{n},i}^{-1} \mathbf{P}_i + \sum_{i=k+1}^{M} \mathbf{C}_{\mathsf{m},i}^{-1} \right)^{-1} \tag{22}$$

where $[\mathbf{P}_i]_{k,l} = \frac{\partial [\mathbf{s}_i(\boldsymbol{\theta})]_k}{\partial [\boldsymbol{\theta}]_l}$ (evaluated at the true value of $\boldsymbol{\theta}$).

In the context of wireless localization, $\mathbf{z}_i$ (for LOS scenarios) and $\hat{\boldsymbol{\theta}}_i$ are often modeled as Gaussian, and then (21) is also the MLE and (22) attains the corresponding CRLB.

B. Measurement Fusion

Replacing all the $\hat{\boldsymbol{\theta}}_i$'s in (19) by $\mathbf{z}_i$'s, we obtain the data model for measurement fusion. For uncorrelated $\mathbf{z}_i$'s, by the WLSE, the covariance matrix of $\hat{\boldsymbol{\theta}}$ is given by

$$\mathbf{C}_{\text{mea}} = \left(\sum_{i=1}^{M} \mathbf{P}_i^T \mathbf{C}_{\mathsf{n},i}^{-1} \mathbf{P}_i \right)^{-1}. \tag{23}$$

Since most of the hybrid methods addressed in the literature use measurement fusions, we will not discuss too much in this part.

C. Estimate Fusion

Replacing all the $\mathbf{z}_i$'s in (19) by $\hat{\boldsymbol{\theta}}_i$'s, we obtain the data model for estimate fusion, which is

$$\mathbf{z} = \begin{bmatrix} \hat{\boldsymbol{\theta}}_1 \\ \vdots \\ \hat{\boldsymbol{\theta}}_M \end{bmatrix} = \begin{bmatrix} \boldsymbol{\theta} \\ \vdots \\ \boldsymbol{\theta} \end{bmatrix} + \begin{bmatrix} \mathbf{m}_1 \\ \vdots \\ \mathbf{m}_M \end{bmatrix} \triangleq \mathbf{G}\boldsymbol{\theta} + \mathbf{m} \tag{24}$$

where $\mathbf{G} = [\mathbf{I}_p \ \dots \ \mathbf{I}_p]^T$ and $\mathbf{I}_p$ is the $p \times p$ identity matrix. The covariance matrix $\mathbf{C}_{\mathsf{m}}$ of $\mathbf{m}$ takes the same form as (20), while $\mathbf{C}_{ij} = \mathbb{E}(\mathbf{m}_i \mathbf{m}_j^T)$.

As can be seen, the data model (24) for estimate fusion is intrinsic linear in $\boldsymbol{\theta}$ no matter the raw measurements $\mathbf{z}_i$'s are or not. By the WLSE, $\hat{\boldsymbol{\theta}}$ always has a closed-form solution, which is

$$\hat{\boldsymbol{\theta}} = (\mathbf{G}^T \mathbf{C}_{\mathsf{m}}^{-1} \mathbf{G})^{-1} \mathbf{G}^T \mathbf{C}_{\mathsf{m}}^{-1} \mathbf{z} \tag{25}$$

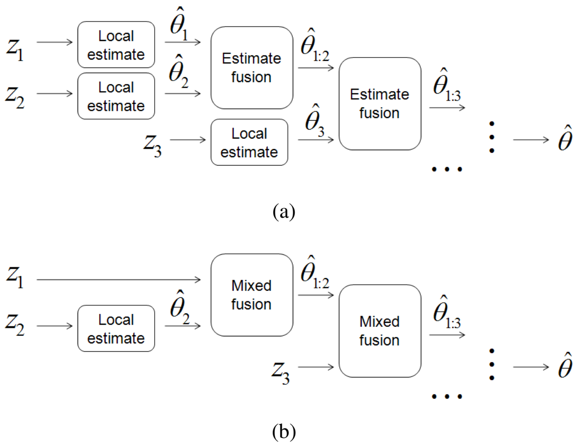


Fig. 2. (a) Distributed estimate fusion. (b) Distributed mixed fusion.

with covariance matrix

$$\mathbf{C}_{\text{est}} = (\mathbf{G}^T\mathbf{C}_{\mathsf{m}}^{-1}\mathbf{G})^{-1}. \tag{26}$$

Since the BLUE is identical to the WLSE for linear data models [15], we obtain the optimal linear (unbiased, if the local estimates are unbiased) estimator (25) for estimate fusion, and it has closed form and can be applied to fuse arbitrary number of local estimates.

If the local estimates are uncorrelated (i.e., $\mathbf{C}_{ij} = \mathbf{0}$), then $\mathbf{C}_{\mathsf{m}}$ is simplified as a block diagonal matrix $\mathbf{C}_{\mathsf{m}} = diag(\mathbf{C}_{\mathsf{m},1}, \ldots, \mathbf{C}_{\mathsf{m},M})$. $\hat{\boldsymbol{\theta}}$ given by (25) can then be simplified as

$$\begin{aligned}\hat{\boldsymbol{\theta}} &= (\mathbf{G}^T\mathbf{C}_{\mathsf{m}}^{-1}\mathbf{G})^{-1}\mathbf{G}^T\mathbf{C}_{\mathsf{m}}^{-1}\mathbf{z}\\ &= \left(\sum_{i=1}^{M}\mathbf{C}_{\mathsf{m},i}^{-1}\right)^{-1}\left[\mathbf{C}_{\mathsf{m},1}^{-1} \ \ldots \ \mathbf{C}_{\mathsf{m},M}^{-1}\right]\mathbf{z}\\ &= \sum_{i=1}^{M}\left(\sum_{i=1}^{M}\mathbf{C}_{\mathsf{m},i}^{-1}\right)^{-1}\mathbf{C}_{\mathsf{m},i}^{-1}\hat{\boldsymbol{\theta}}_i,\end{aligned} \tag{27}$$

with covariance matrix

$$\mathbf{C}_{\text{est}} = \left(\sum_{i=1}^{M}\mathbf{C}_{\mathsf{m},i}^{-1}\right)^{-1}. \tag{28}$$

(27) can be considered as a linear combination of $\hat{\boldsymbol{\theta}}_i$'s with weighting matrix $\mathbf{W}_i = \left(\sum_{i=1}^{M}\mathbf{C}_{\mathsf{m},i}^{-1}\right)^{-1}\mathbf{C}_{\mathsf{m},i}^{-1}$. Obviously, $\sum_{i=1}^{M}\mathbf{W}_i = \mathbf{I}_p$.

