Wireless Scheduling

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Outline

- Structured teaser on wireless scheduling
- Focus on key ideas and 10 open problems
- Biased highlights on 3D tradeoff and CSMA
- Optimization combined with applied probability

- Acknowledgement: coauthors of papers cited in the talk:

Rob Calderbank, Jang-Won Lee, Jiaping Liu, Vince Poor, Alexandre Proutiere, Yung Yi, Junshan Zhang

Book chapter on the subject with Yung Yi

- Apology: for missing references and unbalanced emphasis

The Central Question

In an interference environment, who can talk in each time slot?

The Basic Problem Statement

Given: Who can interfere with whom

- Topology $G = (V, L)$
- Model and representation (graph, set, matrix) of interference

Variables: Who talks when

- Activation vector s, Contention probability p, λ, Holding time μ

Goal: Stable, Fair, Small delay, Big utility

Stochastic optimization: Workload arrival, Algorithm, (Channel)

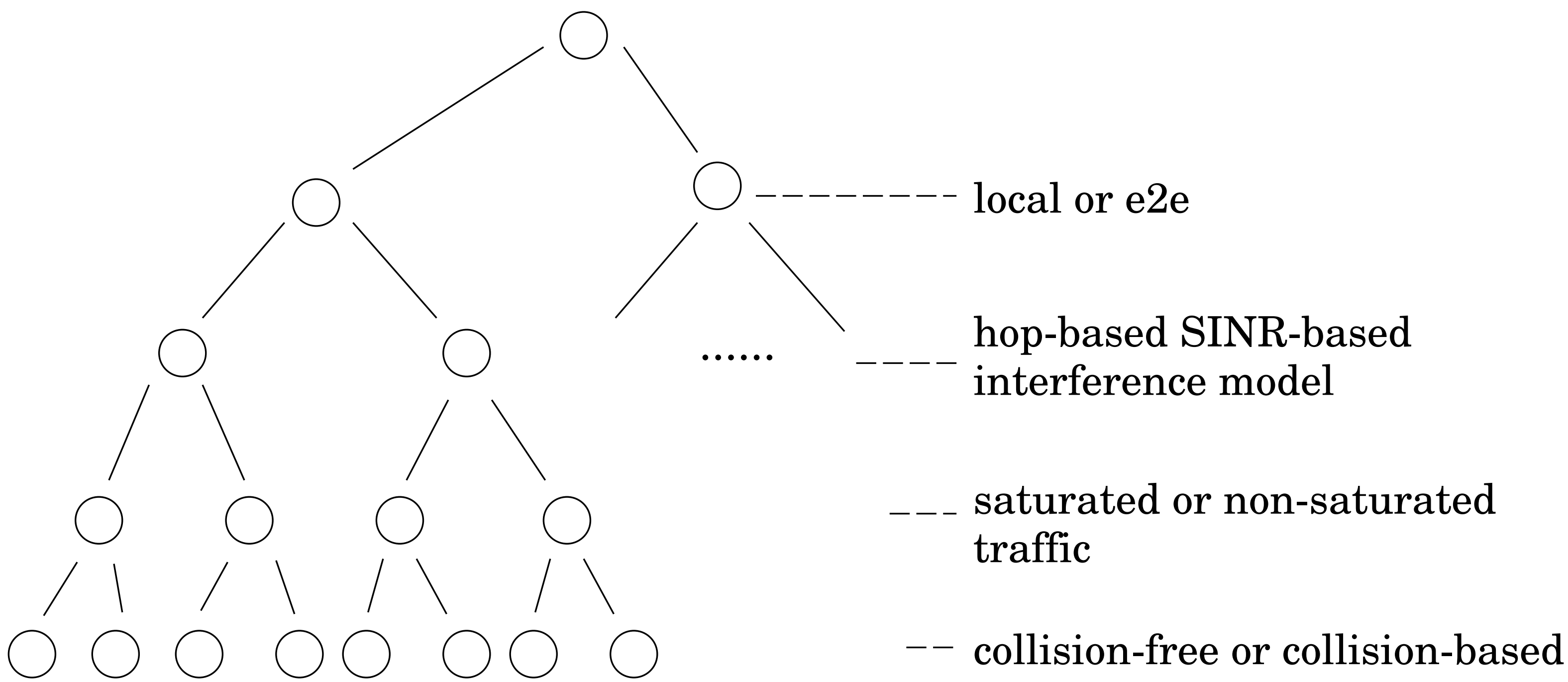
Practice-Theory Dichotomy

Simple ones used, analysis can be very challenging:

- Aloha
- CSMA/CA, CSMA/CD
- RTS/CTS

Sophisticated algorithms based on graph, optimization, game theories

Tree of Problems



Taxonomy of Problems

- Local contention neighborhood
- End-to-end (with routing and rate control)

- K-hop interference model ($K = 1$ bluetooth, $K = 2$ 802.11)
- SIR-based interference model (and adaptive physical layer)

- Saturated traffic (utility, fairness)
- Non-saturated (stability region, delay)

- Contention-free
- Contention-based

End-to-End

- Unsaturated

Joint congestion control, routing, and scheduling: Lin Shroff 2005, Neely Modiano Li 2005, Eryilmaz Srikant 2005, Stolyar 2005, Chen Low Chiang Doyle 2006...

- Saturated

Joint congestion control and contention control: Wang Kar 2005, Lee Chiang Calderbank 2006, Zhang Zheng Chiang 2007...

