Power Control in Cellular Networks: Taxonomy and Recent Results

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RAWNET
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Overview

Three major types of resource constraints:

- Congestion: $x+y\leq 1$ (Distributed gradient and variants)
- Collision: $x+y\leq 1,\ \ x,y\in\{0,1\}$ (Max. weight matching and approx.)
- Interference: $\frac{x}{y}\geq 1$ (Fixed point update and variants)

$$\gamma_1=\frac{p_1}{hp_2+\eta}$$

15 years (at least) of research, tremendous practical impact, still intellectually challenging

Outline

- Part I: Taxonomy
- Part II: 3 Recent Results

Acknowledgement:

- Prashanth Hande, Tian Lan, Chee Wei Tan
- Maryam Fazel, Dennice Gayme, Farhad Meshkati, Dani Palomar, Sundeep Rangan
- Qualcomm

Part I: Taxonomy

Not Covered

Other uses of power control:

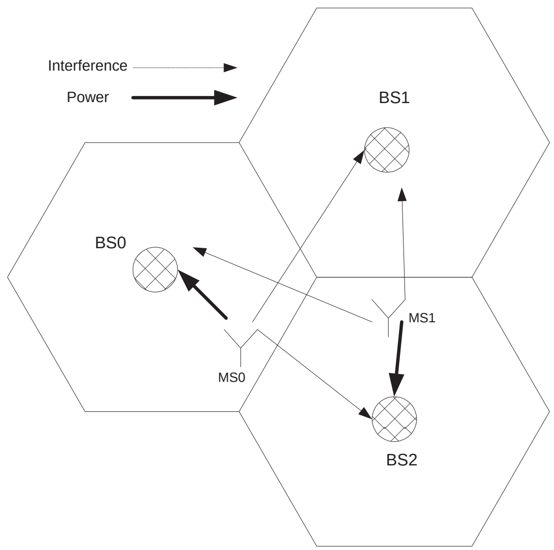
- Channel estimation
- Connectivity management

Other problem formulations:

- Ad hoc network
- Capacity region
- Stochastic stability

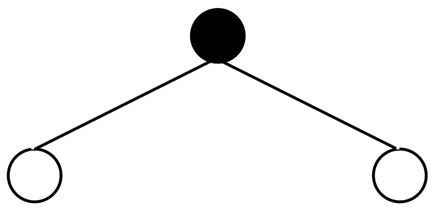
Apology for any missing references

Multi-cellular Wireless Networks



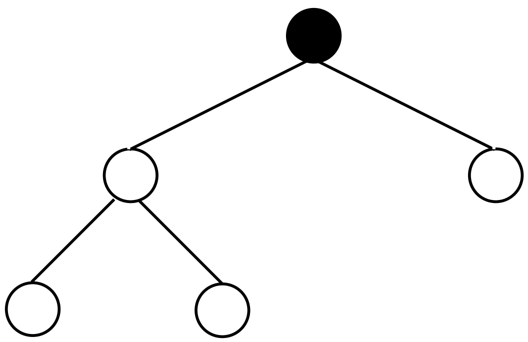
$$\gamma_i(\mathbf{p}) = \frac{p_i h_{ii}}{\sum_{j \neq i} p_j h_{ij} + \eta_i}$$

Problem Tree I: Stationary or Opportunistic



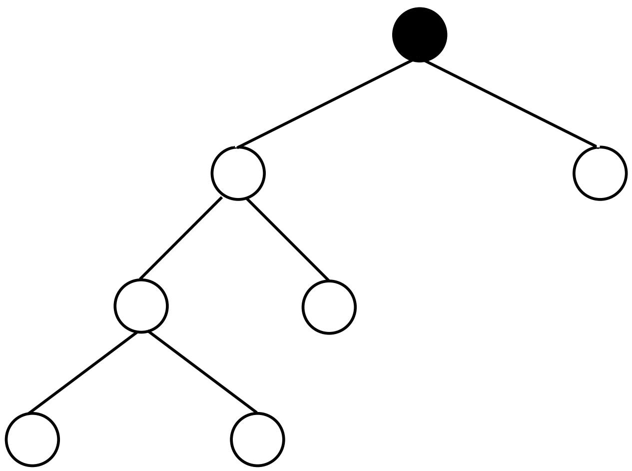
- Stationary: Channel gains are constants in power control algorithm's timescale
- Opportunistic: Time-varying channel gains (due to fast mobility or fading)

Problem Tree II: Cooperative or Non-cooperative



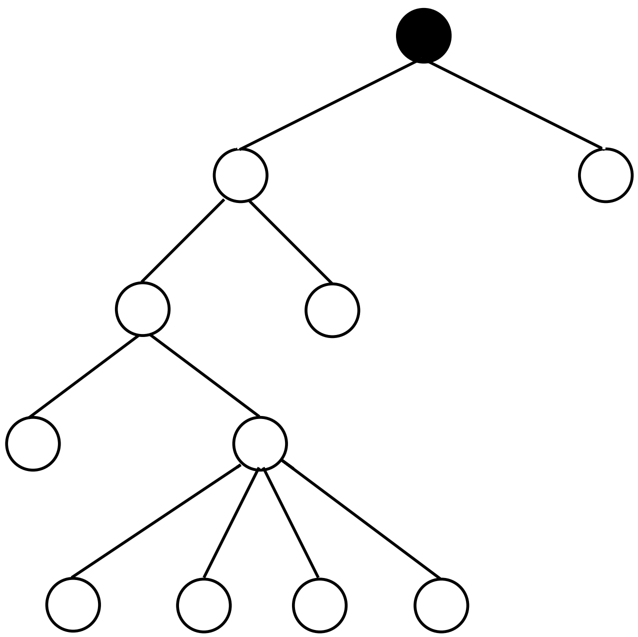
- Cooperative: Optimization theoretic formulations, maximizing a system-wide objective function over feasibility, QoS constraints, and resource constraints
- Non-cooperative: Game theoretic formulations, each user maximizes its selfish utility subject to local constraints

Problem Tree III: Fast Timescale or Slow Timescale



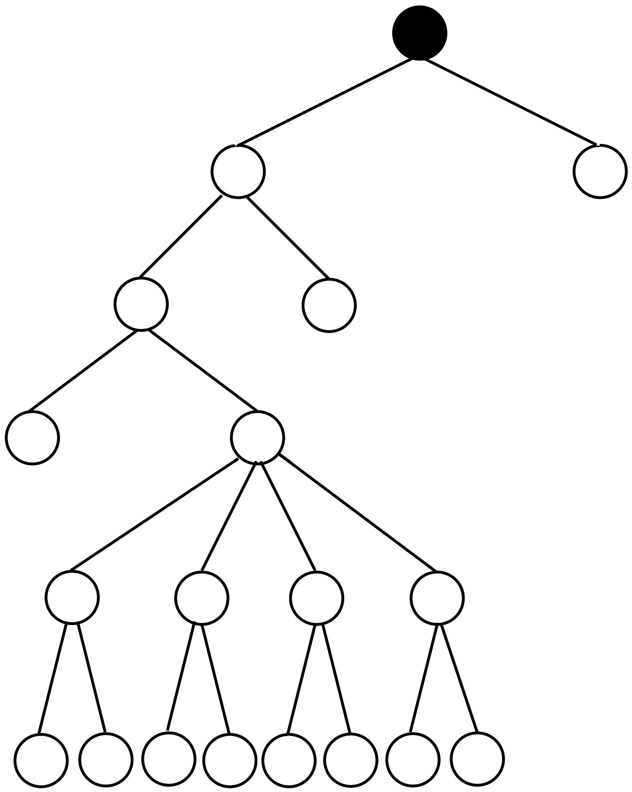
- Fast Timescale: Optimize over powers for fixed SIR targets
- Slow Timescale: Jointly optimize over powers and target SIRs

Problem Tree IV: PC Only or Joint Control



- PC only: Transmit power is only the degree of freedom
- Joint control: Can jointly control other degrees of freedom: multiple-antenna, spectral (bandwidth allocation), spatial (base-station assignment), temporal (scheduling)

Problem Tree V: Uplink or Downlink



- Uplink: from Mobile Stations (MS) to Base Station (BS), multi-cellular interference. Often more difficult
- Downlink: from BS to MS, total power budget. More difficult for beamforming problems, uplink-downlink duality

More On Problem Tree I: Definition of Optimality

In general: problem formulations are indexed by time:

- Allow convexification of the underlying rate region by silencing some users during some time slots
- Converges to a limit cycle rather than a point
- Includes scheduling problem as a special case

A special case considered in almost all PC papers: problem formulation is time-invariant

- No user is silenced at the equilibrium

More On Problem Tree II: Equilibrium or Transience

Questions about equilibrium:

- Convergence
- Properties of equilibrium: Nash equilibrium, local optimum, global optimum

Questions about transience:

- Invariance
- Properties of transience: Rate of convergence

More On Problem Tree III: Definition of Functional Dependencies

Objective function:

