Resonant tunneling of electrons of one or two degrees of freedom

S. Y. Chou, E. Wolak, and J. S. Harris, Jr.
Stanford Electronics Laboratories, Stanford University, California 94305

(Received 30 October 1987; accepted for publication 21 December 1987)

Analytical expressions for the tunneling current of electrons with one or two degrees of freedom (DOF), due to additional quantum confinement transverse to the electron transport direction, are explicitly derived, analyzed, and implemented into computer simulations. The results are compared with the well-known case in which 3-DOF electrons tunnel through a one-dimensional double-barrier well. The results show that the singularity of the density of states in a one-dimensional system will not manifest sharp features in tunneling current, and that when the spacing between the Fermi energy and bottom of conduction band is the same, the tunneling current peak becomes broader and the peak-to-valley ratio becomes smaller as the number of degrees of freedom of the electrons is reduced. The results also show that when scattering is neglected, the energy quantization due to transverse confinement in 1- or 2-DOF systems will not contribute any additional peaks to the tunneling current.

Since the original work by Chang *et al.* in 1974,[1] the resonant tunneling of electrons with three degrees of freedom (DOF) through a one-dimensional double-barrier quantum well has been studied extensively both experimentally[2] and theoretically.[3] As lithography, dry etching, epitaxy, and other semiconductor device fabrication technologies continue to improve, it is possible to fabricate double-barrier tunneling diodes of a cross section with one or both lateral dimensions as small as 0.1 μm or less.[4,5] In these confined small structures, an additional quantization transverse to the electron transport is introduced. As a result, the electrons have only one or two degrees of freedom and tunnel through a three- or two-dimensional double-barrier quantum well. It is important to understand how the transport in such 1- or 2-DOF systems differs from that in a 3-DOF system. It has been speculated that the additional transverse energy levels in a one- or two-dimensional system might create additional peaks in the tunneling current. However, this has not been observed in some recent experiments.[5,6]

In this letter we derive explicit analytical expressions for electrons of 1- or 2-DOF tunneling through a three- or two-dimensional double-barrier quantum well and explain the differences in the current-voltage (*I-V*) characteristics of tunneling electrons with different DOF by analyzing the differences of their electron supply functions and by using computer simulation. Finally, we will show that when scattering can be neglected, no additional current peaks are predicted in a 1- or 2-DOF system.

When scattering is neglected, a general expression for electrons tunneling through a double barrier can be written as

$$J = 2q \int d\mathbf{k}\, g(\mathbf{k}) v_l [f(E) - f(E+qV)] |T(E_l, V)|^2,$$

where q is the magnitude of electron charge, v_l and E_l are, respectively, the electron velocity and energy in the direction of tunneling, E is the total electron energy, f is the Fermi–Dirac distribution function, V is applied voltage, and $|T|^2$ is the tunneling probability and independent of electron transverse energy. The detailed expressions for $d\mathbf{k}$ and density of states for momentum $g(\mathbf{k})$ depend on the degree of freedom of the electrons. Knowing $v_l\, dk_l = dE_l/\hbar$, we have as follows:

for 1-DOF:

$$J_1 = \frac{2q}{h} \int dE_l |T|^2 [f(E) - f(E+qV)], \tag{1}$$

where h is Planck's constant, and $E = E_l + E_{\parallel}$, in which electron energy transverse to the tunneling direction, $E_{\parallel}$, is quantized:

for 2-DOF:

$$J_2 = \frac{q}{\pi h} \int dE_l |T|^2 \int dk_y [f(E) - f(E+qV)], \tag{2}$$

where $E = E_l + E_x + E_y$ in which E_x is quantized:

for 3-DOF:

$$J_3 = \frac{4\pi qmkT}{h^3} \times \int dE_l |T|^2 \ln\left(\frac{1+\exp(E_f - E_l)/kT}{1+\exp(E_f - E_l + qV)/kT}\right), \tag{3}$$

where m is the electron mass in the semiconductor, k is Boltzmann's constant, T is the temperature, and an integration over transverse direction has been carried out.

We can see from Eq. (1) that the singularity of 1-DOF density of states for energy ($\propto 1/\sqrt{E_l}$) is canceled out by the electron velocity and, therefore, will not result in any sharp features in the tunneling current.