Applying (27) to $M = 2$ yields

$$\begin{aligned}\hat{\boldsymbol{\theta}} &= \left(\mathbf{C}_{\mathsf{m},1}^{-1} + \mathbf{C}_{\mathsf{m},2}^{-1}\right)^{-1}\mathbf{C}_{\mathsf{m},1}^{-1}\hat{\boldsymbol{\theta}}_1 + \left(\mathbf{C}_{\mathsf{m},1}^{-1} + \mathbf{C}_{\mathsf{m},2}^{-1}\right)^{-1}\mathbf{C}_{\mathsf{m},2}^{-1}\hat{\boldsymbol{\theta}}_2\\ &= \mathbf{C}_{\mathsf{m},2}\left(\mathbf{C}_{\mathsf{m},1} + \mathbf{C}_{\mathsf{m},2}\right)^{-1}\hat{\boldsymbol{\theta}}_1 + \mathbf{C}_{\mathsf{m},1}\left(\mathbf{C}_{\mathsf{m},1} + \mathbf{C}_{\mathsf{m},2}\right)^{-1}\hat{\boldsymbol{\theta}}_2\end{aligned}$$

which is just the Millman's Formula [19] - the optimal linear estimator of $\boldsymbol{\theta}$ given two uncorrelated local estimates. Obviously, (27) is more general and can be applied to arbitrary M uncorrelated local estimates.

We can show that if the local estimates are uncorrelated, the MSE of the overall estimate from estimate fusion is always smaller than that of any local estimate.

Proposition 1: If local estimates $\hat{\boldsymbol{\theta}}_i$'s (with known invertible covariance matrices $\mathbf{C}_{\mathsf{m},i}$'s) are unbiased and uncorrelated, for estimate fusion (27) with covariance matrix (28),

$$\mathbb{E}\|\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}\|^2 < \min_i \mathbb{E}\|\hat{\boldsymbol{\theta}}_i - \boldsymbol{\theta}\|^2.$$

Proof: For uncorrelated $\hat{\boldsymbol{\theta}}_i$'s, according to (28), we have

$$\begin{aligned}&\mathbf{C}_{\mathsf{m},i} \succ \mathbf{0} \Rightarrow \mathbf{C}_{\mathsf{m},i}^{-1} \succ \mathbf{0}, \ \forall i\\ &\Rightarrow {\mathbf{C}_{\text{est}}}^{-1} = \sum_{i=1}^{M}\mathbf{C}_{\mathsf{m},i}^{-1} \succ \mathbf{C}_{\mathsf{m},i}^{-1} \succ \mathbf{0}, \ \forall i\\ &\Rightarrow \mathbf{C}_{\mathsf{m},i} \succ \mathbf{C}_{\text{est}} \succ \mathbf{0}, \ \forall i\\ &\Rightarrow \mathbf{tr}(\mathbf{C}_{\text{est}}) < \min_i \mathbf{tr}(\mathbf{C}_{\mathsf{m},i}).\end{aligned}$$

Since $\hat{\boldsymbol{\theta}}_i$'s and $\hat{\boldsymbol{\theta}}$ (by the WLSE) are unbiased, $\mathbb{E}\|\hat{\boldsymbol{\theta}}-\boldsymbol{\theta}\|^2 = \mathbb{E}\mathbf{tr}\big((\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})^T\big) = \mathbf{tr}(\mathbf{C}_{\text{est}})$, and thus $\mathbf{tr}(\mathbf{C}_{\text{est}}) < \min_i \mathbf{tr}(\mathbf{C}_{\mathsf{m},i})$ is equivalent to

$$\mathbb{E}\|\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}\|^2 < \min_i \mathbb{E}\|\hat{\boldsymbol{\theta}}_i - \boldsymbol{\theta}\|^2. \ \square$$

D. Relationship among Measurement Fusion, Estimate Fusion and Mixed Fusion

We now discuss the relationship among the three types of fusions. First, we examine the case of uncorrelated raw measurement error vectors.

Proposition 2: If the raw measurement error vectors $\mathbf{n}_i$'s are uncorrelated, and all the estimates (local estimates and data fusions) are done via the WLSE, then for large enough N_i's, measurement fusion, estimate fusion and mixed fusion result in the same estimation covariance matrix. If $\mathbf{n}_i$'s are additionally Gaussian, these three types of fusions attain the same CRLB.

Proof: Uncorrelated raw measurement error vectors $\mathbf{n}_i$'s imply that the raw measurement vectors $\mathbf{z}_i$'s are uncorrelated and the local estimates $\hat{\boldsymbol{\theta}}_i$'s based on respective $\mathbf{z}_i$'s are also uncorrelated. Therefore, all the input vectors ($\mathbf{z}_i$'s and $\hat{\boldsymbol{\theta}}_i$'s) for fusion are uncorrelated. Referring to (7), if local estimates ($\mathbf{z}_i \Rightarrow \hat{\boldsymbol{\theta}}_i$) are done via the WLSE, then $\mathbf{C}_{\mathsf{m},i} = (\mathbf{P}_i^T\mathbf{C}_{\mathsf{n},i}^{-1}\mathbf{P}_i)^{-1}$.

We list the covariance matrices by the three types of fusion schemes for uncorrelated input vectors via the WLSE below (which are given by (23), (28) and (22))

$$\begin{aligned}\mathbf{C}_{\text{mea}} &= \left(\textstyle\sum_{i=1}^{M}\mathbf{P}_i^T\mathbf{C}_{\mathsf{n},i}^{-1}\mathbf{P}_i\right)^{-1}\\ \mathbf{C}_{\text{est}} &= \left(\textstyle\sum_{i=1}^{M}\mathbf{C}_{\mathsf{m},i}^{-1}\right)^{-1}\\ \mathbf{C}_{\text{mix}} &= \left(\textstyle\sum_{i=1}^{k}\mathbf{P}_i^T\mathbf{C}_{\mathsf{n},i}^{-1}\mathbf{P}_i + \sum_{i=k+1}^{M}\mathbf{C}_{\mathsf{m},i}^{-1}\right)^{-1}\end{aligned}$$

It is easy to observe that since $\mathbf{C}_{\mathsf{m},i}^{-1} = \mathbf{P}_i^T\mathbf{C}_{\mathsf{n},i}^{-1}\mathbf{P}_i$, we have $\mathbf{C}_{\text{mea}} = \mathbf{C}_{\text{est}} = \mathbf{C}_{\text{mix}}$. For Gaussian $\mathbf{n}_i$'s, the WLSE is then equivalent to the MLE and the covariance matrices then attains the same CRLB. $\square$

In summary, if the raw measurement vectors $\mathbf{z}_i$'s are uncorrelated, there is no performance degradation (information loss) in the raw measurements → local estimates → estimate fusion or mixed fusion processes compared with measurement fusion which directly fuse all the raw measurements. Therefore, for

Gaussian and uncorrelated measurement vectors, providing only the local estimates (resulted from the WLSE) or any combination of raw measurements and local estimates (resulted from the WLSE) is equivalent to providing all the raw measurements (other necessary information is also assumed provided).