- Combination

Bui Eryilmaz Srikant Wu 2006, Chaporkar Sarkar 2006, Eryilmaz Ozdaglar Modiano 2007, Sharma Shroff Mazumdar 2007...

End-to-End

Joint congestion control, routing, and scheduling:

- Link based formulation
- Node based formulation: per-destination queues, includes routing

$$x_i^k \leq \sum f_{ij}^k - \sum f_{ji}^k \;\rightarrow\; \max_f \sum_{ij} f_{ij} \max_k (q_i^k - q_j^k)$$

Combination of backpressure and congestion pricing

Bottleneck is scheduling

More subtle points:

- Architectural choices: Layering as Optimization Decomposition
- Dual variable not exactly the queue size

SIR Based Interference Model

- Limited work on limited models:

Cruz Santhanam 2003

Johansson Xiao 2006

Yi de Veciana Shakkottai 2007

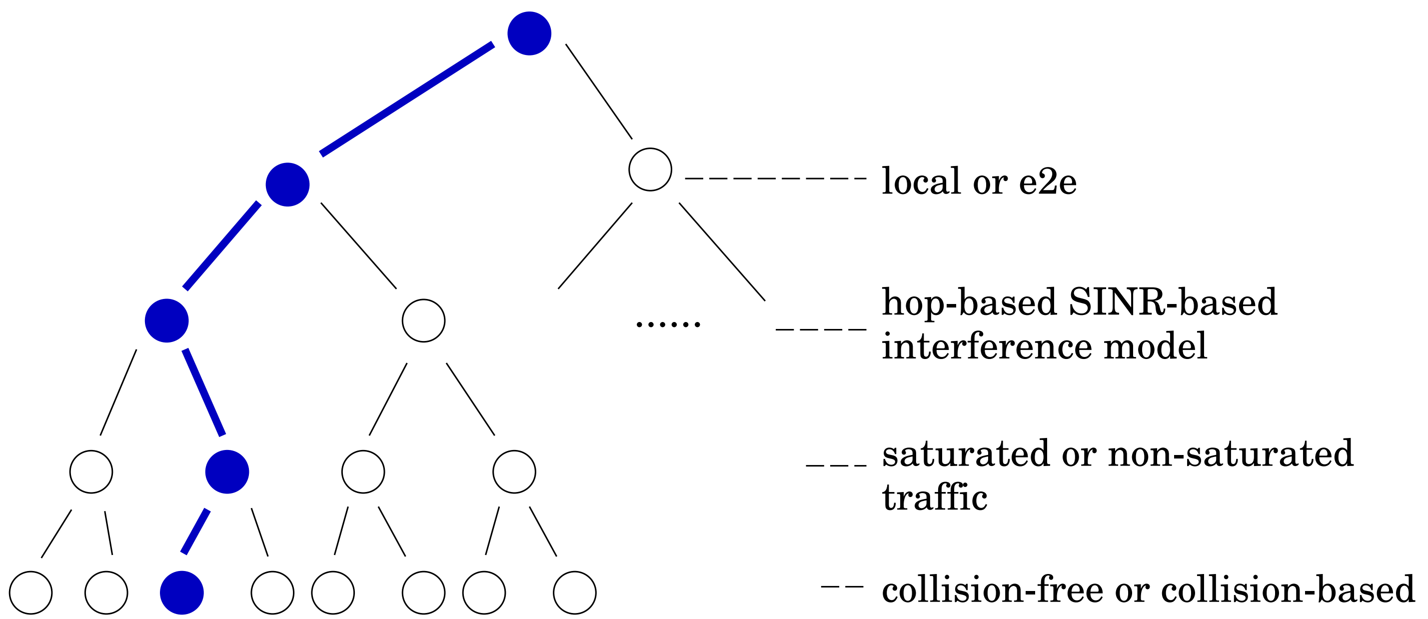
Kompella Wieselthier Ephremides 2008

High SIR models...

Further complications:

- Variable transmit power
- Channel probing
- Capture effect
- Sophisticated decoders

Where We Are In The Tree



Maximum Weight

- Tassiulas Ephremides 1992

The max-weight algorithm is choosing the $s^\star(t)$ at each slot t:

$$s^\star(t) = \arg\max_{s \in \mathcal{S}} W(s), \qquad W(s) \triangleq \sum_{l \in L} Q_l(t) s_l.$$

$\mathcal{S}$: Set of feasible schedules

$Q_l(t)$: Queue size on link l at time t

Throughput-optimal, Maximum stability region

- Connections to:

Prior work: Hajek Sasaki 1988 (known arrivals)

