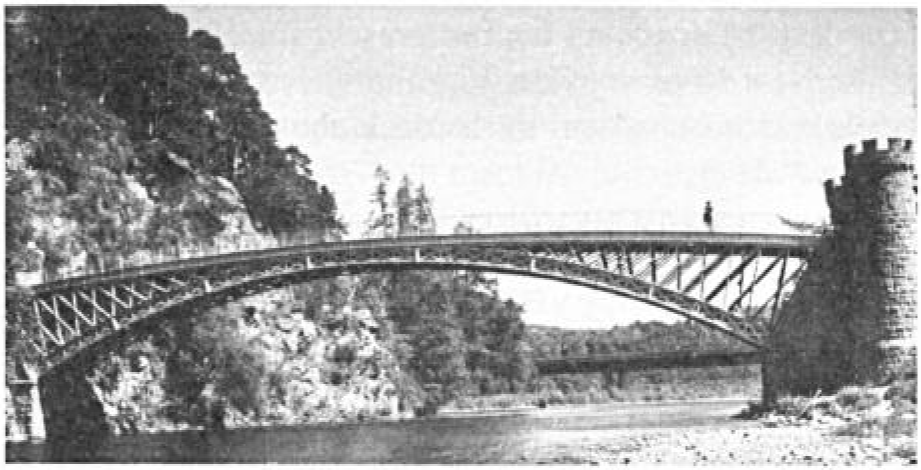
Craigellachie Bridge

Arch



q = 3.0 kips/ft. uniformly distributed load along the bridge deck*
(one kip equals 1,000 pounds)
L = 150 ft. = arch span
d = 20 ft. = arch rise

The horizontal force at each support:

$$H = \frac{qL^2}{8d} = \frac{(3.0)(150)^2}{8(20)} = 422 \text{ kips}$$

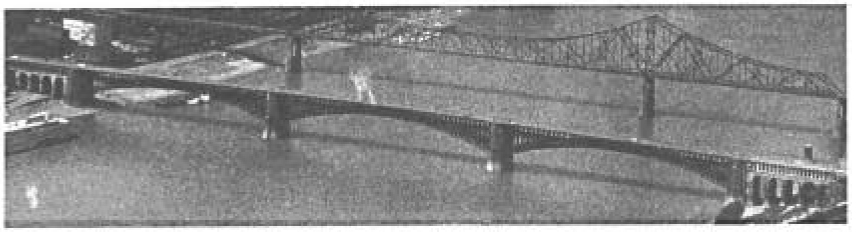
The vertical force V (in this case, half the bridge load) at each support:

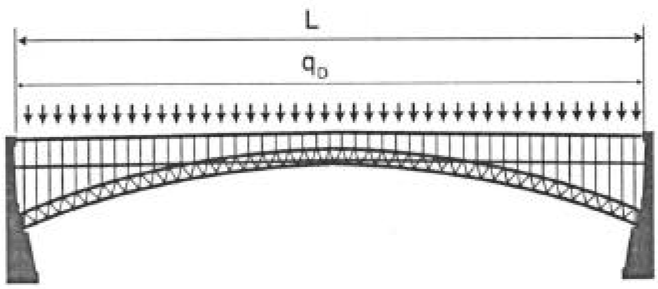
$$V = \frac{qL}{2} = \frac{(3.0)(150)}{2} = 255 \text{ kips}$$

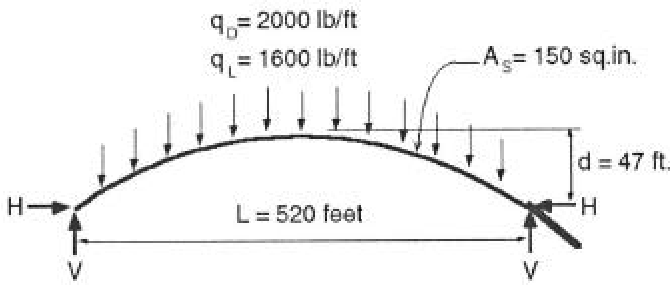
*See *Life of Thomas Telford*, 1854, p. 685. The ironwork weighs 2 kips/ft., and we assume an additional load of 1 kip/ft.

Eads Bridge

Model for Uniform Loads







$$H = \frac{q L^2}{8d}$$

Model for equilibrium of uniform load on each of the four steel arches that make up the central span. Arch is assumed to be three hinged.

Calculations for Uniform Loads

Self-weight (uniform dead load)

$$H_D = \frac{2{,}000(520)^2}{8(47)} = 1{,}438{,}000 \text{ lbs.*}$$

*The axial compression increases from midspan to the supports by the factor $1/\cos\phi$ where ϕ is the slope angle of the arch. In this case the maximum increase is only 7 percent, which we neglect in this discussion. Eads provided more steel area near the supports to carry the increased forces there.

$$C = T = \frac{M}{h}$$

Calculations for Nonuniform Loads

For this case of half-loading, Eads calculated a bending moment at the quarterspan of 3,528 ft.-kips. (See Woodward, note 21.) So the additional force in one chord is

$$C = T = \frac{M}{h} = \frac{3{,}528}{12} = 294 \text{ kips}$$

Therefore, the compressive stress in the top chord would be

$$f_b = \frac{C}{A_S} = \frac{294{,}000}{75} = 3{,}920 \text{ psi*}$$

Had Eads chosen a closer spacing for the chords (such as $h = 3$ ft.), the forces and stresses would have changed as follows:

$$C = \frac{M}{h} = \frac{3{,}528}{3} = 1{,}176 \text{ kips}$$

Therefore, the compressive stress in the top chord would be:

$$f_b = \frac{C}{A_S} = \frac{1{,}176{,}000}{75} = 15{,}680 \text{ psi*}$$

With $h = 3$ ft., the stress increased by a factor of 4. Thus, Eads's design intelligently incorporates a large chord spacing, $h = 12$ ft., to minimize the influence of bending under nonuniform train loads. By splitting the arch into two widely spaced sections he is able to maintain the safety of the bridge without the use of added material (i.e., increased cost).