Now let's discuss the situation if the raw measurement error vectors are correlated. Correlation may arise if different types of raw measurements are derived from the same set of signals. For example, if TOA and RSS measurements are estimated from the same set of signals from certain RSs, they will have correlation since a larger TOA measurement indicates a longer distance the signal traveled and thus a higher power attenuation in the corresponding RSS.

Proposition 3: Given two raw measurement vectors $\mathbf{z}_1$ and $\mathbf{z}_2$, which can be modeled as $\mathbf{z}_i = \mathbf{H}_i\boldsymbol{\theta} + \mathbf{n}_i$ (either naturally linear or by linearization), where $\mathbf{n}_i$ has zero mean and known covariance matrix, and $\mathbf{n}_1, \mathbf{n}_2$ are correlated. If all the estimates (local estimates and data fusions) are done via the WLSE, the covariance matrices of measurement fusion (fusing $\mathbf{z}_1$ and $\mathbf{z}_2$), estimate fusion (fusing $\hat{\boldsymbol{\theta}}_1$ and $\hat{\boldsymbol{\theta}}_2$) and mixed fusion (fusing $\mathbf{z}_1$ and $\hat{\boldsymbol{\theta}}_2$) have the relationship $\mathbf{C}_{\text{mea}} \preceq \mathbf{C}_{\text{mix}} \preceq \mathbf{C}_{\text{est}}$.

If only $\mathbf{H}_1$ is invertible, the above inequality becomes $\mathbf{C}_{\text{mea}} \preceq \mathbf{C}_{\text{mix}} = \mathbf{C}_{\text{est}}$. If only $\mathbf{H}_2$ is invertible, it becomes $\mathbf{C}_{\text{mea}} = \mathbf{C}_{\text{mix}} \preceq \mathbf{C}_{\text{est}}$. If both $\mathbf{H}_1$ and $\mathbf{H}_2$ are invertible, it becomes an equality.

Proof: See the Appendix. □

As can be seen, for correlated raw measurement error vectors, performance degradation (information loss) happens during the raw measurements → local estimates → estimate fusion or mixed fusion processes compared with measurement fusion. In terms of the estimation error covariance (as well as the MSE), measurement fusion performs best, followed by mixed fusion, and estimate fusion is the worst. Proposition 3 tells us if some $\mathbf{z}_i$'s are correlated, it is better to fuse them together to generate a local estimate, rather than produce separate local estimates based on respective $\mathbf{z}_i$'s and then fuse these local estimates.

E. Centralized and Distributed Fusion

In this subsection, we provide a discussion on centralized versus distributed fusion. It is easy to observe that measurement fusion must be done centralized, that is all the $\mathbf{z}_i$'s must be collected together at the fusion center to perform the fusion. While relying on estimate fusion or mixed fusion, the data fusion process can be made distributed as shown in Fig. 2.

Consider there are M uncorrelated local estimates. A typical distributed estimate fusion process is to first fuse $\hat{\boldsymbol{\theta}}_1$ and $\hat{\boldsymbol{\theta}}_2$ to generate $\hat{\boldsymbol{\theta}}_{1:2}$ (with covariance matrix $\mathbf{C}_{1:2}$) and then fuse $\hat{\boldsymbol{\theta}}_{12}$ and $\hat{\boldsymbol{\theta}}_3$ to generate $\hat{\boldsymbol{\theta}}_{1:3}$ (with covariance matrix $\mathbf{C}_{1:3}$). This process is continued until all the local estimates are fused. Of course, other architectures for distributed fusion can also be designed. For example, some intermediate fusions can be made parallel.

By fusing $\hat{\boldsymbol{\theta}}_1$ and $\hat{\boldsymbol{\theta}}_2$, from (27) and (28) we have

$$\hat{\boldsymbol{\theta}}_{1:2} = \left(\mathbf{C}_{\mathsf{m},1}^{-1} + \mathbf{C}_{\mathsf{m},2}^{-1}\right)^{-1} \left(\mathbf{C}_{\mathsf{m},1}^{-1}\hat{\boldsymbol{\theta}}_1 + \mathbf{C}_{\mathsf{m},2}^{-1}\hat{\boldsymbol{\theta}}_2\right)$$

with covariance matrix $\mathbf{C}_{1:2} = \left(\mathbf{C}_{\mathsf{m},1}^{-1} + \mathbf{C}_{\mathsf{m},2}^{-1}\right)^{-1}$.

By fusing $\hat{\boldsymbol{\theta}}_{1:2}$ and $\hat{\boldsymbol{\theta}}_3$, we obtain

$$\begin{aligned}\hat{\boldsymbol{\theta}}_{1:3} &= \left(\mathbf{C}_{1:2}^{-1} + \mathbf{C}_{\mathsf{m},3}^{-1}\right)^{-1} \left(\mathbf{C}_{1:2}^{-1}\hat{\boldsymbol{\theta}}_{1:2} + \mathbf{C}_{\mathsf{m},3}^{-1}\hat{\boldsymbol{\theta}}_3\right)\\ &= \left(\mathbf{C}_{\mathsf{m},1}^{-1} + \mathbf{C}_{\mathsf{m},2}^{-1} + \mathbf{C}_{\mathsf{m},3}^{-1}\right)^{-1} \left(\mathbf{C}_{\mathsf{m},1}^{-1}\hat{\boldsymbol{\theta}}_1 + \mathbf{C}_{\mathsf{m},2}^{-1}\hat{\boldsymbol{\theta}}_2 + \mathbf{C}_{\mathsf{m},3}^{-1}\hat{\boldsymbol{\theta}}_3\right)\end{aligned}$$

with covariance matrix

$$\mathbf{C}_{1:3} = \left(\mathbf{C}_{1:2}^{-1} + \mathbf{C}_{\mathsf{m},3}^{-1}\right)^{-1} = \left(\mathbf{C}_{\mathsf{m},1}^{-1} + \mathbf{C}_{\mathsf{m},2}^{-1} + \mathbf{C}_{\mathsf{m},3}^{-1}\right)^{-1}.$$

Obviously, $\hat{\boldsymbol{\theta}}_{1:3}$ and $\mathbf{C}_{1:3}$ are the same as directly applying (27) and (28) for $M = 3$. As this distributed fusion process continues until all the local estimates are fused, finally we will observe that distributed estimate fusion generate the same result as centralized estimate fusion for uncorrelated local estimates. In other words, estimate fusion can be made distributed while retain the same performance as centralized processing for uncorrelated local estimates. In addition, the Millman's Formula for $M = 2$ is enough for distributed estimate fusion with uncorrelated local estimates since each time only two local estimates are fused.

