

## Charge-ring model for the charge-induced confinement enhancement in stacked quantum-dot transistors

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A model is proposed to explain the charge-induced confinement enhancement observed in a stacked quantum-dot transistor that has a floating dot on top of a channel quantum dot. The model assumes that the charge on the floating dot distributes on its rim, forming a ring and creating a confinement potential that squeezes the electrons in the channel dot toward its center. The charge on the floating dot can be calculated from the device geometry and from the measured threshold voltage difference before and after the charging. Given the charge on the floating dot, the spatial confinement and the energy level spacing increase induced by the charging can be obtained. The calculation based on the model agrees with the observed increase of the energy level spacing. © 1998 American Institute of Physics. [S0003-6951(98)04849-9]

Nanoscale floating gate has been used in the silicon field effect transistors to demonstrate room temperature operation of single-electron memories.<sup>1,2</sup> The principle is based on the screening of gate potential by the charge stored in the floating gate, which causes the threshold voltage shift of the transistor. Recently, we have observed a novel effect in a modified structure—a stacked quantum-dot transistor (QDT), namely the charge-induced confinement enhancement.<sup>3</sup> In this letter, we propose a charge-ring model to account for the enhancement, and compare it with the experiment.

The stacked QDT consists of a polycrystalline silicon (polysilicon) floating gate stacked on top of a silicon dot in the channel (Fig. 1). The floating dot is separated from the channel dot by a thin tunnel oxide and from the control gate by a thick oxide. A sufficiently high positive voltage on the control gate will force electrons to tunnel from the channel to the floating dot. The drain current versus gate voltage characteristics of the device was measured. Figures 2(a) and 2(b) shows the conductance oscillations of a stacked QDT before and after the control gate was pulsed with 13 V at 4.2 K. By comparing the two traces, we can see that not only the threshold voltage of the transistor increased by  $\sim 0.3$  V, but the oscillation peaks also became more pronounced and well separated. The number of electrons inside the silicon dot has decreased from 12 to 4 within the same scanned gate voltage range. Both effects are the result of an increase in the energy level separation inside the silicon channel dot. Such increase can be estimated as follows: since the conductance peak has a full width at half maximum (FWHM) of  $3.5 k_B T$ ,<sup>4</sup> it provides a energy scale that can be used to obtain the energy

level separation ( $\Delta E$ ) by comparing the spacing between the conductance peaks ( $\Delta V_G$ ) with the peak width, i.e.,  $\Delta E = 3.5 k_B T (\Delta V_G / \text{FWHM})$ . Using this method, we estimated that the average energy level separation has increased from 2.5 meV before the charging to 5.5 meV after—an increase by over a fact of 2.

Such an effect can be attributed to the Coulomb interaction between the electrons in the floating gate and those in the channel dot. We propose a charge-ring model to explain the observation. The polysilicon floating gate, being a conductive disk, has a nonuniform charge distribution with charge concentrated near the edge.<sup>5</sup> For simplicity, this model assumes that all the electrons charged into the floating dot are distributed at its rim (Fig. 3). These electrons not only screen the electric field from the control gate, causing a positive shift in the threshold voltage, but also create an additional confining potential that “squeezes” the electrons on the channel dot toward its center, effectively reducing the channel dot size and increasing the energy spacing.

Using this model, we can calculate the change of the energy level spacing inside the channel dot from the device

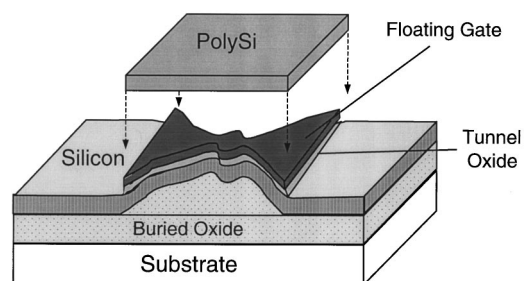


FIG. 1. Schematic view of a stacked quantum-dot transistor.