Graph theory: HP-hard Maximum Weighted Independent Set

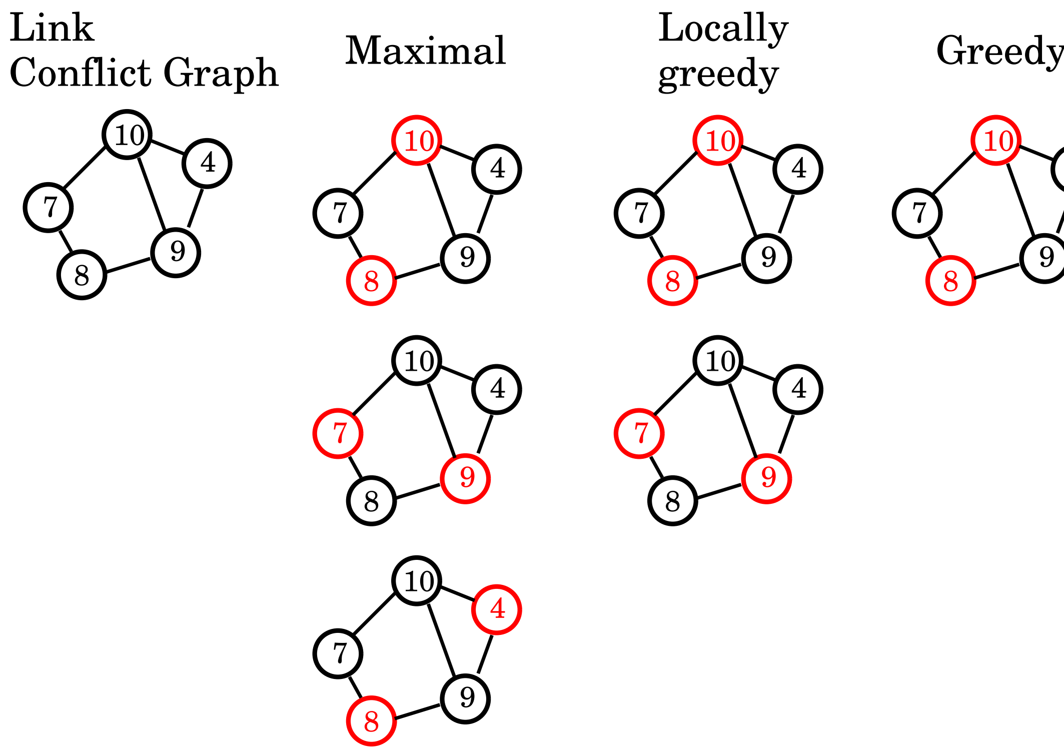
Switching theory

- General yet complex. How to make it simple and distributed?

Approximation: Maximal Weight

Suboptimal matching that can't be increased by activating more links:

- Greedy: the link l with the largest queue length
- Locally-greedy: a random link l with a locally-longest queue length



Approximation: Maximal Weight

$\gamma = 1/2$ ($K = 1$): Chaporkar Kar Sarkar 2006, Wu Srikant 2006

2/3 ($K = 1$, tree): Sarkar Kar 2006

1: NP-hard in general ($K > 1$): Sharma, Mazumdar, Shroff 2006

$1/$(maximum interference degree) Wu Srikant Perkins 2007, Chaporkar Kar Sarkar 2007: 1/8 for geometric graph

Further approx: Gupta Lin Srikant 2007

1 under local pooling condition (tree): Dimaki Walrand 2006, Brzesinski Zussman Modiano 2006, Zussman Brzezinski Modiano 2008, Joo Lin Shroff 2008: 1/6 for 2D geometric graph

Distributed: Israeli Itai 1986, Heopman 2004

Open Problem

Q1: Lower and upper bounds on throughput by maximal weight scheduling for general topology and K? (Also for the next two parts of the talk)

Randomization: Pick and Compare

- Centralized: Tassiulas 1998

At each time slot t, the γ-RPC first generates a random schedule $s'(t)$ satisfying **P**, and then schedule $s(t)$ defined in **C**:

P $\exists 0 < \delta \leq 1$, s.t. $\text{Prob}(s'(t) = s|Q(t)) \geq \delta$, for some schedule s, where $W(s) \geq \gamma W^\star(t)$

C $s(t) = \arg\max_{s=\{s(t-1),s'(t)\}} W(s)$

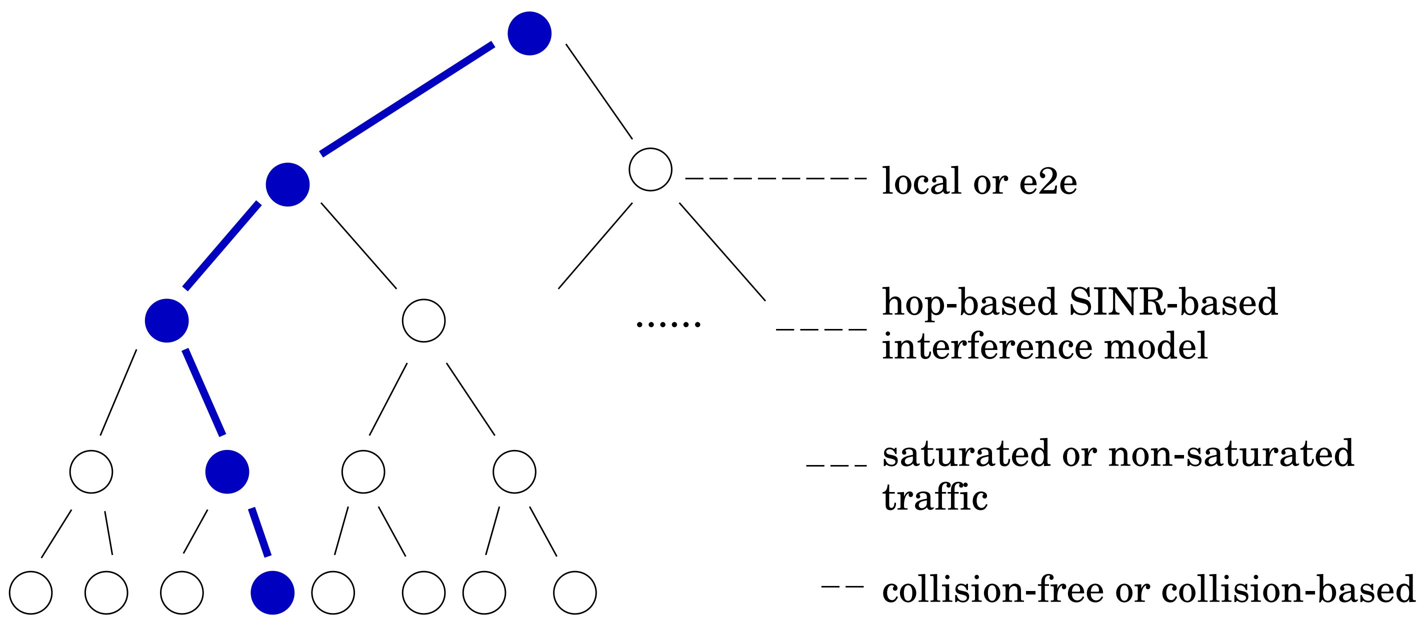
- Message passing with gossip: P and C can be inaccurate