Similarly, for uncorrelated fusion inputs ($\mathbf{z}_i$'s and $\hat{\boldsymbol{\theta}}_i$'s), distributed mixed fusion can attain the same performance as centralized mixed fusion for large enough N_i's (due to the nonlinearity of $\mathbf{s}(\boldsymbol{\theta})_i$'s in $\boldsymbol{\theta}$).

IV. Benefits from Data Fusion for Wireless Localization

In this section, we first discuss the potential benefits from data fusion for wireless localization. Then, we summarize the pros and cons of the three fusion schemes based on our previous discussions.

A. Benefits from Data Fusion

We see that data fusion for wireless localization brings at least the following benefits.

1) Lowered MSE: This is a benefit from all these three fusion schemes.

As long as any new observation (raw measurement or local estimate) is not completely dependent on previous observations, it carries useful information about the unknown parameters, decreasing the uncertainty on estimating the unknown parameters. As a consequence, the MSE, which is equal to the trace of the estimation covariance matrix (if the estimator is unbiased), is lowered.

2) Improved strength with respect to geometric parameters: This is also a benefit from all these three fusion schemes.

For pure TOA, TDOA, AOA and RSS based localization methods, the MSEs of them all depend on two geometric parameters, namely the actual distances d_i's and angles ϕ_i's. For each method, how the MSE depends on the geometric parameters has its own properties. The expressions of the MSE for measurement fusion provides us a convenient way to assess this dependence (estimate fusion and mixed fusion will achieve the same MSE as measurement fusion if the input vectors for fusion are uncorrelated). As what has been discussed in Section II, we have the MSE $\sigma^2 = \mathbf{tr}(\mathbf{C}) = \mathbf{tr}((\mathbf{P}_{\mathsf{x}}^T\mathbf{C}_{\mathsf{x}}\mathbf{P}_{\mathsf{x}})^{-1})$, where $\mathsf{x} \in \{\mathsf{t}, \mathsf{d}, \mathsf{a}, \mathsf{r}\}$.

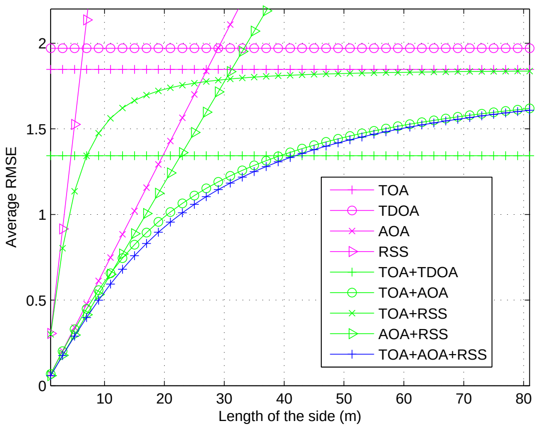


Fig. 3. Average RMSE versus the length of the side L.

We present a numerical example on how the MSE depends on the size of the network (a function of d_i). We set the simulation scenario as follows: $N = 4$ RSs are deployed at the four corners of an $L \times L$ square area and the MS moves randomly inside this area. The models presented in Section II-B are used and the covariance matrices $\mathbf{C}_{\mathsf{x}}$ are set as $\mathbf{C}_{\mathsf{x}} = \sigma_{\mathsf{x}}\mathbf{I}$ with $c\sigma_{\mathsf{t}} = 1.8$ m, $\sigma_{\mathsf{a}} = 5$ degree and $\sigma_{\mathsf{r}}/\gamma = 1.7$ [20]. Different types of measurements are assumed independent. The metric for performance comparison is the average RMSE, which is computed by uniformly choosing 97×97 points over the centric $0.96L \times 0.96L$ area and then calculating the average RMSE. By doing so, the effect of ϕ_i is averaged out.

Fig. 3 plots the average RMSE versus the length of the side L of the deployment area. It is clear that the average RMSEs of the TOA, TDOA and TOA/TDOA based methods are independent of L while the other methods are not. For the AOA and RSS based methods, their average RMSEs are smaller than that of the TOA and TDOA based methods under certain values of L. Once exceeding certain threshold, the performance of the AOA and RSS based methods deteriotes quickly in comparison to the TOA and TDOA based methods as L increases.

Any fusion involving at least one method whose RMSE depends on L will cause the RMSE of the fusion method to depend on L as well. However, fusion has the advantage to integrate the strength from different methods. Take the TOA/RSS based method as an example. When $L \leq 5$, its average RMSE is very close to that of the RSS based method, and is much smaller than that of the TOA based method. For $L \geq 30$, its average RMSE is very close to that of the TOA based method, and tends to be constant regardless of L. When L falls in the interval $[5, 30]$, the TOA/RSS method still exhibit obvious improvement in the average RMSE compared with TOA or RSS based method.

For more examples on how data fusion can gain improved strength with respect to geometric parameters, refer to [21], [22].

3) Enhanced availability of localization: This is a benefit from measurement fusion.

It is well-known that pure TOA, TDOA or RSS based methods require at least 3 RSs, and pure AOA based method requires at least 2 RSs to generate a 2-D location estimate. When the number of available references is less than the minimum required, these methods all fail. Measurement fusion fuses all the available raw measurements from different sources even if they are not the same type. For example, 1 TOA measurement and 1 AOA measurement are enough for measurement fusion to offer a location estimate [5]. As a consequence, measurement fusion can provide a location fix when individual methods all fail and thus enhance the availability of localization. Since both estimate fusion and mixed fusion involve at least one local estimate, they do not enhance the availability of localization (since a local estimate can already be taken as a location fix), but only improve the localization accuracy.

4) Cooperation among different systems with privacy: This is a benefit from estimate fusion.

For commercial or security reasons, a system may choose to disclose only a location estimate $\hat{\boldsymbol{\theta}}_i$ and the associated uncertainty $\mathbf{C}_{\mathsf{m},i}$, rather than the underlying raw measurements $\mathbf{z}_i$, the error covariance $\mathbf{C}_{\mathsf{n},i}$, and especially, the locations of the RSs used $\mathbf{L}_i$. The type of the raw measurements used T_i may or may not be released. In this case, cooperation among different systems requiring measurement fusion is not feasible.

Estimate fusion, however, does not need to access T_i, $\mathbf{C}_{\mathsf{n},i}$, $\mathbf{z}_i$ and $\mathbf{L}_i$. It requires only $\hat{\boldsymbol{\theta}}_i$ and $\mathbf{C}_{\mathsf{m},i}$ at the fusion node. Thus, estimate fusion is more suitable for cooperation among different systems with privacy requirements. Moreover, estimate fusion always has a closed-form expression and is quite easy to perform. It also offers all the superior properties of the BLUE [15].

Similarly, mixed fusion can also provide privacy if the fusion is done by a system which utilizes its own raw measurements and other systems' local estimates.