In the above derivation, we considered only the ground state in a 1- or 2-DOF system. Since there is more than one quantum level, the total current should be the summation of tunneling currents from all states on one side of a diode to those states on the other side. When scattering is neglected and transverse confinement is the same through the entire diodes, the transverse variables in the Schrödinger equations can be separated from variables in the transport direction, and the energy in the transverse direction must be conserved. This means that, for a double-barrier quantum well system, only the resonant tunneling from a state on one side of the diode to the same state on the other side through a state of the same transverse energy in the quantum well is allowed.

Hence the total current density can be written as

$$J_{\text{tot}} = \sum_i J_{i\to i}(E_i),$$

where $J_{i\to i}$ is given by Eqs. (1) and (2), $E_i = E_x + E_y$ for 1-DOF, and $E_i = E_x$ for 2-DOF.

From the above analysis, we can see that when scattering is neglected, the tunneling probability is independent of the transverse momentum. Therefore, electrons in any transverse state in a 1- or 2-DOF system experience the same double barrier in the transport direction as that in a 3-DOF system. The calculation of tunneling current for the different transverse quantum states in 1- or 2-DOF is the same as that for their ground state, except when using an effective Fermi level, $E_F - E_i$, where E_i is the energy of a transverse state relative to that of the ground state.

To see the difference in the *I-V* characteristics of resonant tunneling between electrons with different DOF, we first discuss the case in which only tunneling through the ground state is considered. Results will be valid even when more than one transverse quantum level is considered, as will be discussed later. Equations (1)–(3) can be rewritten in the following form:

$$J = \int dE_l \, |T(E_l)|^2 S(E_l),$$

where S is defined as the electron supply function that represents the electron density available for tunneling. Since $|T|^2$ does not depend on the transverse energy of electrons and is the same for electrons of 1-, 2-, or 3-DOF, the difference of electron supply functions determines the difference of their *I-V* characteristics.

At zero temperature, and when $qV > E_f$, the supply functions are independent of bias V and can be written as follows:

for 3-DOF:

$$S_3 \propto \begin{cases} (1 - E_l/E_f), & E_l \leqslant E_f, \\ 0, & E_l > E_f, \end{cases}$$

for 2-DOF:

$$S_2 \propto \begin{cases} \sqrt{(1 - E_l/E_f)}, & E_l \leqslant E_f, \\ 0, & E_l > E_f, \end{cases}$$

for 1-DOF:

$$S_1 \propto \begin{cases} 1, & E_l < E_f, \\ 0, & E_l \geqslant E_f, \end{cases}$$

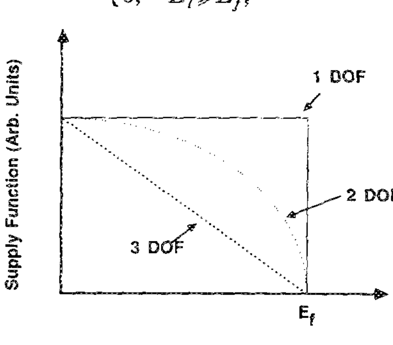

FIG. 1. Schematic of supply functions of electrons of 1-, 2-, and 3-DOF at $T = 0$ and when $qV > E_f$.

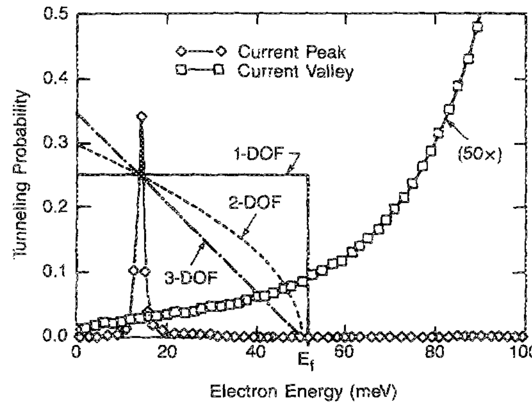

FIG. 2. Supply functions for electrons of 1-, 2-, or 3-DOF, and the tunneling probabilities corresponding to current peak and valley. The supply functions are normalized in such a way that current peaks are the same for 1-, 2-, or 3-DOF.

where E_f is the Fermi level measured from the bottom of the conduction band. Figure 1 shows the supply functions of different degrees of freedom, provided that they have the same Fermi energy.

