Problem Set #5

1. *Discrete-time convolution (3pts).*
   Compute and plot $y[n] = x[n] * h[n]$, where

$$x[n] = \delta[n-2] + 2\delta[n-3] - 2\delta[n-4] - \delta[n-5]$$

$$h[n] = \begin{cases} 1 & \text{if } 3 \le n \le 7, \\ 0 & \text{otherwise.} \end{cases}$$

2. *Continuous-time convolution (6 pts).*
   Define the function $x(t)$ by

$$x(t) = \begin{cases} 2 & \text{if } 0 \le t < 1 \\ -1 & \text{if } 1 \le t < 2 \\ 0 & \text{otherwise} \end{cases}$$

   Please recall the definition of the two continuous-time functions: unit step function $u(t)$, and Rect function $\text{rect}(t)$, and then sketch each of the following convolved signals:

   (a) $u(t) * u(t)$

   (b) $x(t) * u(t)$

   (c) $x(t) * \text{rect}(t)$

3. *Convolution and the Fourier transform (3 pts).*
   What is the Fourier transform of $\text{rect}(t) * \text{sinc}(t)$? The convolution integral will *not* be the easiest way to do this.

4. *Averaging system (6 pts).*
   Suppose $x[n]$ denotes the closing price of a stock on day n. To smooth out fluctuations, a tool often used by technical analysts is the 30-day moving average of the stock price. Let $y[n]$ denote this 30-day moving average, where the average at time n uses the closing price on day n together with the previous 29 days.

   (a) Write an expression for $y[n]$ in terms of $x[\cdot]$.

   (b) $y[n]$ can be thought of as the output of an LTI system when the input is $x[n]$. What is the impulse response of this system?

   (c) How does the impulse response change if instead of the "lagging" average above we use $x[n]$ together with 15 days in the past and 14 days in the future?

   (d) What is a practical problem of using the average of part (c)?

5. *System response (3 pts).*
   A continuous-time LTI system has impulse response $h(t)$ with Fourier transform $H(f)$. What is the output of the system when the input is $\sin(t)$?