5) Rendering the less-than-minimum-required measurement(s) or low-accuracy estimate(s) useful: This is a benefit from mixed fusion.

Consider a scenario that a user obtains a GPS location fix and a terrestrial TOA measurement with respect to certain RS. Without mixed fusion, the only one TOA measurement will become useless since it solely cannot provide a location fix and finally the only choice is to use the GPS location fix. Mixed fusion efficiently utilizes all the available data and it can incorporate the information contained in this only one TOA measurement to the GPS location fix to generate a fusion result with improved accuracy.

In a city canyon, the satellites in view are often less than the minimum required for a GPS location fix, but possibly the terrestrial location service is available. Upon obtaining a coarse terrestrial location fix and one or two satellite pseudo TOA measurements, the user can also perform a mixed fusion to get an improved location estimate.

Mixed fusion is more flexible than measurement fusion and estimate fusion, and it can make use of all the available information to the greatest extent.

6) Distributed localization with retained performance: This is a benefit from estimate fusion and mixed fusion.

TABLE I
PROS AND CONS OF THE THREE FUSION SCHEMES

	Mea fusion	Est fusion	Mix fusion
Lowered MSE	✓	✓	✓
Improved resilience to bad geometries	✓	✓	✓
Enhanced availability	✓		
Closed form expression		✓	
Maximum data usage			✓
Cooperation with privacy		✓	✓
Distributed fusion		✓	✓

As has been shown, for uncorrelated measurement vectors and large enough N_i's, estimate fusion and mixed fusion can make the localization process distributed while retain the same performance as the centralized measurement fusion. Distributed estimate fusion or mixed fusion does not require a centralized fusion center and there is no need to wait until all the available data are transmitted and collected at the fusion center as measurement fusion. Given local estimates with associated covariance matrices, it is easier to compare and decide which ones are more suitable for fusion. In addition, for distributed fusion, the payload in the transmitted signals and the computational load in each fusion node are lowered compared with centralized fusion.

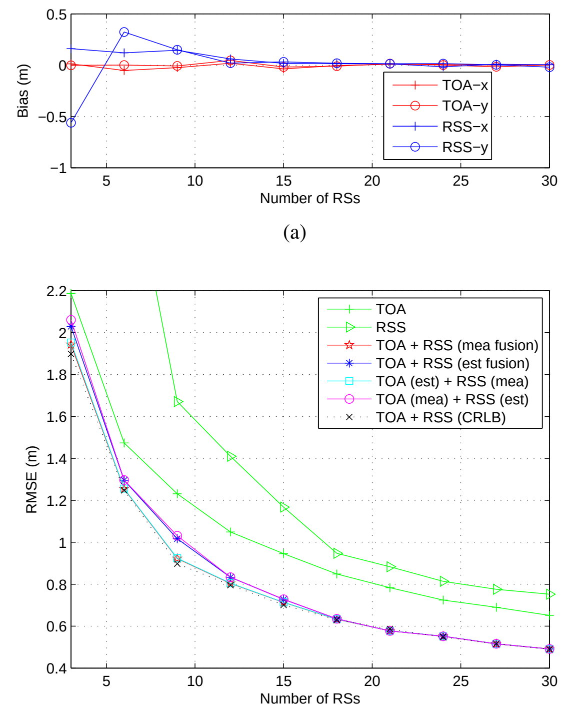
On the other hand, for estimate fusion and mixed fusion, it requires to determine the covariance matrix associated with an estimate, which is not an easy task for an iterative estimator. For a nonlinear WLSE, the covariance matrix can be approximately determined by (7) at a point that is close to the true value of $\boldsymbol{\theta}$ or even at $\hat{\boldsymbol{\theta}}$. For closed-form estimator, it is relatively easy to determine the covariance matrix. For example, the linearized least square estimator [23], the non-parametric estimator [24] and the Unscented Kalman Filter [25] (for parameter estimation). However, there may exist performance degradation since these estimators may not be optimal.

B. Pros and Cons of the Three Fusion Schemes

Based on precious discussions, we summarize the pros and cons of the three fusion schemes in Table I.

V. NUMERICAL CASE STUDIES

This section compares the performance of the three fusion schemes through numerical case studies. The TOA and RSS based methods are chosen for fusion. We denote the TOA measurement vector as $\mathbf{z}_\mathsf{t}$ (a column vector by stacking all the scalar TOA measurements) and the RSS measurement vector as $\mathbf{z}_\mathsf{r}$. The overall measurement vector $\mathbf{z}$ is constructed as $\mathbf{z} = [\mathbf{z}_\mathsf{t}^T \mathbf{z}_\mathsf{r}^T]$. The associated covariance matrix of $\mathbf{z}$ is given by $\mathbf{C}_\mathsf{n} = [\mathbf{C}_\mathsf{t}\ \mathbf{C}_\mathsf{c}; \mathbf{C}_\mathsf{c}^T\ \mathbf{C}_\mathsf{r}]$, where $\mathbf{C}_\mathsf{c}$ denotes the cross correlation matrix between $\mathbf{z}_\mathsf{t}$ and $\mathbf{z}_\mathsf{r}$. The models presented in Section II-B are used. We set $\mathbf{C}_\mathsf{t} = \sigma_\mathsf{t}^2\mathbf{I}$, $c\sigma_\mathsf{t} = 1.8$ m, $\mathbf{C}_\mathsf{r} = \sigma_\mathsf{r}^2\mathbf{I}$, $\sigma_\mathsf{r} = 2.1$ dB and $\gamma = 3$. N RSs are assumed to be evenly located on a circle centered at $(0, 0)$ with radius $rad = 20$ (the unit is meter and the same below). The location of the ith RS is given by $a_i = rad\cos\frac{2\pi(i-1)}{N}$, $b_i = rad\sin\frac{2\pi(i-1)}{N}$. For ease of illustration, the MS is chosen to locate at $(12, 10)$ which represents a typical scenario where the MS is close to several RSs while far away from the others. For each N, the simulations are done via 2000 independent runs. All the estimations (local estimates and data fusion) are done via the WLSE.



(a)

(b)

Fig. 4. (a) Biases of the TOA and RSS based methods versus the number of the RSs. (b) RMSE versus the number of the RSs.

A. Uncorrelated Raw Measurement Vectors

First, we examine the case of uncorrelated raw measurement vectors, i.e., $\mathbf{C}_\mathsf{c} = \mathbf{0}$. The covariance matrices needed for estimate fusion and mixed fusion are calculated using (7) at the true MS location. In practice, this should be calculated at a point which is assumed to be close to the true MS location.

