Multilayer electromechanical composites with controlled piezoelectric coefficient distribution

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ABSTRACT

We have fabricated multilayer electromechanical composites with controlled piezoelectric coefficient distributions using tape casting. Tapes of doped lead zirconate titanate were cut and stacked in accordance with their characteristic electromechanical coupling values and modulus of elasticity. This technique is an extremely versatile method to fabricate displacement actuators to fabricate monolithic ceramic parts with controlled material property gradients. To obtain a quantifiable method to optimize this type of transducer, we have devised a processing model. Given the functional distribution of the electromechanical coupling coefficient, \(d_{31}\), and the functional distribution of elastic modulus through the thickness of the transducer, the analysis predicts the displacement as a function of loading. The tape casting method coupled with the model provides an actuator that maximizes displacement and generated force for the given material properties.

Keywords: actuator, multilayer, piezoelectric, tape casting

1. INTRODUCTION

There has been a continuous effort to improve piezoelectric actuator systems so that simultaneously large displacements, on the order of hundreds of microns, can be obtained while subjected to large forces, on the order of Newtons. Two recently developed piezoelectric actuators, moonies¹ and rainbows² have made significant improvements in obtainable displacements and generated forces. Both contain a shell-like structure and obtain large axial displacements at the apex of the shell through bending stresses. We focus on the rainbow and seek ways to improve the displacement and load bearing properties.

Rainbows achieve their bending stress from a variation in the \(d_{31}\) value of electromechanical coupling across the thickness of the actuator. The gradient in \(d_{31}\) is obtained by selectively reducing one side of a plate (with respect to oxygen content). Rainbows, although achieving large displacements while sustaining moderate loads, are limited from a processing standpoint. Chemical reduction produces only one functional gradient in electromechanical properties across the thickness of the actuator. The resulting functional gradient does not follow the theoretically expected hyperbolic tangent diffusion relation, but contains a pre-reduction zone.³ The grain boundaries within the intermediate pre-reduction zone are modified by the reduction. We present a processing scheme that can customize an actuator for a given function, by providing considerable control over the material property gradients. Our approach permits sharp functional gradients that are not severely altered by solid state diffusion.

Utilizing the benefits of creating displacement through a transverse bending stress, we started to process transducer plates with \(d_{31}\) variations made by the tape casting method. This versatile fabrication route produces 25 to 60 \(\mu m\), uniform sheets of lead zirconate titanate (PZT), that serve as the building block of the actuator. Individual sheets of piezoelectric material can be stacked according to different material properties. We have developed a computational model that serves as
a useful guide in the processing. The analysis supplies quantified design characteristics to produce a transducer with the greatest displacement and load bearing abilities for the given material constants. Incorporating this information with the tape cast processing strategy, we can produce "tailor-made" actuators.

2. PROCESSING OF TRANSDUCERS

Tape casting is frequently utilized in the electronics industry for a variety of applications. Thin, uniform tapes can be made with ceramic contents that exceed 50 volume percent. This process is a natural choice for the fabrication of materials with functional gradients. We have made stable suspensions of various PZT powders that have $d_{31}$ values ranging from $-262 \times 10^{12}$ to $-3 \times 10^{12}$ m/V. The mixing of different powders allows the creation of a continuous array of properties.

An outline of the process is shown in Fig. 1. The suspensions are mixed with polymers and an appropriate solvent. The polymers provide mechanical integrity and flexibility to the tape. A flat doctor blade sweeps the liquid mixture across a substrate. Within several minutes the cast tape is dried, cut, and separated from the backing. Any desired shape could be stamped from the tapes, but preliminary experiments use simple rectangular and circular pieces. The material is subsequently stacked in the desired order. Heat and pressure make the stacked structure into a laminated monolith by interdiffusing the long chain polymers across adjacent tapes.

During the sintering step, the discrete layers create a fully dense monolith without the aid of external pressure, producing a multilayer electromechanical composite with a "custom made" piezoelectric coefficient distribution. We should preface here that the piezoelectric coefficient distribution within the sintered specimen will be slightly different from that of the unsintered piece, due to diffusion of the PZT dopants. This would result in a smoothing of any sharp material property interfaces, as observed by Wu, et al. at the Naval Research Laboratory. Wu, et al noticed that a dopant used to alter resistivity diffused to produce a continuous, nearly linear gradient across the actuator thickness. With our approach of varying the PZT powders, our chemical analysis by energy dispersive spectroscopy shows that sharp material property interfaces remain after sintering. This technique provides a predictable control of material property distributions enabling the desired green structure to remain after sintering. The following sections describe why control is so important to actuator performance.