• $\sum_i U_i$: utility function that can depend on throughput, delay, jitter, energy

Efficiency

Elasticity

User satisfaction

Fairness

• $\sum_i C_i$: cost function of power that can depend on all degrees of freedom, including power

More On Problem Tree III: Definition of Functional Dependencies

Throughput dependency on SIR:

- Capacity formula: $\log(1 + K\gamma)$
- High SIR: $\log(K\gamma)$
- Low SIR: $K\gamma$
- Reliability function: $Rf(\gamma)$
- More complicated formula for multi-user detector and multi-carrier

More On Problem Tree III: Definition of Functional Dependencies

Other degrees of freedom:

- Beamforming: h_{ij} becomes $\mathbf{w}_i^T \mathbf{h}_{ij}$
- Bandwidth allocation: $\log(1+\gamma_i)$ becomes $b_i \log(1+\gamma_i \frac{B}{b_i})$ with $\sum_i b_i = B$
- Base station assignment: G_{ii} becomes $G_{i\sigma_i}$
- Scheduling: $\frac{p_i}{\sum_{j \neq i} p_j + \eta_i}$ becomes $\frac{\theta_i p_i}{\sum_{j \neq i} \theta_j p_j + \eta_i}$, with $\theta_i \in \{0,1\}$

Structures I: Convexity

Constraint set:

• Feasibility set: convex or log-convex (Boche et al, Wong et al)

• QoS requirements: convex after a log change of variable in high SIR regime (Chiang et al)

• Resource constraints: usually affine

Objective function:

• α-fair utility functions $U(x) = x^{1-\alpha}/(1-\alpha)$: concave for all $\alpha \geq 0$, concave after log change of variable for $\alpha \geq 1$

• Convex increasing cost functions

Structures II: Decomposability

Can global optimization be solved distributedly?

Can selfish interactions lead to social welfare maximization?

- Centralized solution: BS collects all information, does all the computation, then broadcasts the solutions.

Often Mobile Switching Center needs to coordinate across multiple BSs

- Distributed solution with explicit feedback: limited message passing between a BS and its MSs, no MSC coordination

- Fully distributed solution with only implicit feedback: no message passing at all, only measures physically meaningful local quantities

Part II: Some Recent Results

References

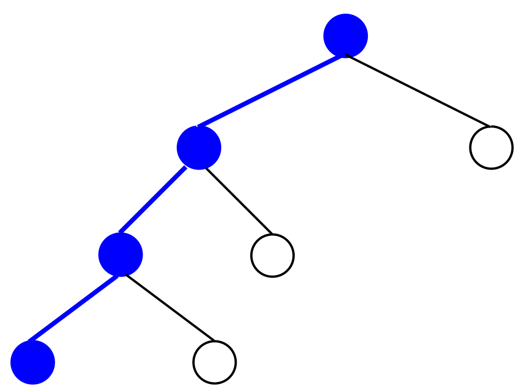
- C. W. Tan, D. Palomar, and M. Chiang, "Exploiting hidden convexity for flexible and robust resource allocation in cellular networks", *Proc. IEEE INFOCOM*, May 2007.

- P. Hande, S. Rangan, M. Chiang, and X. Wu, "Distributed uplink power control for optimal SIR assignment in cellular data networks", *IEEE/ACM Transactions on Networking*, 2008.

- F. Meshkati, M. Chiang, H. V. Poor, and S. Schwartz, "A game-theoretic approach to energy-efficient power control in multi-carrier CDMA systems", *IEEE Journal of Selected Areas in Communications*, vol. 24, no. 6, pp. 1115-1129, June 2006.

Part II.A

Transience: Invariance and Robustness



Foschini Miljanic Distributed Power Control

- Simplest power control solving near-far problem:

One-shot receive power equalization by BS control

- 1992-1993: Zander, Foschini, Mitra:

Iterative distributed power control (DPC), at iteration k:

$$p_l(k+1) = \frac{\gamma_l}{r_l(k)} p_l(k), \ \ \forall l$$

γ_l: target SIR $\quad$ r_l: measured SIR

Linear fixed point equation, Perron-Frobenius theory

$$\mathbf{p}(k+1) = \mathbf{D}(\boldsymbol{\gamma})\mathbf{G}\mathbf{p}(k) + \mathbf{D}(\boldsymbol{\gamma})\text{Diag}(1/h_{ii})\boldsymbol{\eta}$$

$$G_{ii} = 0, \ \ G_{ij} = h_{ij}/h_{ii}, \ \ D_{ii} = \gamma_i$$

Convergence for fixed, feasible $\boldsymbol{\gamma}$: $\rho(\mathbf{D}(\boldsymbol{\gamma})G) < 1$

Difficult Issues

- When is target SIR feasible? (will be answered in Part II.B)
- How to jointly optimize SIR target? (will be answered in Part II.B)
- What happens before convergence? (focus of Part II.A)

Different Levels of SIR

- Protected SIR: $\gamma(1+\epsilon)$
- Target SIR: γ
- Threshold SIR: β

Invariance (Fazel, Gayme, Chiang)

Common Ratio Condition:

A sufficient condition for $r_l(k) \geq \beta_l,\ \forall l \Rightarrow r_l(k+1) \geq \beta_l,\ \forall l$: there is a constant $\delta > 0$ such that

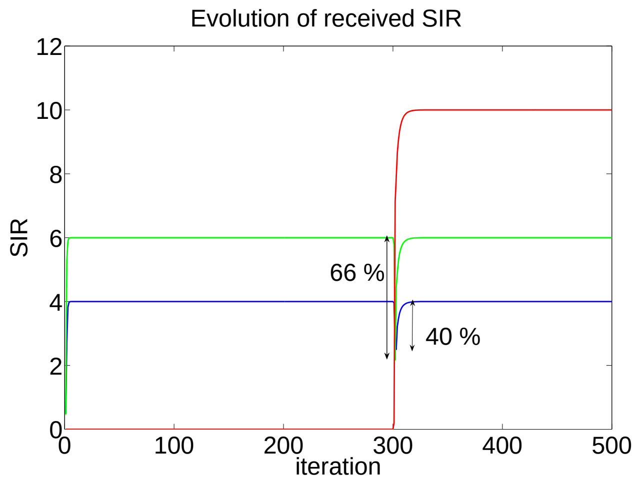
$$\frac{\gamma_l}{\beta_l} = \delta,\ \forall l$$

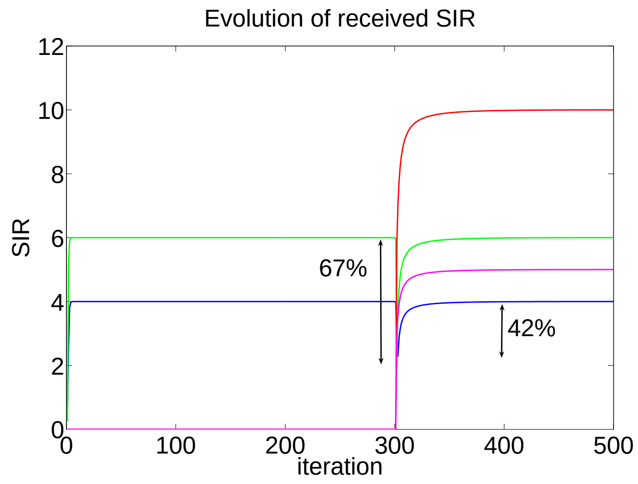
Level sets of the following Lyapunov function:

$$\begin{aligned}\mathcal{V}(\mathbf{r}(k)) &= \max_l \frac{1}{\gamma_l} |r_l(k) - \gamma_l| \\ &= \|\mathbf{D}(\boldsymbol{\gamma})^{-1}(\mathbf{r}(k) - \boldsymbol{\gamma})\|_\infty.\end{aligned}$$

More general test by Linear Programming

SIR Violation When New Users Enter





Optimizing Power Expenditure & Robustness

- $\text{SIR}_l(\mathbf{p}^\star) = \gamma_l$ for all l. Tightening or loosening constraint affects power consumption $\sum_l p_l^\star$

- Introduce protection margin to SIR thresholds:

  - $\text{SIR}_l \geq \gamma_l$ for reliable transmission

  - $\text{SIR}_l \geq (1+\epsilon)\gamma_l$ for robust protection against disturbances in network

Tradeoff between robustness and power saving

DPC/ALP Algorithm

- Distributed Power Control with Active Link Protection

  Bambos et al 2000

- Each user updates the transmitter powers $p_l(k+1)$ at the $(k+1)$th step according to the following rule:

$$p_l(k+1) = \begin{cases} \frac{(1+\epsilon)\gamma_l}{\mathsf{SIR}_l(k)} p_l(k), & \text{if } \mathsf{SIR}_l(k) \geq \gamma_l \\ (1+\epsilon) p_l(k), & \text{if } \mathsf{SIR}_l(k) < \gamma_l \end{cases}$$

- Open issue: How to tune ϵ?
- Tradeoff between admission speed for new users and amount of buffer provided