The biases of the local estimates of the TOA and RSS based methods versus the number of the RSs are plotted in Fig. 4 (a). When the number of the RSs N is small, the local estimate using the RSS based method is obviously biased. However, it turns to be unbiased when $N \geq 13$. For the TOA based local estimate, it is unbiased no matter how large N is. Fig. 4 (b) shows the root MSE (RMSE) versus N for various methods. In addition, the theoretical RMSEs (denoted as CRLBs since the TOA and RSS measurement errors are modeled as Gaussian) are also plotted as benchmarks. As can be observed, measurement fusion and mixed fusion with the TOA based local estimate plus the RSS measurements are the best. They two superpose and attain the theoretical RMSE. However, estimate fusion and mixed fusion with the RSS based local estimate plus the TOA measurements show evident performance degradation for small N. As N becomes large

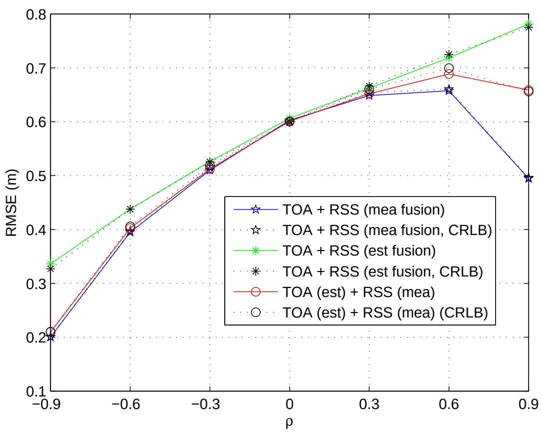


Fig. 5. RMSE versus ρ.

enough, say, $N \geq 13$, they perform close to the previous two schemes and eventually attain the theoretical RMSE.

It is interesting to note that when $N \geq 13$ the RSS based local estimate becomes almost unbiased, and the two fusion schemes using this local estimate also attain the theoretical RMSE. Therefore, the biasedness of a local estimate highly impact the performance of a fusion scheme which utilizes this biased local estimate. Only when the local estimates are unbiased and the associated covariance matrices are correctly determined, estimate fusion and mixed fusion will achieve the same performance as measurement fusion.

B. Correlated Raw Measurement Vectors

Now we examine the case of correlated raw measurement vectors. We set $\mathbf{C}_{\mathsf{c}} = \rho\sigma_{\mathsf{t}}\sigma_{\mathsf{r}}\mathbf{I}$. The covariance matrices needed in fusion are determined by the methods given in the Appendix. The RMSE for three fusion schemes versus ρ is plotted in Fig. 5 where we set $N = 20$ in order for the RSS based local estimate to be unbiased. As can be observed, in terms of the RMSE, measurement fusion performs the best, followed by mixed fusion (TOA based local estimate plus RSS measurements) and estimate fusion is the worst. If $\rho = 0$, i.e., $\mathbf{z}_{\mathsf{t}}$ and $\mathbf{z}_{\mathsf{r}}$ are uncorrelated, then the three fusion schemes achieve the same RMSE. In addition, if $|\rho|$ is small, then performing estimate fusion or mixed fusion will induce only marginal performance degradation compared with measurement fusion. Another observation is that, for given $\mathbf{C}_{\mathsf{t}}$ and $\mathbf{C}_{\mathsf{r}}$, correlation between $\mathbf{z}_{\mathsf{t}}$ and $\mathbf{z}_{\mathsf{r}}$ may enhance the localization performance.

VI. Conclusion

We define three types of fusion schemes: measurement fusion, estimate fusion and mixed fusion, and present a data fusion framework for wireless localization via the WLSE in this paper. Theoretical performance comparison among these schemes in terms of the estimation error covariance matrix is also conducted. We show that, generally, measurement fusion achieves the best performance, followed by mixed fusion and estimate fusion is the worst. However, if the measurement error vectors are uncorrelated, and all the estimates (local estimates and data fusions) are done via the WLSE, then for large enough number of data, these three schemes can achieve the same performance. As for the benefits earned from data fusion, all the three fusion schemes can result in lowered estimation error and improved resilience to bad geometric layouts. Measurement fusion can achieve enhanced localization availability and mixed fusion can improve the data usage to the greatest extend. In addition, estimate fusion and mixed fusion can lead to easily cooperation among different localization systems with privacy and distributed implementation with retained performance.

Appendix
Proof of Proposition 3

Assume $\boldsymbol{\theta} \in \mathbb{R}^{p\times 1}$, $\mathbf{z}_i, \mathbf{n}_i \in \mathbb{R}^{N_i\times 1}$ and $\mathbf{H}_i \in \mathbb{R}^{N_i\times p}$. Denote the covariance matrix of $\mathbf{n}_i$ as $\mathbf{C}_{\mathsf{n},i}$ and the cross-covariance matrix between $\mathbf{n}_1$ and $\mathbf{n}_2$ as $\mathbf{C}_{\mathsf{n},12}$ ($\mathbf{C}_{\mathsf{n},12} \neq \mathbf{0}$).

For measurement fusion, the input data

$$\mathbf{z}_{\text{mea}} = \begin{bmatrix}\mathbf{z}_1\\ \mathbf{z}_2\end{bmatrix} = \begin{bmatrix}\mathbf{H}_1\\ \mathbf{H}_2\end{bmatrix}\boldsymbol{\theta} + \begin{bmatrix}\mathbf{n}_1\\ \mathbf{n}_2\end{bmatrix} \triangleq \mathbf{H}\boldsymbol{\theta} + \mathbf{n}.$$

The covariance matrix of $\mathbf{n}$ (also $\mathbf{z}_{\text{mea}}$) is $\mathbf{C}_{\mathsf{n}} = [\mathbf{C}_{\mathsf{n},1}\ \mathbf{C}_{\mathsf{n},12}; \mathbf{C}_{\mathsf{n},12}^T\ \mathbf{C}_{\mathsf{n},2}]$.

The covariance matrix $\mathbf{C}_{\text{mea}}$ of measurement fusion based on $\mathbf{z}_{\text{mea}}$ via the WLSE is given by

$$\mathbf{C}_{\text{mea}} = (\mathbf{H}^T\mathbf{C}_{\mathsf{n}}^{-1}\mathbf{H})^{-1}. \tag{29}$$

Since $\mathbf{C}_{\mathsf{n},12} \neq \mathbf{0}$, $\mathbf{C}_{\mathsf{n}}^{-1}$ is not block diagonal.

