Problem Set #8

1. *(5 pts).* **Sampling:** Suppose $x(t)$ is real and has a non-zero Fourier transform only on the set of frequencies 5-8Hz (both positive and negative). Draw an illustration of the spectrum of $x(t)$. Also draw an illustration of the spectrum when $x(t)$ is sampled at 16Hz. What is the lowest sampling rate from which the signal can still be recovered?

2. *(6 pts)* **Circular Convolution Property:** Consider the following two finite duration discrete-time signals.

$$x[0] = 1, \quad x[1] = 1, \quad x[2] = 0, \quad x[3] = 0.$$

$$y[0] = 0, \quad y[1] = 1, \quad y[2] = 0, \quad y[3] = 1.$$

   (a) Calculate the circular convolution of $x[n]$ and $y[n]$.

   (b) Calculate the DFT of $x[n]$ and $y[n]$, multiply, and calculate the inverse DFT.

   Note: Make sure to use the DFT rather than the DTFS (the difference is the normalization constant.). If you use the DTFS then you need to include the factor of $N = 4$ is the convolution property.

3. *(4 pts)* **Circular Convolution and Multiplication Property:** Suppose $x[n]$ and $y[n]$ have the same duration $N \geq 3$. Their DFTs are given by $a[k]$ and $b[k]$. We know in advance that

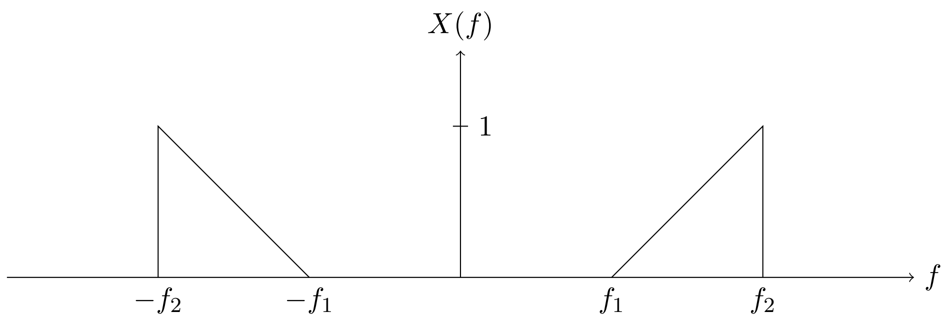
$$\sum_{l=0}^{N-1} a[l]b[k-l \mod N] = 1 + e^{-i\frac{4\pi k}{N}}$$

   for $k = 0, 1, \cdots, N-1$.

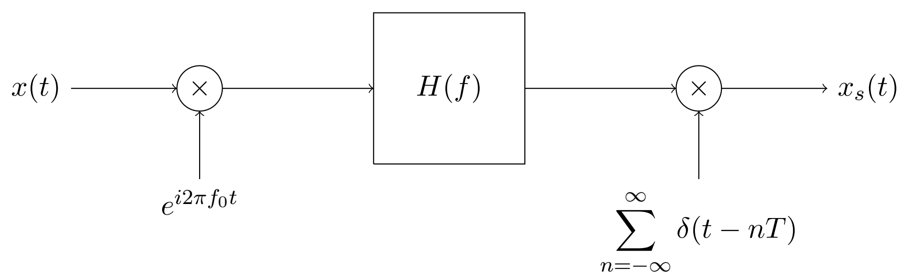
   (a) If $x[n] = 2^{-n}, n = 0, 1, \cdots, N-1$, calculate $y[n]$. Is $y[n]$ unique?

   (b) If $x[n] = 1 + (-1)^n, n = 0, 1, \cdots, N-1$, redo part (a).

4. *(6 pts)* **Band Pass Signal Sampling:** Suppose that the Fourier transform of a real signal $x(t)$ is nonzero only for $f_1 \leq |f| \leq f_2$. A naive application of Nyquist Sampling Theorem will lead to a sampling frequency of at least $2f_2$. However, for such a band pass signal, it is possible to sample at a lower frequency and recover $x(t)$.



The signal $x(t)$ is multiplied by a complex exponential function $e^{i2\pi f_0 t}$ where f_0 is chosen to be $\frac{1}{2}(f_2 + f_1)$. The cutoff frequency of the ideal low pass filter is $\frac{1}{2}(f_2 - f_1)$.



(a) Sketch the Fourier transform of $x_s(t)$.

(b) Determine the largest possible sample period T for recovering $x(t)$.

(c) Design a system that recovers $x(t)$ from $x_s(t)$.