# Problem Set #1

1. *Complex numbers - polar (6 pts).*
   Any complex number $z$ written in rectangular (Cartesian) form

   $$z = x + iy$$

   where $x$ is the real and $y$ is the imaginary part, can also be written in polar (phasor) form

   $$z = |z|(\cos\theta + i\sin\theta) = |z|e^{i\theta}$$

   where $\theta$ is the phase $\measuredangle z$ and $|z|$ is the magnitude of the complex number $z$.

   Using the Euler formula, compute the magnitude and phase of the complex numbers provided in rectangular form below:

   (i) $z = 3 + 2i$ (ii) $z = 1 - i$ (iii) $z = e^{2-i\pi}$

   (Please be careful while calculating the phase of $z = x + iy$. The phase $\measuredangle z$ is not always equal to $\arctan\frac{y}{x}$ . Tangent is periodic with $\pi$ (i.e. $\tan\theta = \tan(\pi + \theta)$), and $\arctan\frac{y}{x} = \arctan\frac{-y}{-x}$. However, that property is not true for $e^{i\theta}$ ! Hence, depending on the signs of $x$ and $y$, you should pick the right angle. Insert your final answers into the polar form and check the equality.)

2. *Complex numbers - rectangular (6 pts).*
   Compute the real and imaginary parts of the complex numbers:

   (a) $e^i$
   (b) $e^{it}(\cos(3t) + sin(2t))$ ($t$ is real)
   (c) $1/(1 + i)$

3. *Imaginary square root (4 pts).*
   Express the square root of $(1 + i)$ in rectangular coordinates.

4. *Phase calculation (8 pts).*
   The complex number z is given by

   $$z = \frac{z_1}{z_2} = \frac{1+3i}{1-i}$$

   (a) Express z in rectangular form.
   (b) Express z in polar form. Compute the magnitude and phase of $z$ in terms of the magnitude and phase of $z_1$ and $z_2$.
   (c) Using the results of (a) and (b), show that $\arctan 3 + \arctan 2 = \frac{3\pi}{4}$.

5. *Shifting and scaling functions (10 pts).*
Let

$$f(t) = \begin{cases} t & 0 \le t < 1 \\ 2 - t & 1 \le t < 2 \\ 0 & \text{else.} \end{cases} \tag{1}$$

Sketch each of the following signals. All sketches must have clearly labeled axes.

(a) $f(t)$

(b) $f(t) + 1$

(c) $f(\frac{t}{2})$

(d) $f(t + 2)$

(e) $f(-t - 1)$