The local estimates via the WLSE are given by

$$\hat{\boldsymbol{\theta}}_i = (\mathbf{H}_i^T\mathbf{C}_{\mathsf{n},i}^{-1}\mathbf{H}_i)^{-1}\mathbf{H}_i^T\mathbf{C}_{\mathsf{n},i}^{-1}\mathbf{z}_i,\ i = 1, 2$$

with covariance matrices $\mathbf{C}_{\mathsf{m},i} = (\mathbf{H}_i^T\mathbf{C}_{\mathsf{n},i}^{-1}\mathbf{H}_i)^{-1},\ i = 1, 2.$

For estimate fusion, the input data

$$\mathbf{z}_{\text{est}} = \begin{bmatrix}\hat{\boldsymbol{\theta}}_1\\ \hat{\boldsymbol{\theta}}_2\end{bmatrix} = \begin{bmatrix}\mathbf{I}_p\\ \mathbf{I}_p\end{bmatrix}\boldsymbol{\theta} + \begin{bmatrix}\mathbf{m}_1\\ \mathbf{m}_2\end{bmatrix} \triangleq \mathbf{G}\boldsymbol{\theta} + \mathbf{m}.$$

Defining $\mathbf{A}_i = (\mathbf{H}_i^T\mathbf{C}_{\mathsf{n},i}^{-1}\mathbf{H}_i)^{-1}\mathbf{H}_i^T\mathbf{C}_{\mathsf{n},i}^{-1}$ ($i = 1, 2$), then $\hat{\boldsymbol{\theta}}_i = \mathbf{A}_i\mathbf{z}_i$ and

$$\mathbf{z}_{\text{est}} = \begin{bmatrix}\hat{\boldsymbol{\theta}}_1\\ \hat{\boldsymbol{\theta}}_2\end{bmatrix} = \begin{bmatrix}\mathbf{A}_1\mathbf{z}_1\\ \mathbf{A}_2\mathbf{z}_2\end{bmatrix} = \begin{bmatrix}\mathbf{A}_1 & \mathbf{0}\\ \mathbf{0} & \mathbf{A}_2\end{bmatrix}\begin{bmatrix}\mathbf{z}_1\\ \mathbf{z}_2\end{bmatrix} \triangleq \mathbf{F}_{\text{est}}\mathbf{z}_{\text{mea}}.$$

The covariance matrix of $\mathbf{m}$ (also $\mathbf{z}_{\text{est}}$) is thus

$$\mathbf{C}_{\mathsf{m}} = \mathbf{F}_{\text{est}}\mathbf{C}_{\mathsf{n}}\mathbf{F}_{\text{est}}^T. \tag{30}$$

From (30), it is easy to verify that $[\mathbf{C}_{\mathsf{m}}]_{i,i} = \mathbf{A}_i\mathbf{C}_{\mathsf{n}i}\mathbf{A}_i^T = (\mathbf{H}_i^T\mathbf{C}_{\mathsf{n}i}^{-1}\mathbf{H}_i)^{-1} = \mathbf{C}_{\mathsf{m},i}$. In addition, the cross-covariance matrix between $\mathbf{m}_1$ and $\mathbf{m}_2$ is $[\mathbf{C}_{\mathsf{m}}]_{1,2} = \mathbf{A}_1\mathbf{C}_{\mathsf{n},12}\mathbf{A}_2^T \neq \mathbf{0}$.

The covariance matrix $\mathbf{C}_{\text{est}}$ of estimate fusion based on $\mathbf{z}_{\text{est}}$ via the WLSE is given by

$$\mathbf{C}_{\text{est}} = (\mathbf{G}^T\mathbf{C}_{\mathsf{m}}^{-1}\mathbf{G})^{-1}. \tag{31}$$

For mixed fusion, the input data

$$\mathbf{z}_{\text{mix}} = \begin{bmatrix}\mathbf{z}_1\\ \hat{\boldsymbol{\theta}}_2\end{bmatrix} = \begin{bmatrix}\mathbf{H}_1\\ \mathbf{I}_p\end{bmatrix}\boldsymbol{\theta} + \begin{bmatrix}\mathbf{n}_1\\ \mathbf{m}_2\end{bmatrix} \triangleq \mathbf{K}\boldsymbol{\theta} + \mathbf{q}.$$

$\mathbf{z}_{\text{mix}}$ can be expressed alternatively as

$$\mathbf{z}_{\text{mix}} = \begin{bmatrix} \mathbf{z}_1 \\ \hat{\boldsymbol{\theta}}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{I}_{N_1}\mathbf{z}_1 \\ \mathbf{A}_2\mathbf{z}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{I}_{N_1} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_2 \end{bmatrix} \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \end{bmatrix} \triangleq \mathbf{F}_{\text{mix}}\mathbf{z}_{\text{mea}}.$$

The covariance matrix of $\mathbf{q}$ (also $\mathbf{z}_{\text{mix}}$) is thus

$$\mathbf{C}_{\mathsf{q}} = \mathbf{F}_{\text{mix}}\mathbf{C}_{\mathsf{n}}\mathbf{F}_{\text{mix}}^T. \quad (32)$$

The covariance matrix $\mathbf{C}_{\text{mix}}$ of mixed fusion based on $\mathbf{z}_{\text{mix}}$ via the WLSE is given by

$$\mathbf{C}_{\text{mix}} = (\mathbf{K}^T\mathbf{C}_{\mathsf{q}}^{-1}\mathbf{K})^{-1}. \quad (33)$$

1. $\mathbf{C}_{\text{mea}}$ *and* $\mathbf{C}_{\text{mix}}$

Since $\mathbf{A}_i\mathbf{H}_i = \mathbf{I}_p$, we can express $\mathbf{K}$ as

$$\mathbf{K} = \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{I}_p \end{bmatrix} = \begin{bmatrix} \mathbf{I}_{N_1}\mathbf{H}_1 \\ \mathbf{A}_2\mathbf{H}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{I}_{N_1} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_2 \end{bmatrix} \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \end{bmatrix} = \mathbf{F}_{\text{mix}}\mathbf{H}. \quad (34)$$

Combing (32), (33) and (34), we have

$$\mathbf{C}_{\text{mix}}^{-1} = \mathbf{H}^T\mathbf{F}_{\text{mix}}^T(\mathbf{F}_{\text{mix}}\mathbf{C}_{\mathsf{n}}\mathbf{F}_{\text{mix}}^T)^{-1}\mathbf{F}_{\text{mix}}\mathbf{H}. \quad (35)$$

Defining $\mathbf{B}_{\text{mix}} = \mathbf{C}_{\mathsf{n}}^{\frac{1}{2}}\mathbf{F}_{\text{mix}}^T(\mathbf{F}_{\text{mix}}\mathbf{C}_{\mathsf{n}}\mathbf{F}_{\text{mix}}^T)^{-1}\mathbf{F}_{\text{mix}}\mathbf{C}_{\mathsf{n}}^{\frac{1}{2}}$, we can then express $\mathbf{C}_{\text{mix}}^{-1}$ as $\mathbf{C}_{\text{mix}}^{-1} = \mathbf{H}^T\mathbf{C}_{\mathsf{n}}^{-\frac{1}{2}}\mathbf{B}_{\text{mix}}\mathbf{C}_{\mathsf{n}}^{-\frac{1}{2}}\mathbf{H}$.