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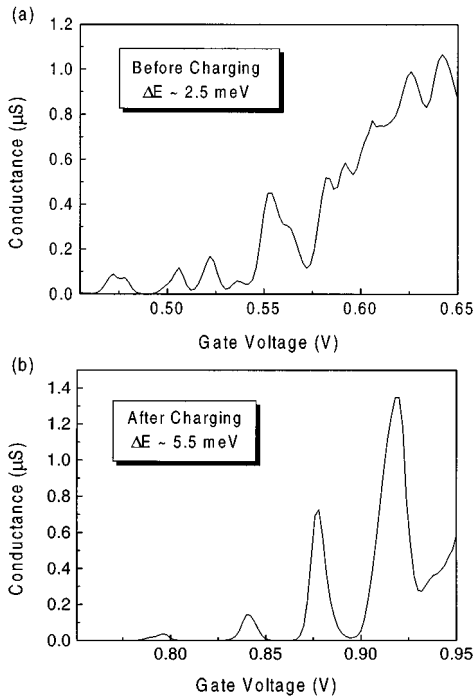


FIG. 2. Current–voltage ( $I$ - $V$ ) characteristics of the device before and after the floating dot is charged by 13 V applied to the control gate.

geometry and the measured threshold voltage shift. The calculation has two steps. The first is to calculate the number of electrons charged into the floating gate (forming a ring) from the fact that at the threshold voltage the potential at the channel dot should be the same before and after the charging of the floating gate. The second step is to calculate the change in the energy level spacing due to the charges stored on the ring using the numerical method discussed in Ref. 6.

First, let us calculate the charge on the floating gate. The electric potential inside the dot before charging the floating gate is determined by the control gate voltage  $V_g$  only, and is given by

$$\begin{aligned} \phi(\xi, \varphi, V_g) = & V_g - \frac{2V_g}{\pi} \left\{ \arctan \sqrt{\frac{\cosh(\xi/2) - \cos(\varphi/2)}{\cosh(\xi/2) + \cos(\varphi/2)}} \right. \\ & - \sqrt{\frac{\cosh \xi - \cos \varphi}{\cosh \xi + \cos \varphi}} \\ & \left. \times \arctan \sqrt{\frac{\cosh(\xi/2) - \sin(\varphi/2)}{\cosh(\xi/2) + \sin(\varphi/2)}} \right\}, \quad (1) \end{aligned}$$

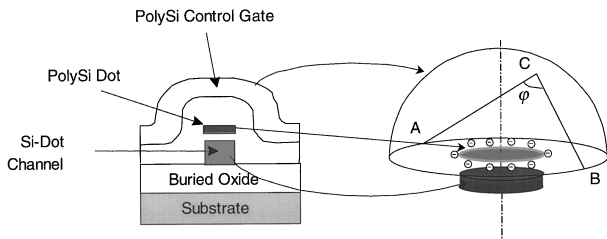


FIG. 3. The stacked QDT modeled as a combination of a hemispherical control gate, charge ring for the floating dot, and a channel quantum dot. Bispherical coordinates  $\xi$  and  $\varphi$  are defined as  $\xi = \ln(AC/BC)$ , and the angle between  $AC$  and  $BC$ , respectively. The charge ring has a radius of  $r_f$  as used in Eq. (3), and the spherical coordinates  $(r, \theta)$  are measured from the center of the ring.

where the control gate is assumed to have an hemisphere shape, and  $\xi$  and  $\varphi$  are the bispherical coordinates<sup>7</sup> defined in Fig. 3. Equation (1) is derived from a well-known formula<sup>8</sup> for the electric potential of a charged metallic two-dimensional “edge,” using the spherical inversion transformation.