$\gamma = 1$ ($K = 1$, not counting complexities): Modiano Shah Zussman 2006

Open Problem

Q2: What's the largest effective throughput of RPC for general K?

Where We Are In The Tree



Message Passing Random Access

$K = 1$. Each slot starts with constant M minislots for control signals

- Compute $0 \le x_l(t) \le 1$ using queue lengths of the interfering neighbors via message passing:

$$x_l(t) = \frac{Q_l(t)}{\max\left[\sum_{k \in L(t(l))} Q_k(t), \sum_{k \in L(r(l))} Q_k(t)\right]}$$

- The link l contends each mini-slot with the probability $p_l = f(x_l(t), M)$ for some f (e.g., $g(M)x/M, 1 - \exp(-g(M)x/M))$
- Successfully contended link transmits during the time slot

$1/3 - 1/M$: Lin Rasool 2006

$1/2 - 1/\sqrt{M}$: Joo Shroff 2007

$1/2 - \log(2M)/2M$: Gupta Lin Srikant 2007

Further stuy: Marbach Eryilmaz Ozdaglar 2007, Joo Lin Shroff 2008

Pause

What about the overhead? Computation and Communication

Detour: Distributed Algorithm in Networking

How distributed is distributed?

Dimensions to quantify explicit message passing:

- How often? Time-complexity
- How far? Space-complexity
- How many bits per message? Bit-complexity

Performance-Distributeness tradeoff:

- Outer bound for benchmarking
- Inner bound by protocol design
- Design ideas and proof techniques

Detour: Optimization Without Optimality

- Optimality-driven design:

Under the constraint of having an optimality proof, find the simplest protocol

- Simplicity-driven design:

Under the constraint of zero message passing, find the best performance protocol

Expand the conditions of convergence, optimality...

Bound the optimality gap, stability region reduction...

- Overhead changes the accounting rule:

Multiplier effect

Sweet spots in the tradeoff

Throughput-Complexity Tradeoff

Local versions of RPC:

Graph partitioning: Ray Sarkar 2007

Link augmentation: Sanghavi Bui Srikant 2007

Extension to general K: Jung Shah 2007, Yi Chiang 2008

Throughput-Delay-Complexity Tradeoff

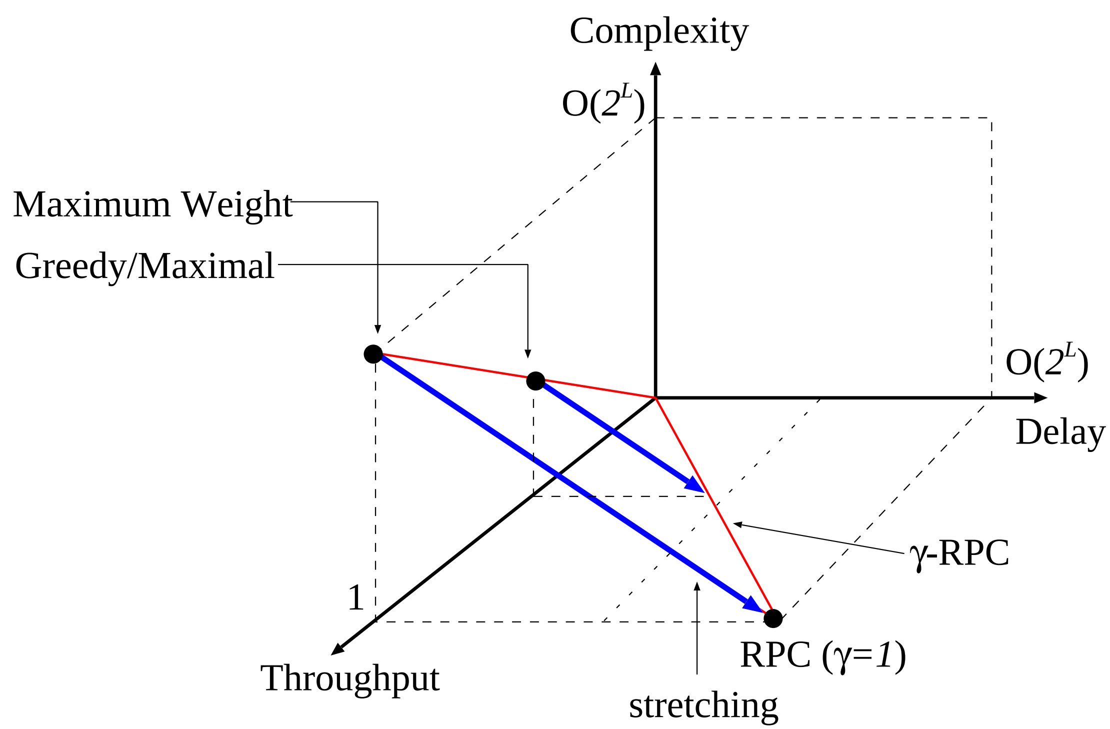
Parameterization: (γ, ξ, χ) approximate algorithm