Since $\mathbf{C}_{\text{mea}}^{-1} = \mathbf{H}^T\mathbf{C}_{\mathsf{n}}^{-1}\mathbf{H} = \mathbf{H}^T\mathbf{C}_{\mathsf{n}}^{-\frac{1}{2}}\mathbf{I}\mathbf{C}_{\mathsf{n}}^{-\frac{1}{2}}\mathbf{H}$, to prove $\mathbf{C}_{\text{mea}}^{-1} \succeq \mathbf{C}_{\text{mix}}^{-1}$, we only need to prove $\mathbf{I} - \mathbf{B}_{\text{mix}} \succeq \mathbf{0}$.

It is interesting to note that $\mathbf{B}_{\text{mix}}$ is symmetric and $(\mathbf{I} - \mathbf{B}_{\text{mix}})(\mathbf{I} - \mathbf{B}_{\text{mix}})^T = (\mathbf{I} - \mathbf{B}_{\text{mix}})(\mathbf{I} - \mathbf{B}_{\text{mix}}) = \mathbf{I} - \mathbf{B}_{\text{mix}}$. Thus, $\mathbf{I} - \mathbf{B}_{\text{mix}}$ is idempotent and positive semidefinite (i.e., $\mathbf{I} - \mathbf{B}_{\text{mix}} \succeq \mathbf{0}$).

We then have

$$\begin{aligned} & \mathbf{I} - \mathbf{B}_{\text{mix}} \succeq \mathbf{0} \\ \Rightarrow & \mathbf{H}^T\mathbf{C}_{\mathsf{n}}^{-\frac{1}{2}}(\mathbf{I} - \mathbf{B}_{\text{mix}})\mathbf{C}_{\mathsf{n}}^{-\frac{1}{2}}\mathbf{H} \succeq \mathbf{0} \\ \Rightarrow & \mathbf{H}^T\mathbf{C}_{\mathsf{n}}^{-\frac{1}{2}}\mathbf{C}_{\mathsf{n}}^{-\frac{1}{2}}\mathbf{H} \succeq \mathbf{H}^T\mathbf{C}_{\mathsf{n}}^{-\frac{1}{2}}\mathbf{B}_{\text{mix}}\mathbf{C}_{\mathsf{n}}^{-\frac{1}{2}}\mathbf{H} \\ \Rightarrow & \mathbf{C}_{\text{mea}}^{-1} \succeq \mathbf{C}_{\text{mix}}^{-1} \Rightarrow \mathbf{C}_{\text{mea}} \preceq \mathbf{C}_{\text{mix}}. \end{aligned} \quad (36)$$

If $\mathbf{H}_2 \in \mathbb{R}^{N_2 \times p}$ is invertible (i.e., $N_2 = p$), then $\mathbf{A}_2$ and $\mathbf{F}_{\text{mix}}$ are all invertible as well. Therefore, $\mathbf{B}_{\text{mix}} = \mathbf{I}$, and $\mathbf{I} - \mathbf{B}_{\text{mix}} = \mathbf{0}$. Then $\mathbf{C}_{\text{mea}} = \mathbf{C}_{\text{mix}}$. Otherwise, $\mathbf{I} - \mathbf{B}_{\text{mix}} \succeq \mathbf{0}$ and $\mathbf{C}_{\text{mea}} \preceq \mathbf{C}_{\text{mix}}$ but $\mathbf{C}_{\text{mea}} \neq \mathbf{C}_{\text{mix}}$ (i.e., the eigenvalues of $\mathbf{C}_{\text{mix}} - \mathbf{C}_{\text{mea}}$ are all nonnegative and at least one eigenvalue is positive).

2. $\mathbf{C}_{\text{mix}}$ *and* $\mathbf{C}_{\text{est}}$

Similarly, we can express $\mathbf{G}$ as

$$\mathbf{G} = \begin{bmatrix} \mathbf{I}_p \\ \mathbf{I}_p \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1\mathbf{H}_1 \\ \mathbf{A}_2\mathbf{H}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_2 \end{bmatrix} \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \end{bmatrix} = \mathbf{F}_{\text{est}}\mathbf{H}. \quad (37)$$

As has been shown in the previous part, it is straightforward to conclude that $\mathbf{C}_{\text{mea}} \preceq \mathbf{C}_{\text{est}}$ and the equality holds only if $\mathbf{H}_1$ and $\mathbf{H}_2$ are all invertible.

But what is the relationship between $\mathbf{C}_{\text{mix}}$ and $\mathbf{C}_{\text{est}}$?

$\mathbf{F}_{\text{est}}$ can be expressed as

$$\mathbf{F}_{\text{est}} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{N_2} \end{bmatrix} \begin{bmatrix} \mathbf{I}_{N_1} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_2 \end{bmatrix} \triangleq \mathbf{T}\mathbf{F}_{\text{mix}} \quad (38)$$

Combing (30), (31), (37) and (38), we have

$$\mathbf{C}_{\text{est}}^{-1} = \mathbf{H}^T\mathbf{F}_{\text{mix}}^T\mathbf{T}^T(\mathbf{T}\mathbf{F}_{\text{mix}}\mathbf{C}_{\mathsf{n}}\mathbf{F}_{\text{mix}}^T\mathbf{T}^T)^{-1}\mathbf{T}\mathbf{F}_{\text{mix}}\mathbf{H}. \quad (39)$$

Defining $\mathbf{B}_{\text{est}} = \mathbf{C}_{\mathsf{q}}^{\frac{1}{2}}\mathbf{T}^T(\mathbf{T}\mathbf{C}_{\mathsf{q}}\mathbf{T}^T)^{-1}\mathbf{T}\mathbf{C}_{\mathsf{q}}^{\frac{1}{2}}$, we can then express $\mathbf{C}_{\text{est}}^{-1}$ as $\mathbf{C}_{\text{est}}^{-1} = \mathbf{H}^T\mathbf{F}_{\text{mix}}^T\mathbf{C}_{\mathsf{q}}^{-\frac{1}{2}}\mathbf{B}_{\text{est}}\mathbf{C}_{\mathsf{q}}^{-\frac{1}{2}}\mathbf{F}_{\text{mix}}\mathbf{H}$, where $\mathbf{C}_{\mathsf{q}}$ is given in (32). Similarly, $\mathbf{I} - \mathbf{B}_{\text{est}}$ is idempotent and positive semidefinite and finally, we have $\mathbf{C}_{\text{mix}} \preceq \mathbf{C}_{\text{est}}$. If $\mathbf{H}_1 \in \mathbb{R}^{N_1 \times p}$ is invertible (i.e., $N_1 = p$), then $\mathbf{C}_{\text{mix}} = \mathbf{C}_{\text{est}}$.

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