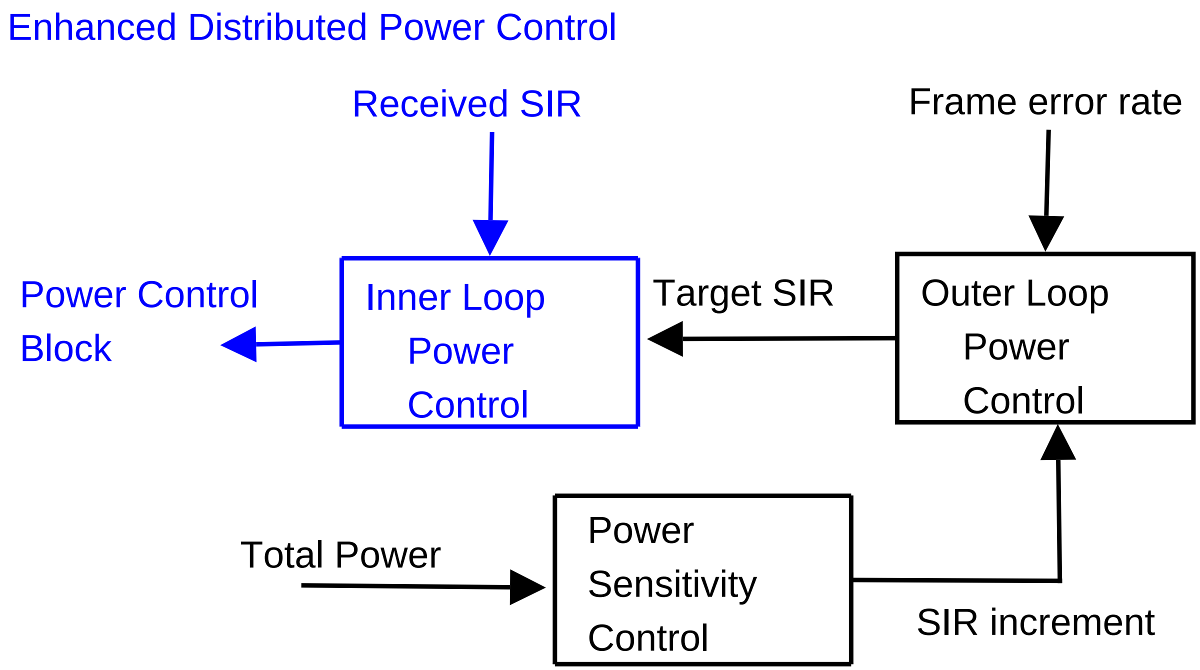
Robust Power Control Problem

- Formulation:

$$\begin{array}{ll} \text{minimize} & \sum_l p_l + \phi(\epsilon) \\ \text{subject to} & \text{SIR}_l(\mathbf{p}) \geq \gamma_l(1+\epsilon) \quad \forall l, \\ & \epsilon \geq 0, p_l \geq 0 \quad \forall l \\ \text{variables:} & p_l \, \forall l, \ \epsilon \end{array}$$

- Problem is nonconvex, but convex after log change of variables (both $\mathbf{p}$ and ϵ) provided $\frac{\partial^2 \phi(z)/\partial z^2}{\partial \phi(z)/\partial z} \geq -1/z$
- Solution: Enhanced DPC/ALP

E-DPC/ALP Block Diagram (Mobile Station)

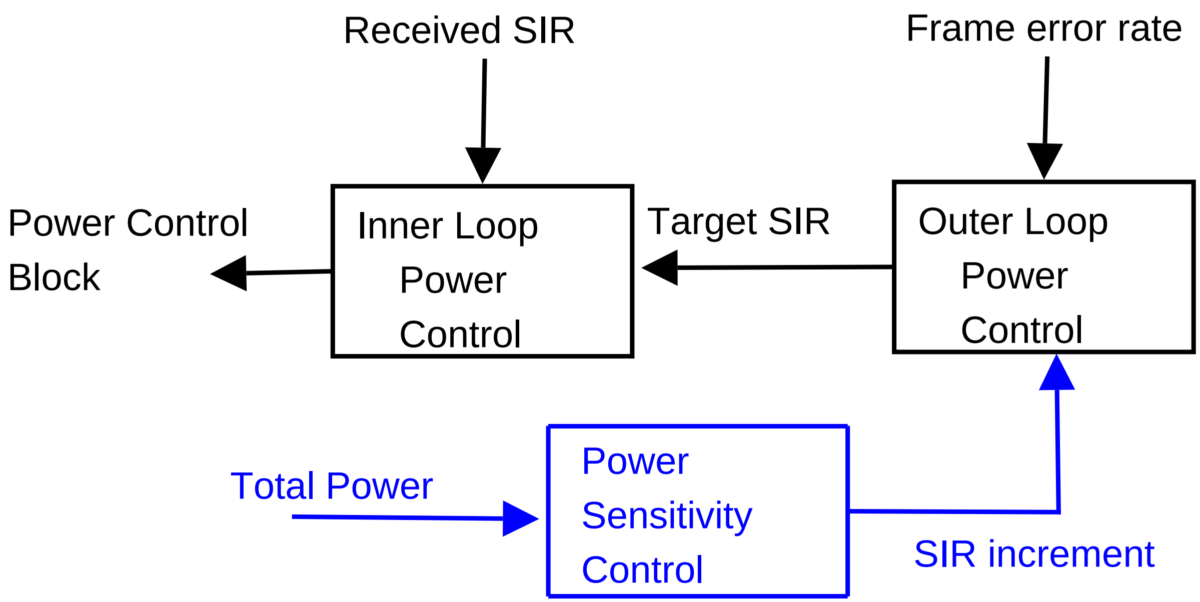


Algorithm E-DPC/ALP (Mobile Station)

- updates the transmitter powers $p_l(k+1)$ at the $(k+1)$th step according to the following rule:

$$p_l(k+1) = \begin{cases} \frac{(1+\epsilon(k))\gamma_l}{\mathsf{SIR}_l(k)} p_l(k), & \text{if } \mathsf{SIR}_l(k) \geq \gamma_l \\ (1+\epsilon(k)) p_l(k), & \text{if } \mathsf{SIR}_l(k) < \gamma_l \end{cases} \quad (1)$$

E-DPC/ALP Block Diagram (Base Station)



Enhanced Active Link Protection

Algorithm E-DPC/ALP (Base Station)

- computes $x_l(k+1)$, the lth component of $\mathbf{x}(k+1)$, using

$$\mathbf{x}(k+1) = (1+\epsilon(k))(\mathbf{DG})^T\mathbf{x}(k) + \mathbf{1}$$

- computes

$$\nu_l(k+1) = x_l(k+1)p_l(k+1) \quad \forall l$$

- updates $\epsilon(k+1)$ by solving

$$-\left.\frac{\partial\phi(\epsilon)}{\partial\epsilon}\right|_{\epsilon=\epsilon(k+1)}(1+\epsilon(k+1)) = \mathbf{1}^T\boldsymbol{\nu}(k+1)$$

Properties of E-DCP/ALP

Distributed version with BS-MS message passing

Theorem: If E-DCP/ALP converges, it converges to the globally optimal solution to Robust Power Control Problem

Theorem: A sufficient condition on spectral radius for convergence

Choice of Cost Function

- If network can tolerate at most an increase of $\delta/(\mathbf{1}^T\mathbf{p}^\star)$ percent in total power,

$$\phi(\epsilon) = \delta \log(1 + 1/\epsilon)$$

- A family of $\phi(\epsilon)$ for $\epsilon \in (0, 1]$:

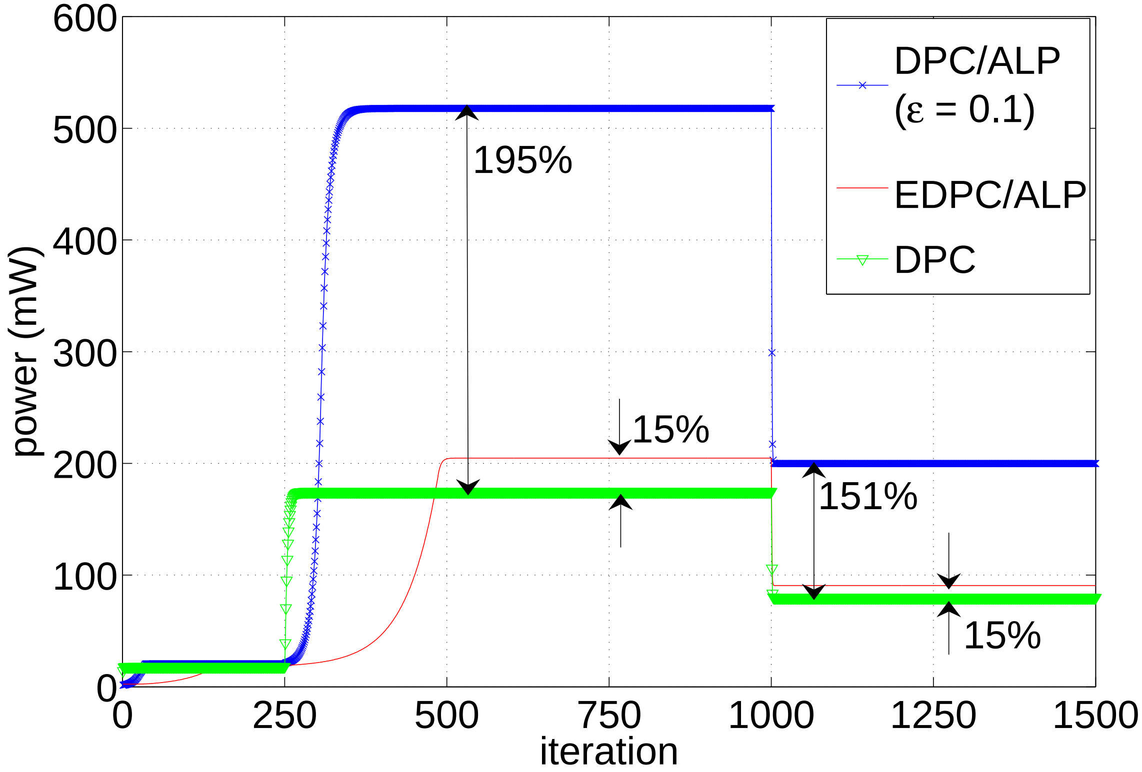
$$\phi(\epsilon) = \delta \left(\sum_{j=1}^{q} (-1)^{q-j} \epsilon^{-j}/j + \log(1 + 1/\epsilon) \right),$$

  parameterized by a nonnegative integer q to control rate of convergence

- Different $\phi(\epsilon)$ controls the exact relationship between care and congestion:

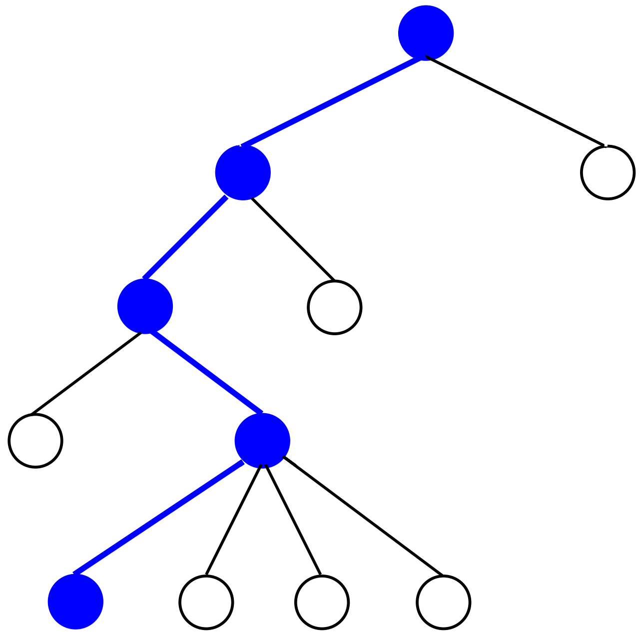
$$\epsilon(k) \;\propto\; \frac{1}{\mathbf{1}^T \boldsymbol{\nu}(k)}$$

Numerical Example



Part II.B

Joint SIR Assignment and Power Control



Power Control With Variable SIR Targets

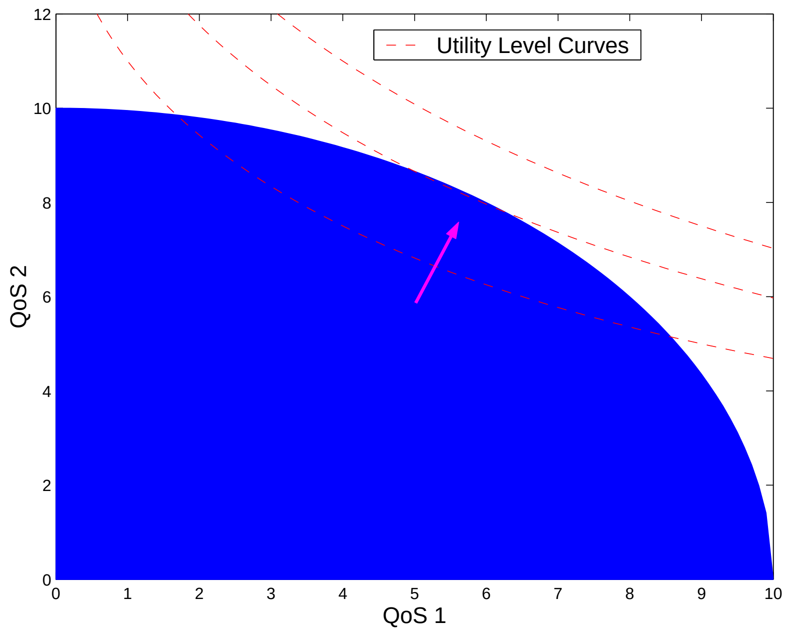
Solve the problem of distributed and jointly optimal power control and QoS assignment in multi-cellular uplink

- Difficulty: coupled feasibility constraint set
- Key idea: find the right re-parametrization

Implementation: Qualcomm Flarion Technologies Flash-OFDM Network

Uplink Power Control in Multi-cellular Networks

Maximize: utility function of powers and QoS assignments
Subject to: QoS assignments feasible
Variables: transmit powers and QoS assignments



Some Related Work

- 1989: CDMA for voice wireless networks

- Late 1980s: Qualcomm's received power equalization for near-far problem

- 1992-2000 Fixed SIR: distributed power control:

Zander 1992, Foschini Miljanic 1993, Mitra 1993, Yates 1995, Bambos Pottie 2000 ...

- Late 1990s: 3G for data wireless networks

- 2001-2004 Nash equilibrium for joint SIR assignment and power control:

Saraydar, Mandayam, Goodman 2001, 2002, Sung Wong 2002, Altman 2004 ...

- 2004-2005 Centralized computation for globally optimal joint SIR assignment and power control:

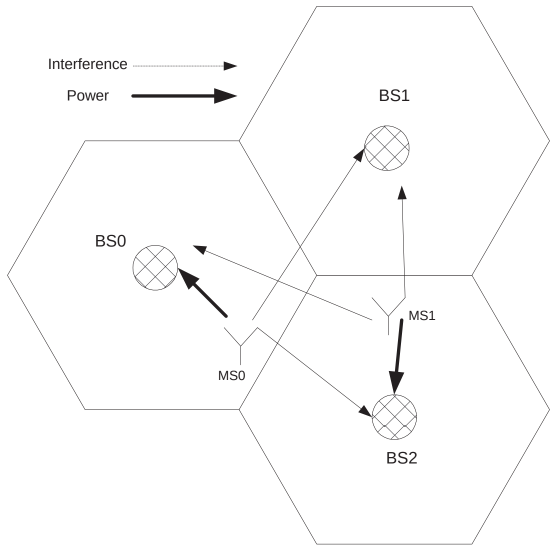
Chiang 2004, O'Neill, Julian, and Boyd 2004, Boche and Stanczak 2005

Open Issues

Distributed and optimal joint SIR assignment and power control?

- Convexity assumed
- Coupled and complicated constraint set is the difficulty

System Model



M MS and N BS

Each MS i served by a BS σ_i

Each BS k serving a set of MS: S_k

C_i: set of interference links

- Non-orthogonal system: $C_i = \{j \mid j \neq i\}$
- Orthogonal system: $C_i = \{j \mid \sigma_j \neq \sigma_i\}$

Feasible Regions

Assume $\boldsymbol{\eta} \neq 0$, feasible region:

$$\mathbf{B} = \{\boldsymbol{\gamma} \succ 0 : \rho(\mathbf{GD}(\boldsymbol{\gamma})) < 1\}$$

Finite power case: given $\rho \in [0, 1)$

$$\mathbf{B}_\rho = \{\boldsymbol{\gamma} \succ 0 \mid \rho(\mathbf{GD}(\boldsymbol{\gamma})) \leq \rho\}$$

Can extend to power or interference-constrained cases

Conditions for feasible region to be convex well-understood by now

Distributed solution to a fixed, feasible SIR target is well-known

Question: How to attain a point on the Pareto-boundary in a distributed way?

