1. **Discrete-time convolution (3pts).**
   Compute and plot $y[n] = x[n] * h[n]$, where
   
   $x[n] = \delta[n - 2] + 2\delta[n - 3] - 2\delta[n - 4] - \delta[n - 5]$
   
   $h[n] = \begin{cases} 
   1 & \text{if } 3 \leq n \leq 7, \\
   0 & \text{otherwise}.
   \end{cases}$

2. **Continuous-time convolution (6 pts).**
   Define the function $x(t)$ by
   
   $x(t) = \begin{cases} 
   2 & \text{if } 0 \leq t < 1 \\
   -1 & \text{if } 1 \leq t < 2 \\
   0 & \text{otherwise}
   \end{cases}$
   
   Please recall the definition of the two continuous-time functions: unit step function $u(t)$, and Rect function $\text{rect}(t)$, and then sketch each of the following convolved signals:
   
   (a) $u(t) * u(t)$
   
   (b) $x(t) * u(t)$
   
   (c) $x(t) * \text{rect}(t)$

3. **Convolution and the Fourier transform (3 pts).**
   What is the Fourier transform of $\text{rect}(t) * \text{sinc}(t)$? The convolution integral will not be the easiest way to do this.

4. **Averaging system (6 pts).**
   Suppose $x[n]$ denotes the closing price of a stock on day $n$. To smooth out fluctuations, a tool often used by technical analysts is the 30-day moving average of the stock price. Let $y[n]$ denote this 30-day moving average, where the average at time $n$ uses the closing price on day $n$ together with the previous 29 days.
   
   (a) Write an expression for $y[n]$ in terms of $x[n]$.
   
   (b) $y[n]$ can be thought of as the output of an LTI system when the input is $x[n]$. What is the impulse response of this system?
   
   (c) How does the impulse response change if instead of the “lagging” average above we use $x[n]$ together with 15 days in the past and 14 days in the future?
(d) What is a practical problem of using the average of part (c)?

5. *System response (3 pts).*
   A continuous-time LTI system has impulse response $h(t)$ with Fourier transform $H(f)$. What is the output of the system when the input is $\sin(t)$?