Problem Set #11

1. *(6 points)* **Hamming codes for error correction:**

   (a) Hamming codes correct single bit errors. For any integer k there is a Hamming code of size $n = 2^k - 1$ which consists of k bits of redundancy. What is the rate of Hamming codes as a function of n (rate means bits of message per bit of transmission)? What constant does this approach as n goes to infinity?

   (b) Suppose each transmitted bit has probability p of being received in error. What is the probability that the Hamming code is decoded in error, as a function of n? (Hint: The probability of error is one minus the probability of being correct. For correct decoding there must be either no bit errors or one error, which can occur in n possible places. Add the contributions of the probability of each of these sequences. For example, for the no error sequence, the probability is $(1-p)^n$) What constant does this approach as n goes to infinity?

2. *(4 points)* **One-time pad:** Apply a one-time pad to the message $m = 0110100101$ using the key $k = 0011011010$.

3. *(20 points)* **Laplace Transform and Z-transform:**

   (a) Use the Laplace transform integral to calculate the Laplace transform of $x(t) = 5te^{-2t}u(t) - 3e^{-t}u(2-t)$ and corresponding region of convergence (ROC).

   (b) Calculate the z-transform of $x[n] = 4^n u[n-5]$. What is the ROC here ?

   (c) Calculate the z-transform and its ROC of

$$x[n] = \begin{cases} 3, & n = 2, \\ -5, & n = 7, \\ 1, & n = 10, \\ 0, & \text{else.} \end{cases}$$