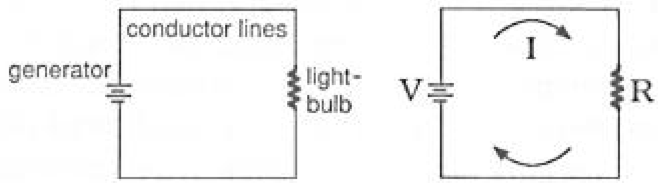
Networks for Electric Lighting



Ohm's law (1)

V	=	I	R
voltage at the generator		current flowing in the circuit	resistance of the lamp
volts		**amps**	**ohms**

Joule's law (2)

P	=	V	I
power from the generator		voltage at the generator	current flowing in the circuit
watts		**volts**	**amps**

Substitute (1) into (2)

I^2	R	=	P
current squared	resistance of the lamp		power converted to light

Note: Formulas for this simplified network neglect the power losses in the lines.

Current in One Coil

Ohm's law, $V = IR$ **Resistance** $R = \rho \frac{L}{A}$

V = *The voltage* produced by the battery which we assume, for Henry's experiment 15, to have been 1 volt.

R = *The resistance* throughout each of the 9 circuits of length L.

L = 60 ft., *the length* for one wire going from the battery to the magnet, coiled around the iron core, and returning to the battery (one circuit).

A = 0.00159 sq. in. the *cross-sectional* area of the .045-in. diameter copper wire.

ρ = The *resistivity* of the wire material, which for copper is 0.67×10^{-6} ohm-in.

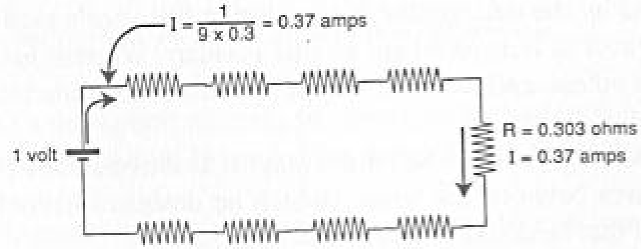
Hence, the resistance in each of the 9 parallel circuits is

$$R = 0.67 \times 10^{-6} \frac{60 \times 12}{1590 \times 10^{-6}} = 0.303 \textit{ ohms}$$

and, from Ohm's law, we calculate the current as

$$I = \frac{V}{R} = \frac{1}{0.303} = 3.3 \textit{ amps} \text{ in each of 9 parallel circuits.}$$

Series Circuit for 9 Coils



$$I = \frac{V}{R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7 + R_8 + R_9} = \frac{1}{9 \times 0.303} = \mathit{0.37\ amps}$$

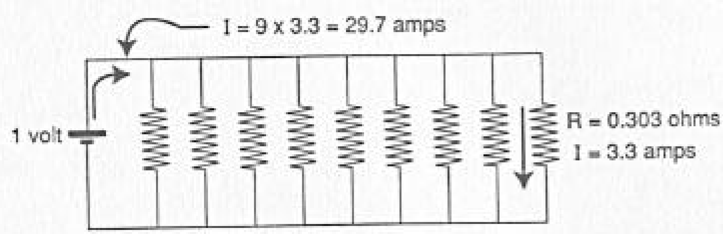
$$\frac{IN}{L_C} = \frac{0.37(9 \times 80)}{23} = \frac{266}{23} = 11.6\ \frac{\text{ampere–turns}}{\text{in.}}; \quad B \approx 30{,}000\ \frac{\text{lines of flux}^*}{\text{sq. in.}}$$

$$2F = \frac{2(30{,}000)^2\,4}{7.213 \times 10^7} = 100 \text{ lbs.}$$

Henry got over seven times the strength by using a parallel instead of a series circuit for his electromagnet.

*Note that B is not linearly proportional to $\frac{IN}{L_c}$.