To see how the supply functions affect the *I-V* characteristics, we show, in Fig. 2, the supply functions at zero temperature and the distribution of the electron tunneling probabilities corresponding to the peak and valley of the tunneling current. The probability distribution for the current valley is enlarged by 50 times. We normalized the supply functions in such a way that the peak currents for different DOF are the same. As shown in Fig. 2, the peak value of tunneling current is determined mainly by the peak of tunneling probability and the supply function's value in the region where the probability peak is; the distribution of the supply functions in other regions is insignificant. At the tunneling current valleys, the peak of tunneling probability is far away from the Fermi energy and therefore has little effect on the tunneling current. The entire supply function distribution plays an important role in determining the total valley current. Clearly, the 3-DOF electrons have the smallest valley current, because their supply function is smaller than that of the other two. For the same reasons, the 1-DOF electrons have the largest valley current. As a consequence, for a

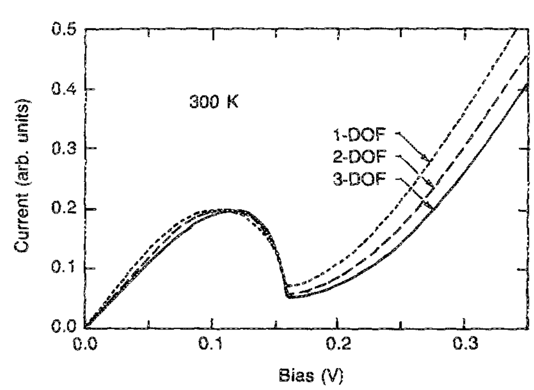

FIG. 3. Currents for electrons of 1-, 2-, or 3-DOF tunneling through a 5nm/5nm/5nm GaAs/$Al_{0.3}Ga_{0.7}As$/GaAs double-barrier quantum well at 300 K. The current peaks are normalized.

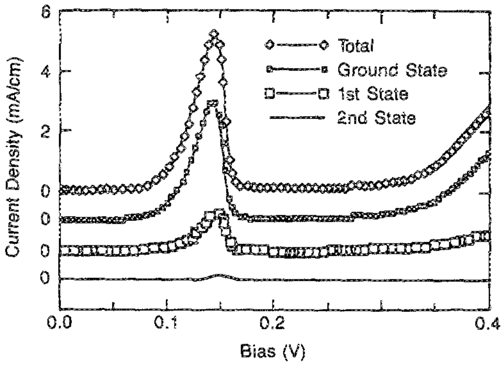

FIG. 4. At 77 K, the total tunneling current of a 2-DOF system of three transverse quantum states and the currents of each of the individual states. The system has a 5nm/5nm/5nm GaAs/$Al_{0.3}Ga_{0.7}As$/GaAs double-barrier quantum well.

given spacing between the Fermi level and the bottom of the conduction band, the peak-to-valley (PTV) ratio is largest for 3-DOF and smallest for 1-DOF. The peak width in 3-DOF is smaller than that of 1- or 2-DOF, because the supply function of electrons of 3-DOF decreases faster with energy than that of electrons with 1- or 2-DOF. By similar analysis, the peak width in 1-DOF should be the broadest.

Figure 3 shows the computer simulation for the tunneling current for electrons of 1-, 2-, and 3-DOF tunneling through a GaAs/$Al_{0.3}Ga_{0.7}As$ double-barrier quantum well with the following properties: barrier thickness, 5 nm; barrier height, 0.23 eV; and well width, 5 nm. The simulation was done by assuming their Fermi levels were 43 meV above their own conduction-band bottom for all three cases. For comparison, the values of current peaks were normalized. Indeed, it can be seen that as the number of DOF is reduced, the valley current and peak width increases, and the peak-to-valley ratio decreases.

It must be pointed out that, for a given degree of freedom, reduction of Fermi level can increase the PTV ratio and reduce current peak width.[7] This implies that if the contact doping concentration of a tunneling diode is fixed, and the transverse confinement is introduced to reduce the electron's DOF, the PTV ratio may increase, and peak width may decrease. This results because the spacing between the Fermi level and the bottom of the conduction band becomes smaller as number of degrees of freedom is reduced with fixed doping, and the improvement in the PTV ratio and peak width from the reduction of the Fermi level may overcome the degradation from the effect of supply functions.