Load-Spillage Characterization

Lemma: $\boldsymbol{\gamma} \succ 0$ is feasible (and ρ-optimal) iff there exists a $\mathbf{s} \succ 0$ and $\rho \in [0, 1)$ such that

$$\mathbf{s}^T \mathbf{G} \mathbf{D}(\boldsymbol{\gamma}) = \rho \mathbf{s}^T$$

Let $\mathbf{r}(\mathbf{s}) = \mathbf{G}^T \mathbf{s}$

A new parametrization on SIR: $\boldsymbol{\gamma}(\mathbf{s}, \rho) = \rho \mathbf{s} / \mathbf{r}(\mathbf{s})$

$\mathbf{s}$ and $\mathbf{r}$ are left eigenvectors of the matrices $\mathbf{G}\mathbf{D}(\boldsymbol{\gamma})$ and $\mathbf{D}(\boldsymbol{\gamma})\mathbf{G}$ (corresponding to eigenvalues ρ)

$\mathbf{s}$: Load vector: $s_i = r_i \gamma_i / \rho$

$\mathbf{r}$: Spillage vector: $r_i = \sum_j G_{ji} s_j$

Alternative to power-interference characterization

Key to distributed algorithm

Attaining Pareto-Optimality

Algorithm:

Initialize: Fixed $\mathbf{s} \succ 0$ and $\rho \in [0, 1)$.

1. BS k broadcasts the BS-load factor $\ell_k = \sum_{j \in S_k} s_j$.
2. Compute the spillage factor $r_i = \sum_{j \neq i, j \in S_{\sigma_i}} s_j + \sum_{k \neq \sigma_i} h_{ki} \ell_k$.
3. Assign SIR values $\gamma_i = \rho s_i / r_i$.

Stop. The resulting SIR vector $\boldsymbol{\gamma} = \boldsymbol{\gamma}(\mathbf{s}, \rho)$.

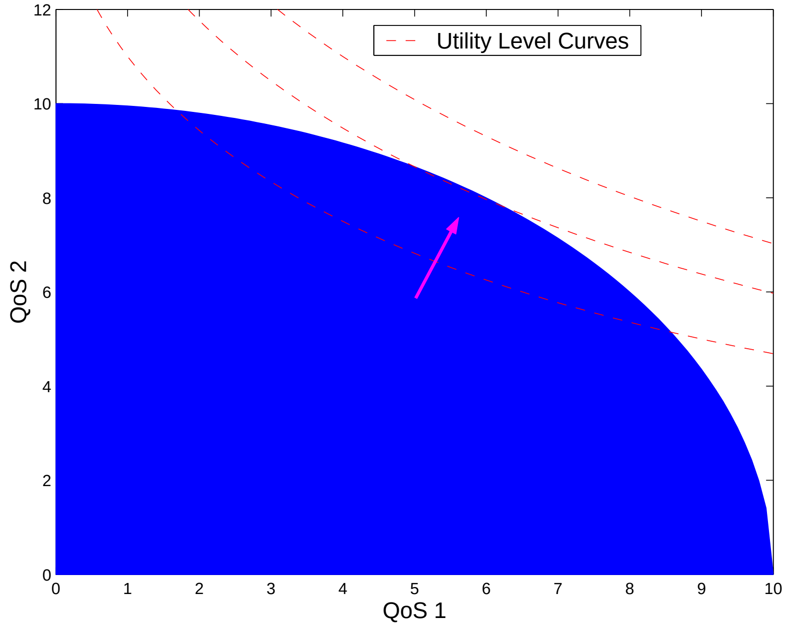
Alternative versions: MS-Control or BS-Control

Question: Which Pareto-optimal point will be obtained?

A distributedly computable ascent direction coming up next

Uplink Power Control in Multi-cellular Networks

Maximize: utility function of powers and QoS assignments
Subject to: QoS assignments feasible
Variables: transmit powers and QoS assignments



Utility Maximization

Which Pareto-optimal $\boldsymbol{\gamma}$ to pick?

Maximize concave utility functions over Pareto-optimal boundary

Utility functions $U(\boldsymbol{\gamma}) = \sum_i U_i(\gamma_i)$:

• Strictly increasing, twice differentiable with bounded derivatives, strictly concave in $\log \gamma_i$

• No starvation: As $\gamma_i \to 0$, $U_i(\gamma_i) \to -\infty$

Intuition: Assign higher SIR to

• MS with good channel condition (power-interference view)

• MS with worse interfering channel condition (load-spillage view)

Distributed Algorithm

Load-Spillage Power Control (LSPC) Algorithm:

Initialize: Arbitrary $\mathbf{s}[0] \succ 0$.

1. BS k broadcasts the BS-load factor $\ell_k[t] = \sum_{i \in S_k} s_i[t]$.
2. Compute the spillage-factor $r_i[t]$ by $\sum_{j \neq i, j \in S_{\sigma_i}} s_j + \sum_{k \neq \sigma_i} h_{ki} \ell_k$.
3. Assign SIR values $\gamma_i[t] = s_i[t]/r_i[t]$.
4. Measure the resulting interference $q_i[t]$.
5. Update (in a distributed way) the load factor $s_i[t]$:

$$s_i[t+1] = s_i[t] + \delta \Delta s_i[t].$$

where $\Delta s_i = \frac{U_i'(\gamma_i)\gamma_i}{q_i} - s_i$

Continue: $t := t + 1$.

Convergence and Optimality

Theorem: For sufficiently small step size $\delta > 0$, Algorithm converges to the globally optimal solution of

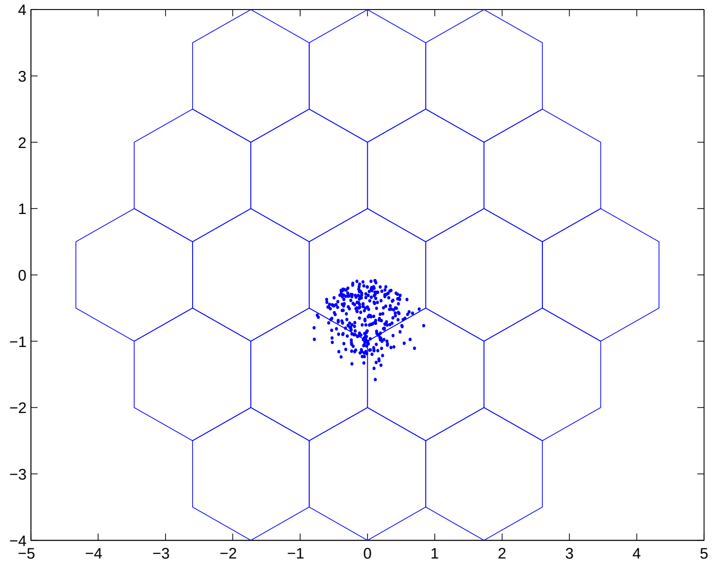
$$\begin{array}{ll} \text{maximize} & U(\boldsymbol{\gamma}) \\ \text{subject to} & \rho(\mathbf{D}(\boldsymbol{\gamma})\mathbf{G}) \leq 1 \end{array}$$

Proof: Key ideas:

- Develop a locally-computable ascent direction (most involved step)
- Evaluate KKT conditions
- Guarantee Lipschitz condition