Parallel Circuit for 9 Coils

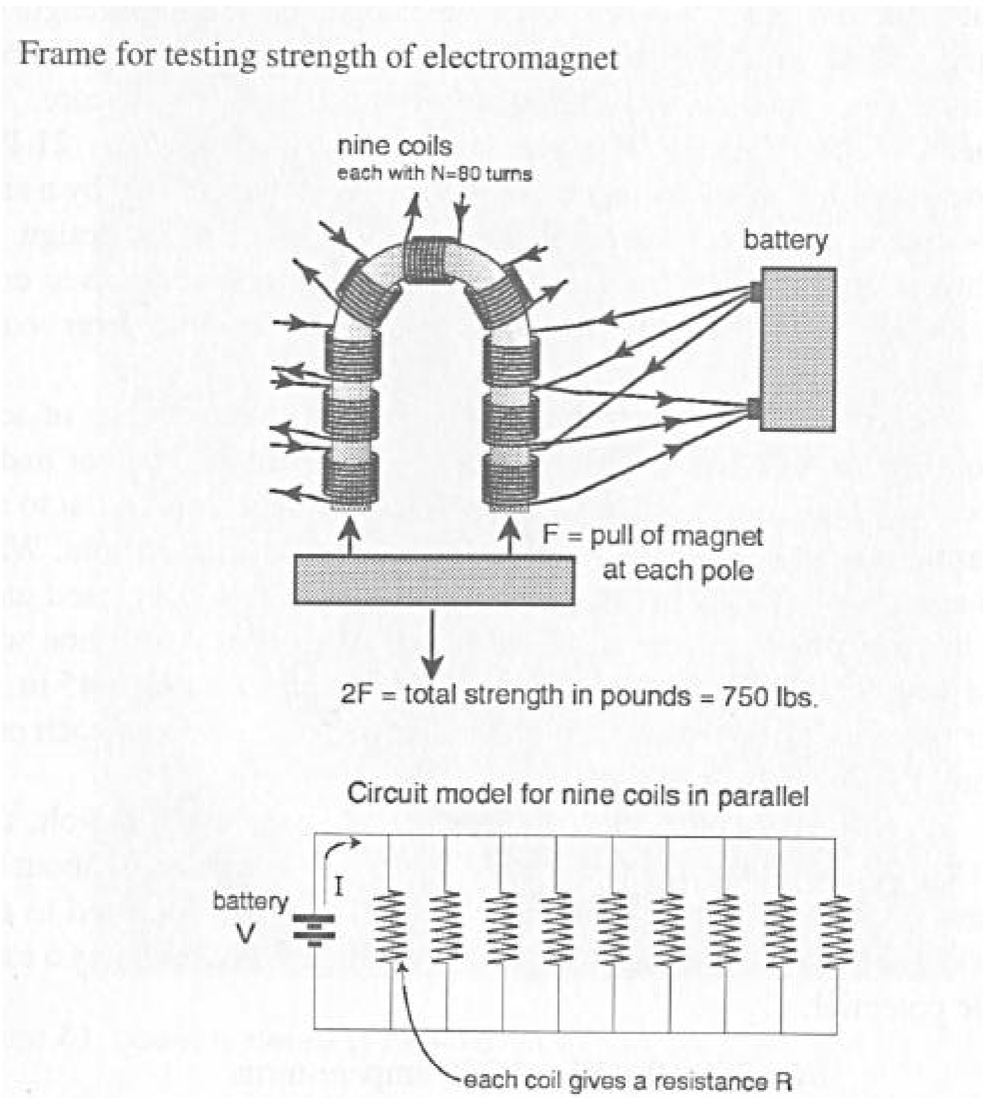


$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \frac{V}{R_4} + \frac{V}{R_5} + \frac{V}{R_6} + \frac{V}{R_7} + \frac{V}{R_8} + \frac{V}{R_9} = 9 \times \frac{1}{0.303} = \mathit{29.7\ amps}$$

$$\frac{IN}{L_C} = \frac{(3.3)(720)}{23} = 103\ \frac{\text{ampere–turns}}{\text{in.}} \quad B \approx 82{,}000\ \frac{\text{lines of flux}}{\text{sq. in.}}$$

$$2F = \frac{2(82{,}000)^2\,4}{7.213 \times 10^7} = 746 \text{ lbs.}$$

Joseph Henry's Magnet



Magnet Strength

The strength of the electromagnet, F, depends upon the flux density B and the pole area A.

$$F = \frac{B^2 A}{72{,}130{,}000} \text{ lbs.}$$

The flux density B depends upon the number of turns of wire ($N = 9 \times 80 = 720$ turns) coiled around a core of length $L_C = 23$ in., the current flowing through the wires ($I = 3.3$ amps in each turn) and the material properties of the core:

$$\frac{IN}{L_C} = \frac{3.3(720)}{23} = 103 \text{ ampere-turns per in.}$$

The flux density varies with the quantity IN/L_C as well as with the materials; it must be taken from experimentally derived magnetization curves. For the soft iron used by Henry, we can estimate, taking standard curves for iron and steel with $IN/L_C = 103$,

*B = 82,000 lines of flux per sq. in.**

The pole area A is the cross-sectioned area of one end of the horseshoe core or $2 \times 2 = 4$ sq. in. Therefore, the total strength of both poles

$$2F = \frac{2(82{,}000)^2(4)}{7.213 \times 10^7} = 746 \text{ lbs.}$$

is about what Henry measured.

*For such curves see, for example A. L. Cook and C. C. Carr, *Elements of Electrical Engineering*, 5th ed., New York, 1947, p. 30. We have interpolated and picked the value that gives Henry's result.