Stretching by m: stability unaffected, delay grows linearly in m

- From (γ, ξ, χ) to $(\gamma, \xi + mV\Omega(1+\gamma), \chi/m)$
- Each scheduling algorithm is one point in 3D tradeoff space
- Parameterize into tradeoff curves
- Three 2D projections: e.g., Stability-delay tradeoff for a fixed complexity

Yi Proutiere Chiang 2008

3D Tradeoff



Complexity
O(2^L)
Maximum Weight
Greedy/Maximal
O(2^L)
Delay
γ-RPC
1
Throughput
RPC (γ=1)
stretching

Open Problem

Q3: Only achievable curves. What about achievability surface or converse?

Q4: Tradeoff with spatial-complexity and bit-complexity (event-triggered, differential-coded)?

Q5: Only a bound on delay. Can we understand delay better and minimize delay? (Tight bounds for various algorithms in general graph and for general K)

Delay Charaterization

- The challenge of dimensionality
- Switching literature sometimes helpful

- Lyapunov bound:

Neely 2006, Neely 2008, Chaporkar et al 2008, Gupta Shroff 2009

$$Q(t+1) = [Q(t) - D(t) + A(t)]^{+}$$

Upper bound $\mathcal{O}(\log \max_l N(l))$ Maximal Weight and Markov bursty traffic

Lower bound for multihop backpressure with fixed routing

- Large deviation:

Venkataramanan and Lin 2006, Ying Srikant Dullerud 2006

Delay bound violation probability constraint

Related: scheduling under deadline constraints

Delay Charaterization

- Heavy traffic approximation:

Shakkottai Srikant Stolyar 2004, Shah Wischik 2007

Assume heavy traffic regime and diffusion scale $\hat{x}^n(t) = X(n^2 t)/n$

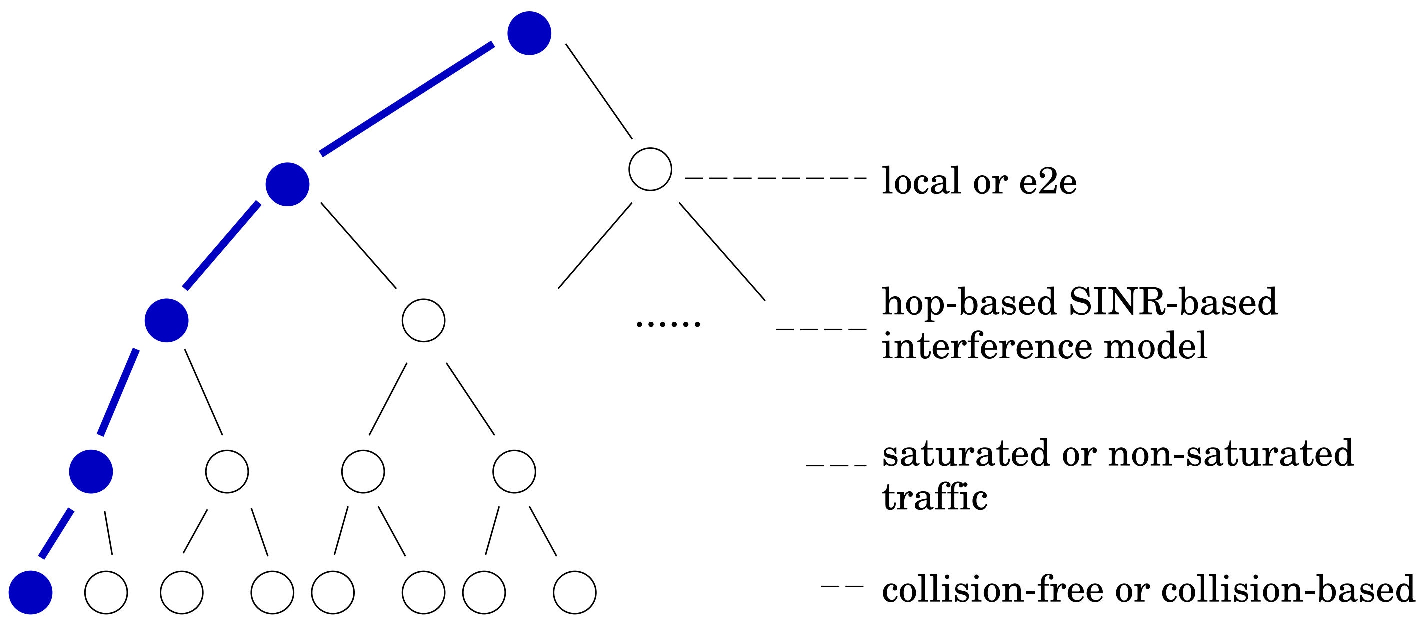
Prove state space collapse and characterize workload process