Extend to power and interference constrained cases

Simulation



3GPP Evaluation Tool in industry: 19 cells in three hexagons

Each cell divided into three 120 degree sectors, 57 base station sectors

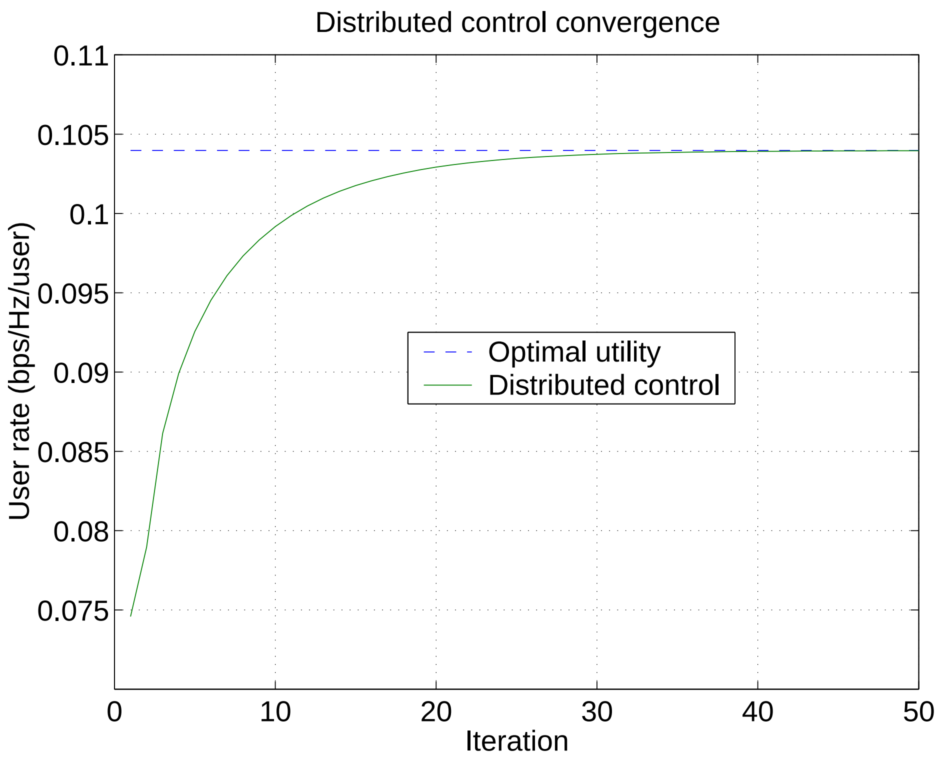
Uniform distribution of MS

Antenna: 65 degree 3 dB bandwidth, 15 dB antenna gain

Channel: Pass loss exponent: 3.7, log-normal shadowing: 8.9 dB

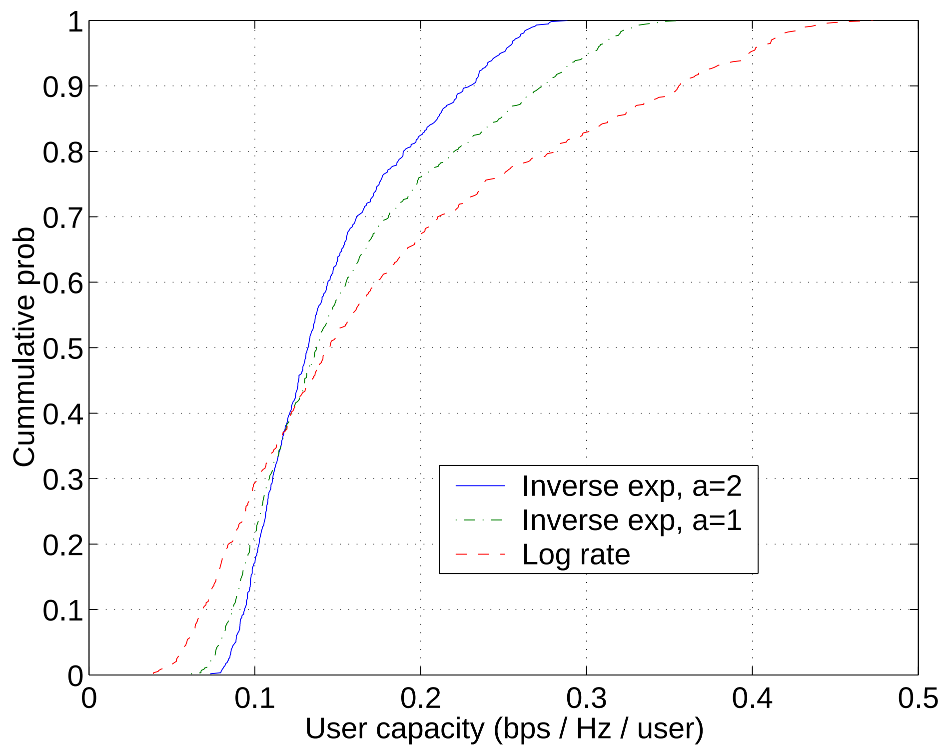
Convergence

10 MS per sector, 570 MS in total
Fast convergence with distributed control



Impacts of Utility Functions

Effects of shapes of utility function



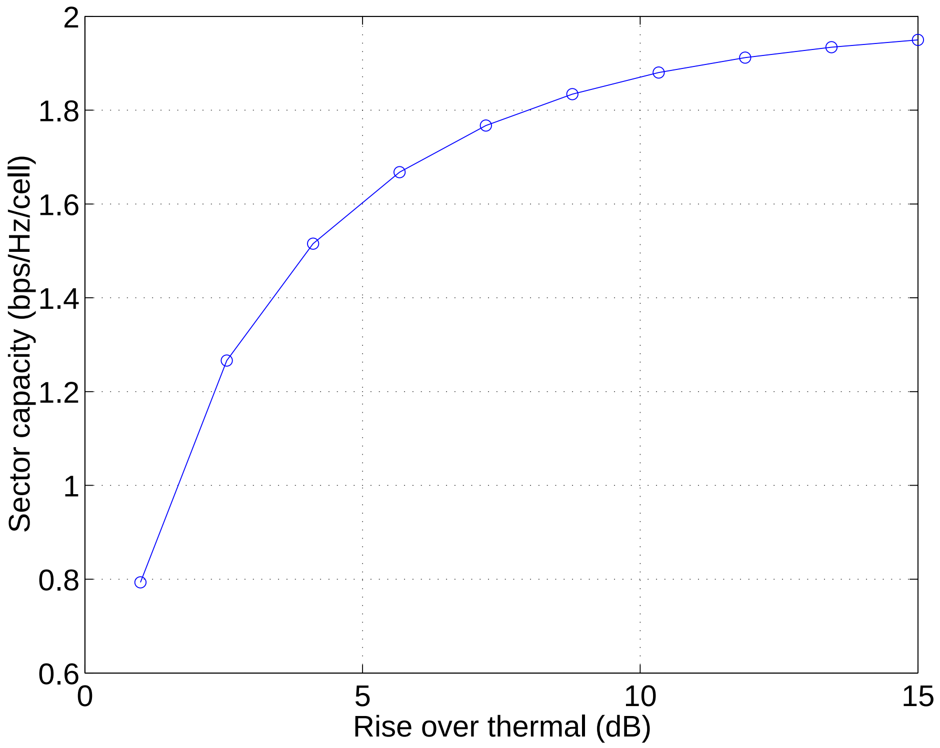
Tradeoff between Sector Capacity and Fairness

Tradeoff between efficiency and fairness

Utility function	Sector capacity (bps / Hz / sector)	10% Worst User capacity (bps / Hz)
Log	1.90	0.055
α-fair, $\alpha = 2$	1.58	0.086
α-fair, $\alpha = 3$	1.46	0.094
α-fair, $\alpha = 4$	1.46	0.097

Spectral Efficiency and MS Power Consumption

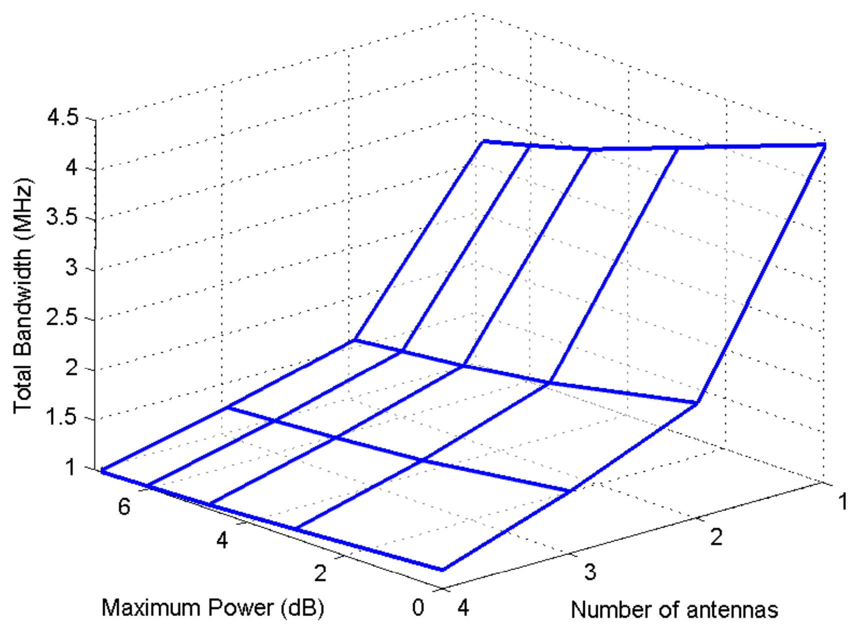
Interference-limited version of distributed algorithm
Tradeoff between sector capacity and Rise-Over-Thermal limit



Extensions

Joint bandwidth allocation, beamforming, power control for utility-optimal SIR assignment by distributed algorithm

Economic implication: Pareto-optimal tradeoff surface among three degrees of design freedom that achieve the same utility



Summary

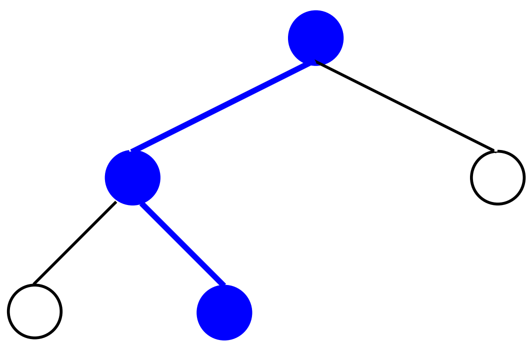
A distributed and jointly optimal QoS assignment and power control (for convex formulations) has now been obtained using load-spillage characterization

Easy extensions: implementation alternatives, joint bandwidth assignment, other modulation schemes, ad hoc networks ...