*At the quarterspan, the axial stress due to dead load is 9,740 psi and due to half-span live loads is 3,900 psi (assuming $A_S = 150$ sq. in.). Adding these values to the bending stress of 3,920 psi produces a total stress of 17,560 psi. So, in this case, the additional stress due to bending does not increase the arch stress much above the 17,260 psi found for the full uniformly distributed loads.

$$f_D = \frac{H_D}{A_S} = \frac{1{,}438{,}000}{2(75)} = 9{,}590 \text{ psi at midspan}$$

Full train load (uniform live load on whole span)

$$H_L = \frac{1{,}600(520)^2}{8(47)} = 1{,}151{,}000 \text{ lbs.}$$

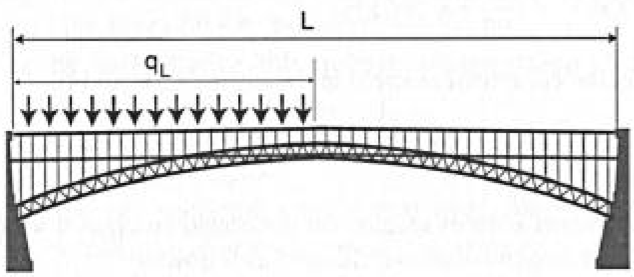
$$f_L = \frac{H_D}{A_S} = \frac{1{,}151{,}000}{2(75)} = 7{,}670 \text{ psi at midspan}$$

Total stress

$$f_{\text{TOTAL}} = f_D + f_L = 9{,}590 + 7{,}670 = 17{,}260 \text{ psi at midspan}$$

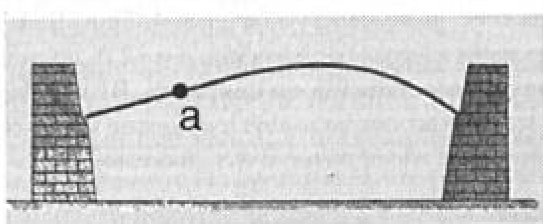
This is a safe value because Eads considered the maximum allowable stress for steel to be 30,000 psi.

Model for Nonuniform Load



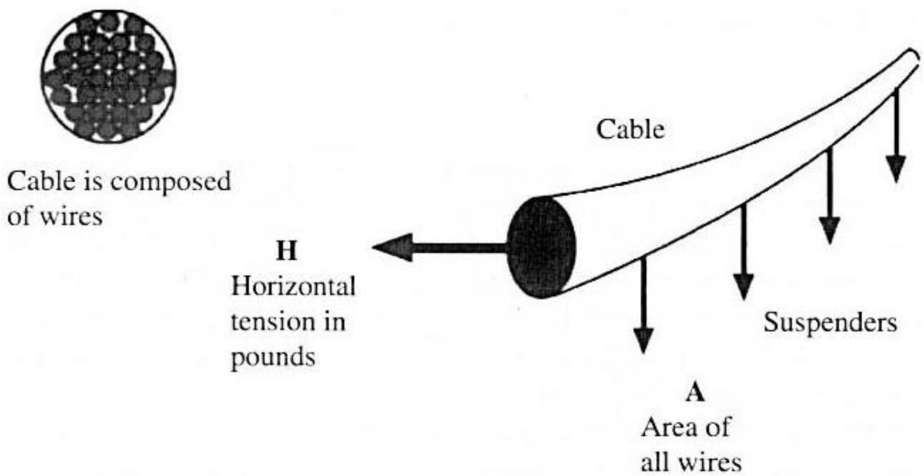
Bending in the Arch

Eads considered the possibility that heavy trains nonuniformly distributed on the bridge deck might lead to additional stresses in the arch. The following drawings illustrates the bending Eads anticipated in the half-loaded span:



This bending, which is described by a "bending moment," M, introduces additional forces into the arch chords: compression (C) in the top chord and tension (T) in the bottom chord. These forces are inversely proportional to the distance between the chords, h. Eads chose h = 12 ft. a indicates the quarterspan.

How a Cable Works



$$f = \frac{H}{A} \text{ or } A = \frac{H}{f}$$

A = area (square inches)

H = tension (pounds)

f = allowable wire stress (pounds/square inch)

f_B = wire breaking strength (pounds/square inch)

Where A (area) = .05 square inches for each wire and H_B = 10,000 pounds breaking strength per wire

$$f_b = \frac{H_B}{A} = \frac{10{,}000}{.05} = 200{,}000 \text{ psi (pounds per square inch) breaking strength}$$

$$\text{Safety factor} = \frac{\text{breaking strength}}{\text{allowable stress}} = \frac{f_B}{f}$$

When one chooses a safety factor of 2: $\frac{f_B}{f} = 2$

Thus, allowable stress: $f = \frac{f_B}{2} = \frac{200{,}000}{2} = 100{,}000$ psi

If one decides to reduce the safety factor to 1.33, the allowable stress would be:

$$f = \frac{f_B}{1.33} = \frac{200{,}000}{1.33} = 150{,}000 \text{ psi}$$