3. MODELING FOR FLEXTENSIONAL TRANSDUCERS

The following is a model that we are currently using to guide our fabrication. This multilayer thin plate model provides insight on the key phenomena. The theory is a natural extension to tape cast processing. First, we define a multilayer laminate composed of "n" layers. Each layer has defined values for the modulus of elasticity, the electromechanical coupling value of $d_{31}$, and the thickness.

The effect of the electromechanical coupling, $d_{33}$ is assumed insignificant. In the case that the length of the actuator is large relative to the thickness, this assumption is true. The model is limited to the linear region of displacement vs. voltage relationship. Finally, the present analysis uses the beam geometry of which bending effects along the width are neglected. For a transducer that is much longer than it is wide, this assumption will not introduce appreciable error. For the case that the width of the plate is comparable to the length, the model can be generalized to include the contribution from the whole dimension.

This section borrows ideas from a recent publication by Shih, et al. Shih, et al.'s two layer model is made more general to include many thin layers that can closely approximate a continuum. We examine the specific case of placing a
distributed force that spans the width of the transducer at the length’s mid-point and finding the resulting displacement at this location. Figures 2 and 3 define the geometry. There are two boundary conditions that must be satisfied:

(i) The sum of the bending stress must equal zero,

\[ \sum_{x=1}^{n} \int_{t_{x-1}}^{t_x} E_x (z - t_{np}) \, dz = 0, \quad \text{and} \]

(ii) the sum of the lateral stresses equals zero,

\[ \sum_{x=1}^{n} \int_{t_{x-1}}^{t_x} E_x (c - d_{31} \varepsilon) \, dz = 0. \]

We solve for \( t_{np} \) and \( \varepsilon \), where

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>( n )</td>
<td>total number of layer</td>
</tr>
<tr>
<td>( t_x )</td>
<td>the position at the top of layer ( x )</td>
</tr>
<tr>
<td>( d_{31,x} )</td>
<td>the ( d_{31} ) value of layer ( x )</td>
</tr>
<tr>
<td>( t_{np} )</td>
<td>the position of the neutral plane</td>
</tr>
<tr>
<td>( c )</td>
<td>the constrained in plane strain</td>
</tr>
<tr>
<td>( L )</td>
<td>length of the beam</td>
</tr>
<tr>
<td>( E )</td>
<td>modulus of elasticity</td>
</tr>
<tr>
<td>( v )</td>
<td>deflection of the beam</td>
</tr>
<tr>
<td>( F )</td>
<td>distributed force along mid-point</td>
</tr>
<tr>
<td>( z )</td>
<td>distance from ( t_0 )</td>
</tr>
<tr>
<td>( E_x )</td>
<td>modulus of elasticity of layer ( x )</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>the electric field</td>
</tr>
<tr>
<td>( r )</td>
<td>the radius of curvature</td>
</tr>
<tr>
<td>( t_b )</td>
<td>the position at the bottom of the first layer,</td>
</tr>
<tr>
<td></td>
<td>which is taken as the origin.</td>
</tr>
<tr>
<td>( I )</td>
<td>moment of inertia</td>
</tr>
<tr>
<td>( b )</td>
<td>width of beam</td>
</tr>
<tr>
<td>( x )</td>
<td>position on the actuator</td>
</tr>
</tbody>
</table>

Following the fundamental equation of beam theory:

\[ \frac{d^2 v}{dx^2} = -\frac{M}{EI}, \]

the displacement is expressed as a function of the bending moment.

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**Figure 2**: Geometry without load (symbols defined above).

**Figure 3**: Geometry with load (symbols defined above).
Using the results from Eqs. (1) and (2), we have values for \( t_{ep} \) and \( c \). The bending moment generated from the piezoelectric effect is calculated as follows:

\[
M = b \sum_{x=1}^{n} \int_{t_{x-1}}^{t_{x}} E_x \left( d_{31} x E_e - c \right) \left( z - t_{np} \right) dz
\]

(4)

The displacement of a point on the actuator can be calculated from substituting (3) into (4), and is shown below:

\[
EI v(x) = \frac{M}{2} \left( x^2 - Lx \right) - \frac{F}{48} \left( 4x^3 - 3L^2x \right) \text{ for } 0 \leq x \leq \frac{L}{2}.
\]