Difficult extension: distributed convexification

Part II.C

Multi-Carrier Energy-Efficiency Power Control Game



Some Related Work

- MacKenzie and Wicker 2001
- Xiao, Shroff and Chong 2001
- Alpcan, Basar, Srikant, Altman 2002
- Saraydar, Mandayam, and Goodman 2002
- Yu, Ginis, and Cioffi 2002
- Sung and Wong 2003

Open issues:

- Energy efficiency as utility function $\Rightarrow$ Non-quasiconcave utilities
- Multiple carriers $\Rightarrow$ Vector strategy

Energy Efficiency Utility Function

l: carrier index. D carriers

k: user index. K users

$\gamma_{kl} = \frac{p_{kl} h_{kl}}{\eta + \frac{1}{N} \sum_{j \neq k} p_{jl} h_{jl}}$: SIR for user k on carrier l

$f(\gamma_{kl})$: reliability function (sigmoidal function)

Throughput: $T_{kl} = R_k f(\gamma_{kl})$

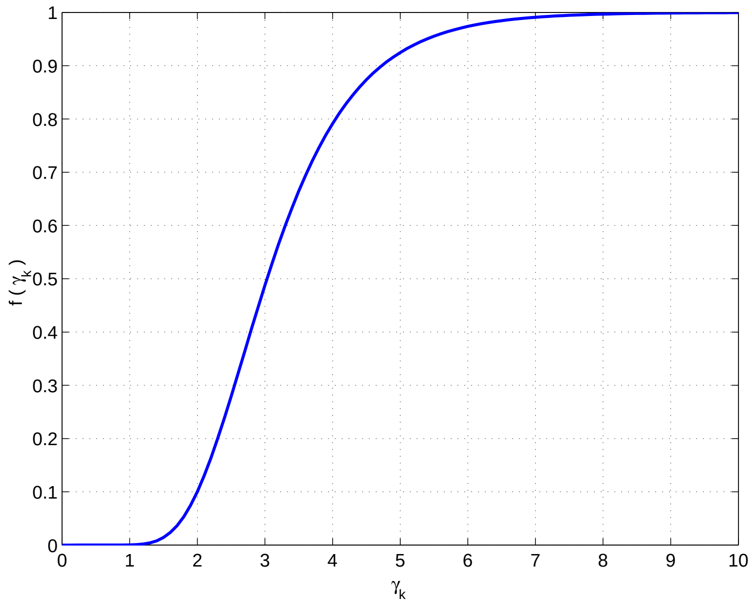
Power: p_{kl}

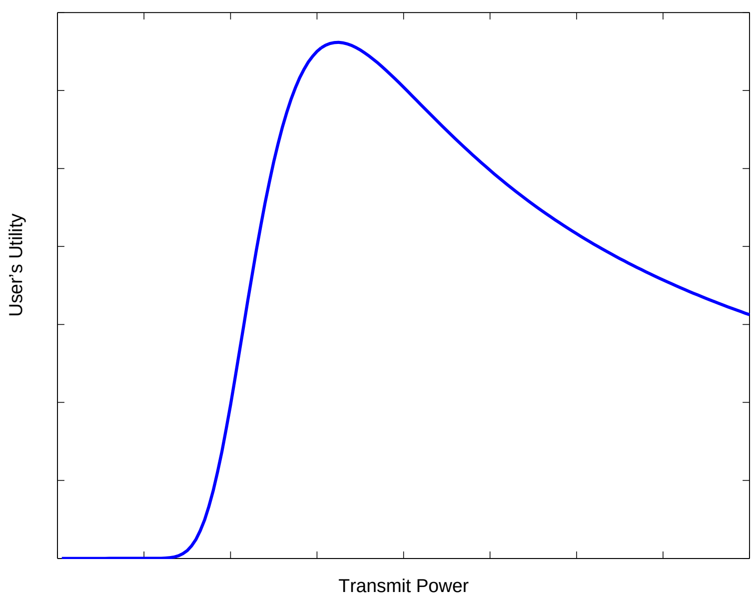
Energy efficiency utility function: $u_k = \frac{\sum_{l=1}^{D} T_{kl}}{\sum_{l=1}^{D} p_{kl}}$

Local and selfish utility maximization: $\max_{\mathbf{p}_k} u_k$

Game: $[\{1, 2, \ldots, K\}, \{[0, P_{max}]_k^D\}, \{u_k\}]$

Reliability Function and Energy Efficiency Utility





Multi-Carrier Energy Efficiency Maximization

γ^*: unique positive solution of $f(\gamma) = \gamma f'(\gamma)$

p^*_{kl}: transmit power needed to achieve SIR γ^* (or P_{max} if γ^* is not attainable)

L_k: $\mathrm{argmin}_l\, p^*_{kl}$ ('best' carrier)

Theorem: Energy efficiency maximizer is $p_{kl} = p^*_{kL_k}$ for $l = L_k$ and 0 otherwise

Only transmit on the 'best' carrier

Reduces number of possibilities of NE to D^K

Characterization of NE

Channel gains $\{h_{jl}\}$ determine NE possibilities

First assume that γ^* is attainable by all users (large enough processing gain N)

Define $\Theta_n = \frac{1}{1-(n-1)\frac{\gamma^*}{N}}, \;\; n = 0, 1, \ldots, K$
$(0 < \Theta_0 < \Theta_1 = 1 < \Theta_2 < \ldots < \Theta_K)$

$n(i)$: number of users transmitting on carrier i

Theorem: For user k to transmit on carrier l at NE:

$$\frac{h_{kl}}{h_{ki}} > \frac{\Theta_{n(l)}}{\Theta_{n(i)}}\Theta_0, \;\; \forall i \neq l$$

and in this case, $p^*_{kl} = \gamma^* \sigma^2 \frac{\Theta_{n(l)}}{h_{kl}}$

Existence and Uniqueness of NE

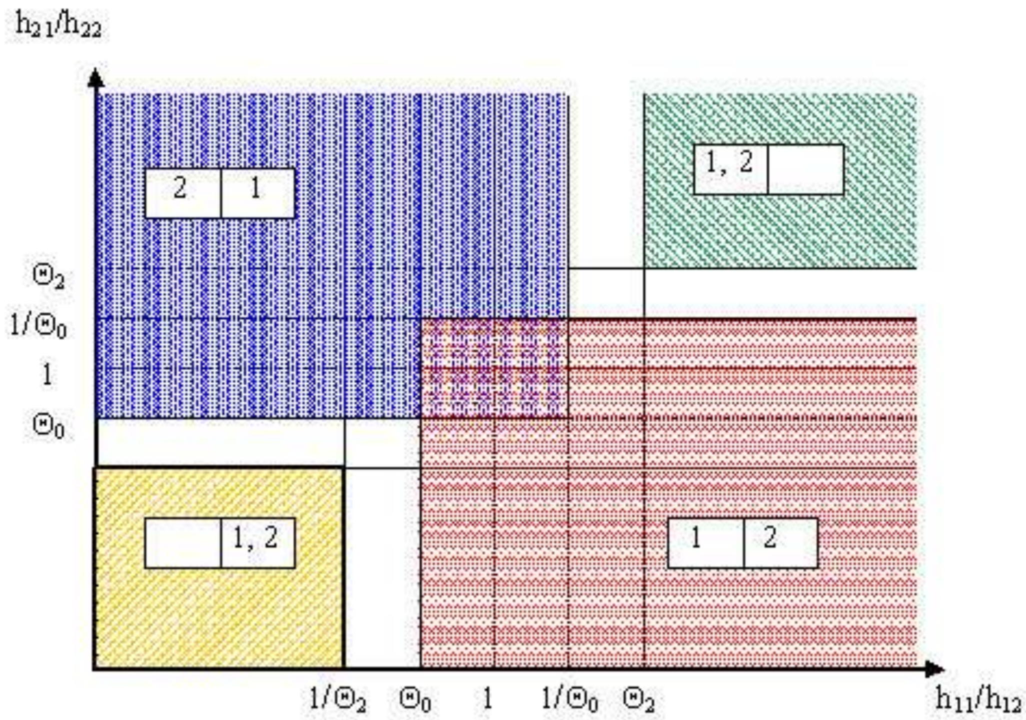
Existence of NE: Sufficient condition is that channel gains satisfy $K(D-1)$ inequalities simultaneously

Uniqueness of NE: Not guaranteed

$(K=2, D=2)$ case. Four possibilities:

- $(1,2|)$: $\frac{h_{11}}{h_{12}} > \Theta_2$ and $\frac{h_{21}}{h_{22}} > \Theta_2$
- $(|1,2)$: $\frac{h_{11}}{h_{12}} < \frac{1}{\Theta_2}$ and $\frac{h_{21}}{h_{22}} < \frac{1}{\Theta_2}$
- $(1|2)$: $\frac{h_{11}}{h_{12}} > \Theta_0$ and $\frac{h_{21}}{h_{22}} < \frac{1}{\Theta_0}$
- $(2|1)$: $\frac{h_{11}}{h_{12}} < \frac{1}{\Theta_0}$ and $\frac{h_{21}}{h_{22}} > \Theta_0$

Example



Homogeneity of channel gains: If either h_{11}/h_{22} or h_{22}/h_{11} belongs to $[1/\Theta_2^2, \Theta_0^2]$, then there does not exist NE