Derive inference to the original problem

Yi Zhang Chiang 2009

Vacation model for complexity: exponential growth

Where We Are In The Tree



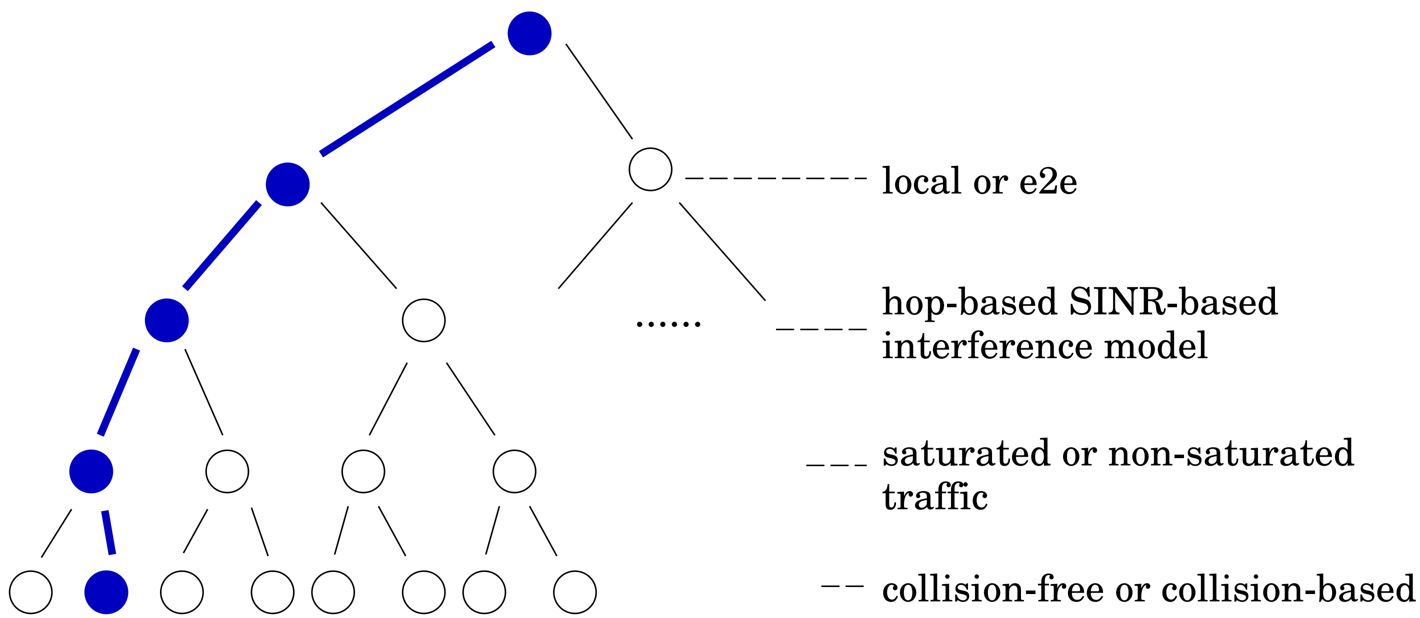
Contention Graph

Nandagopal Kim Gao Bharghavan 2000

Chen Low Doyle 2005

Turns the problem to one similar to congestion control

Where We Are In The Tree



Contention Probability for Slotted Aloha

Proportional fair: Kar Sarkar Tassiulas 2004

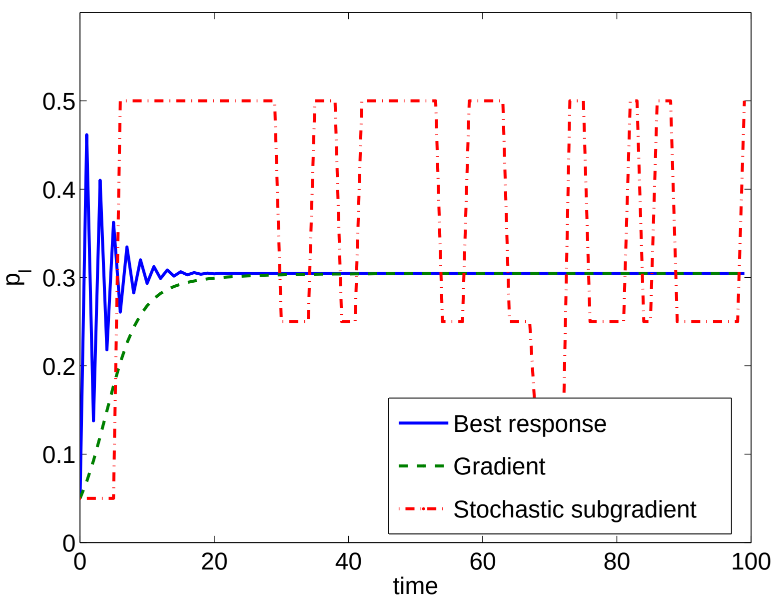
General utility: Lee Chiang Calderbank 2006

Queue backpressure: Gupta Stoylar 2006, Stoylar 2008, Liu Stoylar Chiang Poor 2008

Reverse Engineering Exponential Backoff

- Reverse engineer as a game (derive utility function)
- Nash equilibrium exists but suboptimal
- Existing protocol is stochastic subgradient
- Converges under conditions on how interfered the topology is

Lee Chiang Calderbank 2007



Reverse Engineering Exponential Backoff

- Contrast to reverse engineering of TCP congestion control into NUM
- Self interests not aligned
- How to align them? Maybe with the help of message passing?