Before the charging, the potential at the center of the channel dot at the threshold voltage,  $V_{g1}$ , is given by Eq. (1), with  $\xi=0$  and  $\varphi=\pi$ :

$$\phi(V_{g1}) = V_{g1} \left( \frac{1}{2} + \frac{1}{\pi} \right). \quad (2)$$

After the floating dot is charged, the electric potential at the channel dot has an additional contribution from the excess charge,  $Q_f$ , distributed on the rim of the floating gate (i.e., in a ring shape):

$$\phi_f(r, \theta, Q_f) = \frac{Q_f}{\pi \epsilon \sqrt{r} r_f \sin \theta} kK(ik), \quad (3)$$

where  $r$  and  $\theta$  are the spherical coordinates with the origin located at the center of the ring,  $r_f$  is the radius of the ring,  $K(x)$  is a complete elliptic integral of the first kind,  $\epsilon$  is the dielectric constant, and  $k^2 = 4r r_f \sin \theta / (r^2 + r_f^2 - 2r r_f \sin \theta)$ .

Upon charging of the floating gate, the potential at the center of the channel dot at the new threshold voltage,  $V_{g2}$ , is given by the sum of the two terms represented by Eqs. (2) and (3) with  $\theta=0$ :

$$\phi(V_{g2}) = V_{g2} \left( \frac{1}{2} + \frac{1}{\pi} \right) + \frac{Q_f}{\epsilon \sqrt{a^2 + r_f^2}}. \quad (4)$$

Here  $a$  is the thickness of the oxide layer separating the floating dot and the quantum dot channel (see Fig. 3).

Since at the threshold voltages  $V_{g1}$  and  $V_{g2}$ , the potential  $\phi$  in the channel dot is the same, Eqs. (2) and (4) must equal to each other. This gives us a relation between the charge on the floating gate and the threshold voltage difference before and after charging,  $\delta V_g = V_{g2} - V_{g1}$ :

$$Q_f = \delta V_g \left( \frac{1}{2} + \frac{1}{\pi} \right) \epsilon \sqrt{a^2 + r_f^2}. \quad (5)$$

Now let us calculate the energy level spacings in the channel dot before and after charging  $Q_f$  into the floating gate. The ground state energy of two interacting electrons in a two-dimensional parabolic quantum potential has been numerically calculated by Merkt *et al.*<sup>6</sup> It was done by exact numerical diagonalization of the two-particle Hamiltonian of the system written in the effective-mass approximation. The ground state energy was plotted against the characteristic frequency of the parabolic confining potential. This means that we can obtain the ground state energy of the two-particle interacting system if the parabolic confinement potential is known, or vice versa.

In our calculation, we obtain the initial confining parabolic potential before charging the floating gate from the measured energy level spacing in the channel dot. Then we add to the Hamiltonian the additional confining potential due to the charge-ring on the floating gate, to obtain a new parabolic potential. From the new potential and Merkt’s graph,

we obtain the new energy level separation. The new energy level spacing can be compared with the experiment.

Before the floating dot is charged, electrons are already confined within the channel quantum dot by a potential created by the walls of the quantum dot. We model this potential with a parabolic well,  $U_L(\rho) = (m\omega_L^2/2)\rho^2$ , where  $\rho$  is the distance from the center of the dot,  $m$  is the effective electron mass, and  $\omega_L$  is the characteristic frequency of oscillations in such a well. The spacing between the first and the second energy levels in the conductance oscillations is proportional and thus determined by the difference between the ground state energy of one electron in the quantum dot, given by  $\hbar\omega_L$ , and the ground state energy after adding a second electron into the quantum dot (i.e., the energy difference between the one-particle ground state and the two-particle ground states). Using the curve obtained in Ref. 6, the measured value of the channel quantum dot level spacing before charging of the floating dot which is 2.5 meV, and the value of the effective Rydberg constant in silicon, we obtain the characteristic energy of the lateral confining potential to be  $\hbar\omega_L \approx 0.6$  meV.