Two-Carrier Two-User Case

Rayleigh fading channel: $h_{kl} = \frac{c}{d_k^{-4}} a_{kl}^2$

a_{kl}: i.i.d. and have Rayleigh distribution with mean 1

X_1: number of users transmitting over first carrier at NE

$$P_{X_1}(0) = P_{X_1}(2) = \begin{cases} 0 & \text{if } N \leq \gamma^* \\ \left(\frac{1}{1+\Theta_2}\right)^2 & \text{if } N > \gamma^* \end{cases},$$

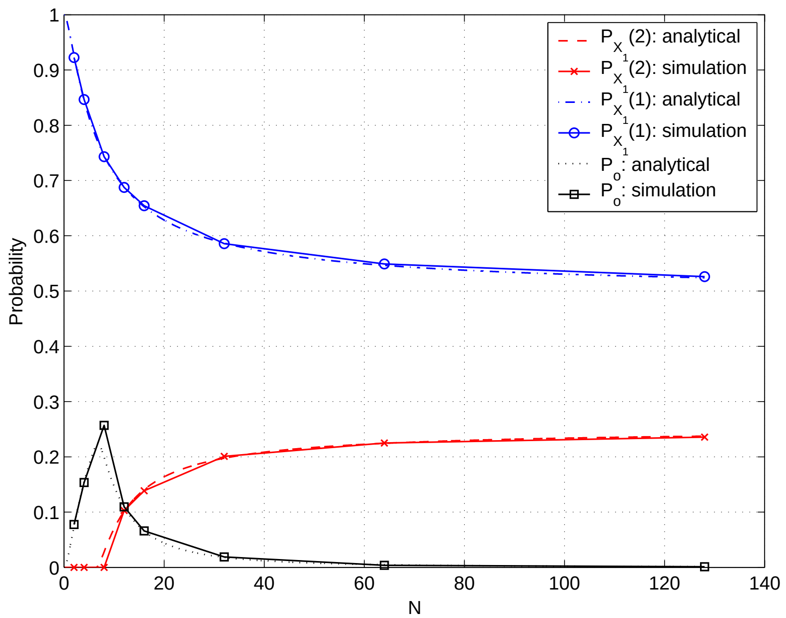
$$P_{X_1}(1) = 2\left(\frac{1}{1+\Theta_0}\right)^2 - \left(\frac{1-\Theta_0}{1+\Theta_0}\right)^2,$$

$$P_o = \begin{cases} 2\left(\frac{\Theta_0}{1+\Theta_0}\right)^2 & \text{if } N \leq \gamma^* \\ 2\left[\left(\frac{\Theta_0}{1+\Theta_0}\right)^2 - \left(\frac{1}{1+\Theta_2}\right)^2\right] & \text{if } N > \gamma^* \end{cases}.$$

Two-Carrier Case

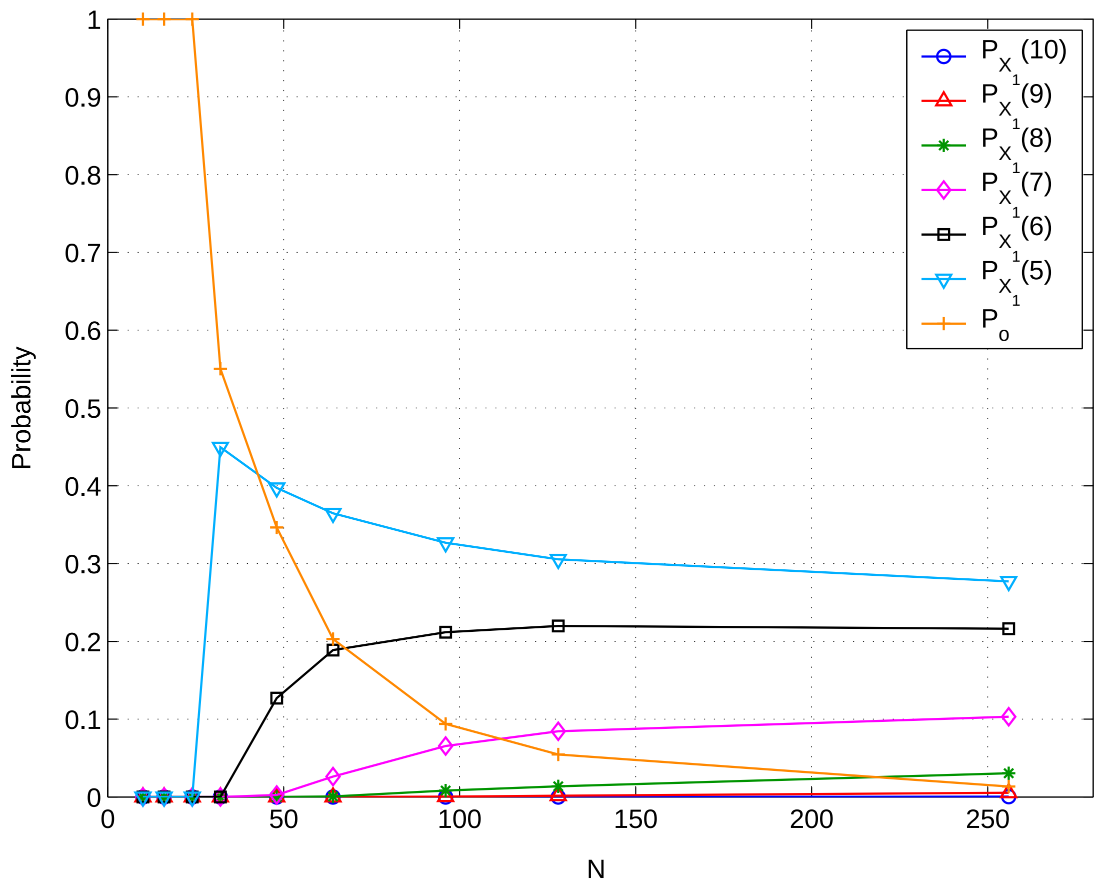
As processing gain N becomes large, there exists a unique NE well approximated by:

$$Pr\{X_1 = m\} \approx C_m^K (0.5)^K, \;\; m = 0, 1, \ldots, K$$



Example (2 carriers, 10 users)

No NE when N is too small, and always exists NE as $N \to \infty$



Distributed Algorithm

Sequential best response algorithm:

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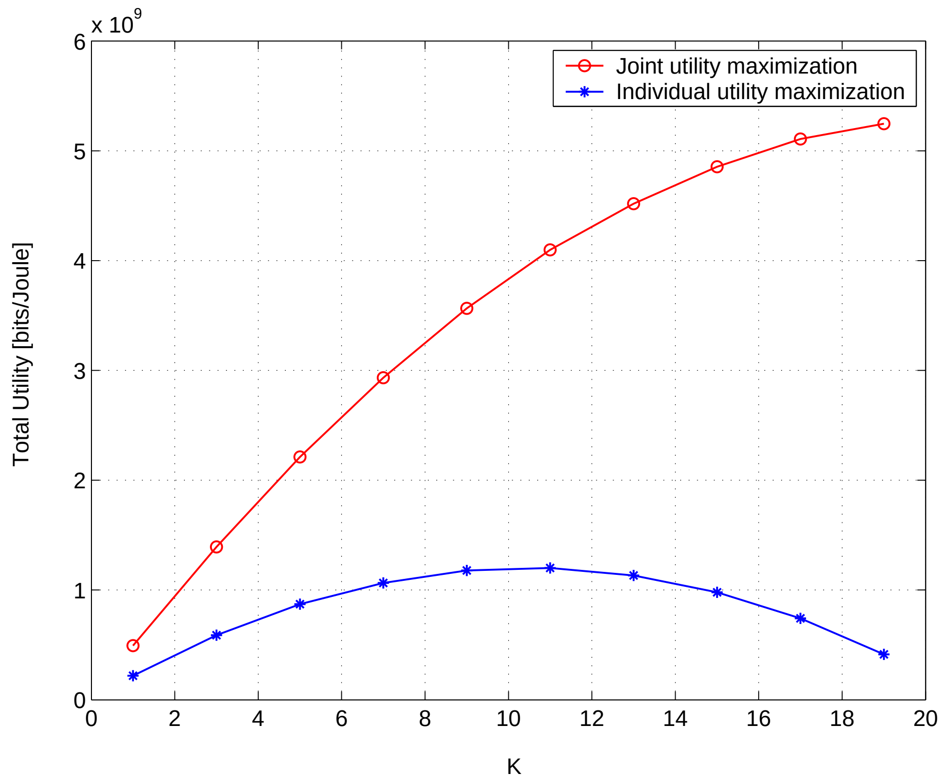
1. $k = 1$
2. User k picks "best" carrier and transmits on it only, at power level to attain SIR γ^*
3. $k = (k+1)$ mod K

Theorem: Above algorithm converges to NE (when it exists) for all two-user and three-user cases

Observation: Always converges to NE in all cases

Performance Gain

Comparison between our vector-valued strategy game and optimization over individual carriers



Summary

- Power control in cellular networks has a long history, rich taxonomy, structured understanding, and verifiable applications

- Many branches extensively studied and open problems solved, building up an intellectual basis for this area

- Many more branches still under-explored and some open problems unresolved