Problem Statement

$L_{out}(n)$: set of logical links where node n is transmitter

$N(l)$: set of nodes whose transmission collide with that on l

Each link with a utility function $U_l(x_l)$ and fixed rate c_l

$$x_l = c_l p_l \prod_{k \in N(l)} (1 - P^k)$$

Optimization over variables $(\mathbf{p}, \mathbf{P})$:

$$\begin{array}{ll} \text{maximize} & \sum_l U_l(c_l p_l \prod_{k \in N(l)} (1 - P^k)) \\ \text{subject to} & x_l^{min} \leq c_l p_l \prod_{k \in N(l)} (1 - P^k) \leq x_l^{max}, \ \ \forall l \\ & \sum_{l \in L_{out}(n)} p_l = P^n, \ \ \forall n \\ & P^{min} \leq P^n \leq P^{max}, \ \ \forall n, \ 0 \leq p_l \leq 1, \ \ \forall l \end{array}$$

How Distributed Can Solution Be

- Step 1: log change of variable to decouple
- Step 2: dual decomposition
- Step 3: $\alpha \geq 1$ utility function to ensure global optimality

- How to make it converge faster?

Stepsize-free algorithm

- How to reduce message passing to zero?

Learn from historical record of collisions

Optimal for fully-interferred topology and sufficient number of nodes

Mohsenian-Rad Huang Chiang Wong 2009

Open Problem

Q6: How suboptimal is utility maximization by Aloha with no message passing?

Utility-Optimal CSMA

No message passing (think converse point in 3D tradeoff)

- Utility in saturated case
- Rate stability in non-saturated case

Adaptive CSMA:

- Jiang Walrand 2008
- Rajagopalan Shah 2008
- Liu Yi Proutiere Chiang Poor 2008

Related: Marbach Eryilmaz 2008, Liew et al 2008

Key background: Kelly 1987, Hajek 1988, Borkar 2006

Problem Statement

- $\boldsymbol{\gamma} = (\gamma_l, l \in \mathcal{L})$: long-term throughputs
- Γ: throughput region

$$\Gamma = \left\{ \boldsymbol{\gamma} : \exists \boldsymbol{\tau} \in \Upsilon, \forall l \in \mathcal{L}, \gamma_l \leq \sum_{s \in \mathcal{S} : s_l = 1} \tau_s \right\}$$

where $\Upsilon = \{\boldsymbol{\tau} = (\tau_s, s \in \mathcal{S}), \forall s, \tau_s \geq 0, \sum_{s \in \mathcal{S}} \tau_s = 1\}$

Optimization problem:

$$\max \sum_l U(\gamma_l), \text{ s.t. } \boldsymbol{\gamma} \in \Gamma$$

Two Timeslot Models

- Poisson clock contention

Mathematically, no collision

More tractable starting point

Turns out optimality can be asymptotically approached arbitrarily tightly

- Discrete time contention and backoff

Represent the reality and incorporate collision

Need to bound both algorithm inefficiency and collision degradation

Can form a sequence of systems converging to Poisson clock model

Throughput gap and efficiency-fairness tradeoff

Timescale Assumption

Two interacting components:

- Continuous time: defines at each instant which links are transmitting
- Discrete time: periodically updates the CSMA transmission parameters (λ_l, μ_l) used in the first component

Two timescales:

- Easy: Freeze CSMA parameters over a frame of timeslots, wait for stochastic network state converge to stationary distribution
- Hard: Underlying stochastic network and CSMA transmission parameters evolve simultaneously

Algorithm

Parameters: $V > 0,\ W(\cdot),\ b(t)$

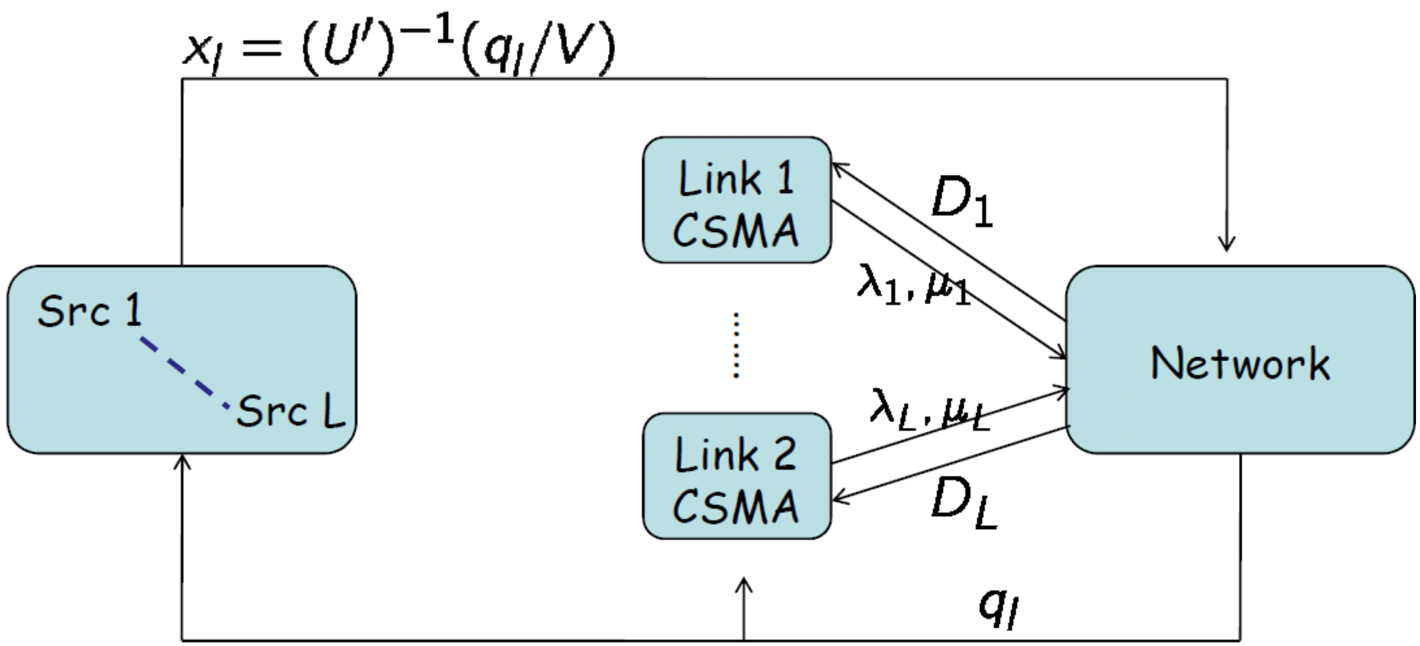
(e.g., $V = 10,\ W(x) = \log\log(x+e)$ or $W(x) = x, b(t) = 1/t$)

