Problem Set #11

1. (6 points) **Hamming codes for error correction:**

   (a) Hamming codes correct single bit errors. For any integer \( k \) there is a Hamming code of size \( n = 2^k - 1 \) which consists of \( k \) bits of redundancy. What is the rate of Hamming codes as a function of \( n \) (rate means bits of message per bit of transmission)? What constant does this approach as \( n \) goes to infinity?

   (b) Suppose each transmitted bit has probability \( p \) of being received in error. What is the probability that the Hamming code is decoded in error, as a function of \( n \)? (Hint: The probability of error is one minus the probability of being correct. For correct decoding there must be either no bit errors or one error, which can occur in \( n \) possible places. Add the contributions of the probability of each of these sequences. For example, for the no error sequence, the probability is \((1 - p)^n\).) What constant does this approach as \( n \) goes to infinity?

2. (4 points) **One-time pad:** Apply a one-time pad to the message \( m = 0110100101 \) using the key \( k = 0011011010 \).

3. (20 points) **Laplace Transform and Z-transform:**

   (a) Use the Laplace transform integral to calculate the Laplace transform of \( x(t) = 5te^{-2t}u(t) - 3e^{-t}u(2 - t) \) and corresponding region of convergence (ROC).

   (b) Calculate the z-transform of \( x[n] = 4^n u[n - 5] \). What is the ROC here?

   (c) Calculate the z-transform and its ROC of

   \[
   x[n] = \begin{cases} 
   3, & n = 2, \\
   -5, & n = 7, \\
   1, & n = 10, \\
   0, & \text{else}.
   \end{cases}
   \]