(5)

The product of elastic modulus and moment of inertia of a laminated composite can be calculated as:

\[
EI = b \sum_{x=1}^{n} \int_{t_{x-1}}^{t_{x}} E_x \left( z - t_{np} \right)^2 dz
\]

(6)

Simplifying the above analysis to a two layer composite without a load, yields exact agreement with Timoshenko’s analytic model of bi-metal thermostats. 8

4. RESULTS

The following three examples, Figs. 4, 5, and 6, display different functional variations of \( d_{31} \) across the thickness of the actuator. Figure 7 compares the results. Figures 4, 5, and 6, are a step function, linear function, and diffused step function, respectively. Each type of actuator has a graph that displays the relationship between displacement and generated force along with the corresponding work under these conditions. The following conditions were held constant for each sample calculation:

(i) length = 25.4 mm, width = 12.7 mm, thickness = 0.6 mm,
(ii) modulus of elasticity is constant at 6.4x10^10 N/m^2,
(iii) voltage drop = -500 V; producing an electric field = -8.3 \times 10^5 V/m,
(iv) variation is \( d_{31} \) from maximum to minimum absolute value is 167x10^12 m/V, and
(v) the plane of neutral stress is constant at the center of the thickness = 0.3 mm.

A number of important design criteria were obtained from the modeling of the transducers. An actuator that moves a large distance while supporting a large load is more desirable. Typically this quantity, the maximum amount of obtainable work for a given electric field, provides a value to judge the usefulness of a given actuator. The following are the conditions for the axial \( d_{31} \) distributions that optimize the obtainable work:

(i) maximize the difference between the \( d_{31} \) values of the top and bottom layers,
(ii) a step-wise distribution of \( d_{31} \) is the optimal functional distribution, and
(iii) place the steepest part of the slope, or the step in \( d_{31} \) on the plane of neutral stress.

Condition (i) is trivial and needs no explanation. Condition (ii) is verified by Table 1. Condition (iii) can be obtained analytically by maximizing M in Eq. (5). The maximum generated moment is obtained only when the plane of neutral stress lies on the interface of the two layers. The computational model, outlined above, will verify condition (iii) for the functional form of the gradient in \( d_{31} \) that is displayed in Fig. 6.

The step function is shown to be the optimal design to achieve maximum amount of work. However this \( d_{31} \) profile is not readily achievable with our process, due to diffusion of the PZT dopants at sintering temperatures. 6 We expect a \( d_{31} \) that more closely approximates the diffused step function shown in Fig. 6.
5. CONCLUSIONS

The tape casting method to fabricate multilayer electromechanical composites with controlled piezoelectric coefficient distributions has been demonstrated. "Tailor-made" actuator designs can be accomplished to achieve the optimal displacement and load bearing properties. A computational model has been developed to guide the processing for given material constants and comparisons between different designs can be made and judged without the need for numerous experiments and testing. A step functional variation of $d_{31}$, with the interface placed on the plane of neutral stress has been shown to be the optimized arrangement. The tape casting method is a processing method that is well suited to ensure that these design requirements are made with a high degree of accuracy.

ACKNOWLEDGMENTS

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Figure 4: Step function in $d_{31}$, with accompanying displacement and work vs. force diagram.

Figure 5: Linear function in $d_{31}$, with accompanying displacement and work vs. force diagram.

Figure 6: Diffused step function in $d_{31}$, with accompanying displacement and work vs. force diagram.
Table 1: Outline of results from Figs. 4, 5, and 6.

<table>
<thead>
<tr>
<th></th>
<th>Step Function</th>
<th>Linear Function</th>
<th>Diffused Step Function</th>
</tr>
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<tbody>
<tr>
<td>Displacement at zero load</td>
<td>28.1 μm</td>
<td>20.6 μm</td>
<td>25.8 μm</td>
</tr>
<tr>
<td>Force at zero displacement</td>
<td>1.20 N</td>
<td>0.88 N</td>
<td>1.10 N</td>
</tr>
<tr>
<td>Maximum amount of obtainable work</td>
<td>8.43x10⁻⁶ J</td>
<td>4.54x10⁻⁶ J</td>
<td>7.12x10⁻⁶ J</td>
</tr>
</tbody>
</table>

REFERENCES


5. EDO Corporation, Ceramic Operations, Salt Lake City, Utah.