$$\bullet q_l[t+1] = \left[q_l[t] + \frac{b[t]}{W'(q_l[t])} \left(U_l'^{-1}\left(\frac{W(q_l[t])}{V}\right) - D_l[t] \right) \right]_{q^{\min}}^{q^{\max}},$$

$$\bullet \rho_l[t+1] = \exp\{W(q_l[t+1])\}$$

- The corresponding $\lambda_l[t+1]$ and $\mu_l[t+1]$ updated such that $\rho = \lambda/\mu$

Algorithm With Congestion Control



$$q_l[t+1] = \left[q_l[t] + b[t]\Big((U')^{-1}(q_l[t]/V) - D_l[t] \Big) \right]_{q^{\min}}^{q^{\max}}$$

$$\lambda_l[t+1] = \mu^{-1} \exp(q_l[t+1])$$

Performance

Convergence to: $\lim_{t\to\infty} \boldsymbol{q}[t] = \boldsymbol{q}^\star$

The corresponding throughput $\boldsymbol{\gamma}(\boldsymbol{\rho}(\boldsymbol{q}^\star))$ solves:

$$\begin{array}{ll} \text{maximize} & V\sum_{l\in\mathcal{L}} U(\gamma_l) - \sum_s \tau_s \log \tau_s \\ \text{subject to} & \gamma_l \le \sum_{s\in\mathcal{S}:s_l=1} \tau_s \\ & \sum_s \tau_s = 1 \end{array}$$

Approximately solves utility maximization. Max error: $\log|\mathcal{S}|/V$

As $V \to \infty$ with speed $\mathcal{O}(L)$, it solves utility maximization

Proof

- As a stochastic subgradient algorithm modulated by a Markov chain
- Main step 1: show averaging over fast timescale is valid

Interpolation of discrete q converges a.s. to a continuous q solving a system of ODE

- Main step 2: show the resulting averaged process converge

The system of ODE describes the trajectory of subgradient to solve the dual problem

- Main step 3: Standard methods in convex optimization and duality

Based on our approach. See also other proofs that modify the algorithm

Detour: A General Lemma

Given sequence x_n of random real numbers, and random variable Y_n,

$$x_{n+1} = x_n + b_n h(x_n, Y_n)$$

h is bounded, continuous, Lipschitz (to first variable)

Y_n is Markov chain whose kernel evolves in time and depends on x_n:

$$\text{Prob}[Y_{n+1} = z | Y_n = y, x_n = x] = p(z|y,x)$$

Kernel p of a stationary, ergodic Markov chain with stationary distribution π_x

Let $\bar{x}$ be interpolated x, and $\tilde{x}^s$ be solution to the following ODE:

$$\frac{dx(t)}{dt} = \sum_y \pi_{x(t)}(y) h(x(t), y), \quad \tilde{x}^s(0) = \bar{x}(s)$$

Then, a.s.,

$$\lim_{s \to \infty} \sup_{t \in [s, s+T]} |\bar{x}(t) - \tilde{x}^s(t)| = 0$$

See Borkar proof and Proutiere proof

Discrete Slot Model: Efficiency-Fairness Tradeoff

Contention probability: $p_l = \epsilon\lambda_l$. Channel holding $1/\epsilon\mu_l$

Average number of periods during which link l do not transmit successfully: $E_l = \frac{1}{\epsilon\mu} \times \frac{1-\gamma_l(\boldsymbol{\rho}^\star)}{\gamma_l(\boldsymbol{\rho}^\star)}$

Short-term fairness index: $\beta = 1/\max_l E_l$ (worst transient delay)

Contrast to long-term fairness (equilibrium throughput utility)

For fully-interferred network, to guarantee a loss of utility of δ,

without RTS/CTS: $\beta \le \frac{\delta}{C_1 \exp(C_2/\delta)}$,

with RTS/CTS: $\beta \le \frac{\delta}{C_3}$.

Based on our approach. See also Ni Srikant 2009

Open Problem

Q7: 3D tradeoff and transient behavior of utility-optimal adaptive CSMA?

Q8: Queue stability for non-saturated arrival: can CSMA with zero message passing be optimal?

Q9: Implementation and deployment of utility optimal CSMA?

Open Problem

Q10: Is it better to control when to talk or how loud to talk?

Centralized: when to convexify power controlled throughput region?

Distribute: even harder

Final Thoughts

- Wireless scheduling is hard, even for simple models:

High dimensionality and queueing dynamics

Non-convexity and computation complexity

Coupling and communication complexity

- New tools and results are making fast progress

Form an intellectual heritage with clear open problems

Need to demonstrate impact in commercial design

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