Finally, we examine the possibility of observing additional current peaks in a 1- or 2-DOF system when scattering is neglected. In order to observe the tunneling current peaks due to tunneling through the excited states, both of the following conditions must be satisfied: (1) the position of the additional current peak must be separated enough from the ground-state peak to be resolved; (2) the magnitude of the additional tunneling peak has to be larger than the background tunneling current of the ground state. When scattering is neglected, the tunneling current for excited transverse states can be calculated, as pointed out earlier, in the same way as that for the ground state, except with the use of a smaller effective Fermi level. Reduction of the effective Fermi level hardly shifts the position of peak current, but decreases the amplitude of the tunneling current peak almost exponentially.[7] This means that for a transverse quantum level that is well above the ground state, its current peak is too small to make an observable contribution, and that for a transverse quantum level that is close to the ground state, its peak is too close to the ground-state peak to be distinguishable. For an example, we consider a 2-DOF GaAs/AlGaAs double-barrier diode consisting of three transverse states at 77 K. The first and second excited states are, respectively, 11.5 and 28.5 meV above the ground state. If there are 3×10^{11} cm^{-2} electrons in the ground state, then there are 10^{11} cm^{-2} electrons in the first excited state and 10^{10} cm^{-2} electrons in the second. Compared with the ground-state current peak, the first excited-state peak position shifts only 4 mV, and its amplitude is 3.3 times smaller; and the position of the second excited peak shifts 6 mV, and its amplitude is 35 times smaller. Figure 4 shows the tunneling currents for each state and the total current calculated with the use of Eq. (2). Clearly, the peaks due to the first and second excited states cannot be distinguished from the ground-state peak. It can be shown that the same situation is also true for resonant tunneling of electrons with 1-DOF.

In summary, by deriving explicit analytical expressions for the tunneling current of electrons with one or two degrees of freedom, analyzing their supply functions, and using computer simulations, we have shown that for a given spacing between the Fermi energy and the bottom of conduction band, as the degrees of freedom of the electrons decreases, the tunneling current peak becomes broader, and peak-to-valley ratio becomes smaller. For a fixed contact doping concentration, the reduction of DOF may result in a narrower peak width and better PVT ratio, because of the improvement from reducing the spacing between the Fermi level and the conduction-band bottom. We have also shown that when scattering is neglected, the energy quantization caused by transverse confinement in a 1- or 2-DOF system will not contribute any additional distinguishable peaks in tunneling current.

S. Chou would like to thank R. F. W. Pease, and T. W. Sigmon for their encouragement. E. Wolak would like to thank A. Harwit, T. C. L. G. Sollner, and S. Ullah for useful discussions. The work was supported by Defense Advanced Research Projects Agency under contract No. N00014-86-K-0530 and Joint Services Electronics Program under contract No. DAAG-29-85-K0048.

[1] L. L. Chang, L. Esaki, and R. Tsu, Appl. Phys. Lett. **24**, 593 (1974).
[2] For example, T. C. G. Sollner, W. D. Goodhue, P. E. Tannenwald, C. D. Parker, and D. D. Peck, Appl. Phys. Lett. **43**, 588 (1983); M. Tsuchiya, H. Sakaki, and J. Yoshino, Jpn. J. Appl. Phys. **24**, part 2, L-466–468 (1985).
[3] For example, Hiroaki Ohnishi, Tsuguo Inata, S. Muto, N. Yokoyama, and A. Shibatomi, Appl. Phys. Lett. **49**, 10 (1986); B. Ricco and M. Y. Azbel, Phys. Rev. **29**, 1970 (1984).
[4] J. Cibert, P. M. Pertroff, G. J. Dolan, S. J. Pearton, A. C. Gossard, and J. H. English, Appl. Phys. Lett. **49**, 10 (1986).
[5] J. N. Randall, M. A. Reed, T. M. Moore, R. J. Matyi, and J. W. Lee, in Proceedings of the 31st International Symposium on Electron Ion and Photon Beams, Woodland Hills, LA, May 1987.
[6] M. Read, J. Superlattices Microstruct. (in press).
[7] S. Y. Chou and J. S. Harris, Jr. (unpublished).