After the floating dot is charged, the energy level spacing in the channel quantum dot changes due to the additional confining potential caused by the excess charge ( $Q_f$ ) on the floating gate. This potential is expressed as  $U_E(x) = e\phi(x) - e\phi(0)$ . In the vicinity of the minimum, the potential is essentially parabolic and can be obtained by expansion of Eqs. (1) and (3) as  $U_E(\rho) = (m\omega_E^2/2)\rho^2$ . Here the characteristic frequency,  $\omega_E^2 \approx e/m [Q_f/\epsilon r_f^3 - 4\delta V_g/\pi R_g^2]$ , depends on the amount of the excess charge on the floating dot;  $R_g$  is the radius of the control gate. Note that there are also electrons distributed on the side wings of the floating gate, but the additional confinement due to repulsion from these charges is over a magnitude weaker as compared with that from the charge ring on the polysilicon dot. After charging of the floating dot, the total confining potential for the electrons inside the channel dot is a sum of two terms,  $U(\rho) = U_L(\rho) + U_E(\rho) = (m\omega_0^2/2)\rho^2$ , where the renormalized characteristic frequency of the confining potential is given by  $\omega_0^2 = \omega_L^2 + \omega_E^2$ . This means that from  $Q_f$  and  $\delta V_g$  the new confinement potential can be obtained, which in turn gives the new energy level spacing through Merkt's graph.

Now we put the experimental data for the device geometry and the threshold voltage shift into the equations derived above, and calculate the charges on the floating dot and the energy level spacing in the channel quantum dot. From the measurements we know that the threshold voltage is shifted by  $\delta V_g = 0.3$  V after charging the floating dot. Taking the radius of the floating dot and oxide thickness to be  $r_f = 25$  nm and  $a = 2$  nm, respectively, we obtain from Eq. (5) that the charge accumulated on the floating gate,  $Q_f$ , is  $3.3 \times 10^{-18}$  C, i.e., about 20 electrons. Using the radius of the control gate  $R_g \approx 40$  nm, the  $r_f$ , and the  $Q_f$ , the characteristic energy becomes  $\hbar\omega_0 \approx 2.2$  meV, which is significantly larger than it was before the charging of the floating dot. Now, provided with the value of  $\omega_0$  and with the curve obtained in Ref. 6, we can find the energy separation corresponding to the second Coulomb blockade peak from the first one in the conductance, which appears to be  $\delta E = 6.2$  meV. This value is rather close to the interpeak separation of 5.5 meV observed experimentally after the charging of the floating dot.

In summary, we propose a model explaining the significant enhancement of the conductance oscillations in the stacked quantum-dot transistor that followed after charging of the floating dot. The effect is explained as a result of the electrostatic "squeezing" of the Si quantum dot by the charged floating gate, which increases the energy level separation in the Si dot.

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- <sup>1</sup>L. J. Guo, E. Leobandung, and S. Y. Chou, *Science* **275**, 649 (1997).
- <sup>2</sup>A. Nakajima, T. Futatsugi, K. Kosemura, T. Fukano, and N. Yokoyama, *Appl. Phys. Lett.* **70**, 1742 (1997).
- <sup>3</sup>L. J. Guo and S. Y. Chou, *Electron. Lett.* **34**, 1031 (1998).
- <sup>4</sup>C. W. J. Beenakker, *Phys. Rev. B* **44**, 1646 (1991).
- <sup>5</sup>Charge distribution in a conductive disk of radius  $a$  is given by  $\sigma = (e/4\pi a^2)(1 - r^2/a^2)^{-1/2}$ , see L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media*, 2nd ed. (Pergamon, New York, 1984).
- <sup>6</sup>U. Merkt, J. Huser, and M. Wagner, *Phys. Rev. B* **43**, 7320 (1991).
- <sup>7</sup>P. Moon and D.E. Spencer, *Field Theory Handbook* (Springer, New York, 1971).
- <sup>8</sup>See, e.g., J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975).