

Problem Set #9

1. (6points) **Entropy:**

Estimate the number of bits of information (entropy) in each of the following observations. Briefly justify your answers.

- (a) The day of the maximum temperature in Princeton in the coming July.
- (b) Whether or not it snows in Princeton on the day in part (a).
- (c) A random persons birthday.
- (d) Among 10 twenty-year-old students, whether or not any of them were born on the same day.
- (e) Among 23 twenty-year-old students, whether or not any of them were born on the same day. (search online for birthday problem for a hint)
- (f) Among 40 twenty-year-old students, whether or not any of them were born on the same day.

2. (6 points) **SNR from quantization:**

- (a) Suppose a signal with values $x[n] \in [-1, 1]$ is uniformly quantized with b bits. Calculate the signal-to-noise ratio (SNR) of the quantized signal (power of the signal divided by power of the noise). For simplicity, assume that the samples $x[n]$ are uniformly distributed on the range $[-1, 1]$.
- (b) Its common to measure ratios of powers on a log-scale using decibels, especially for signal-to-noise ratios. A decibel (1dB) is $10 \log_{10}$ of the ratio. Calculate the SNR of the quantized signal in dB. What is the value of each quantization bit in terms of dB?

3. (6points) **Group Delay:**

Consider an LTI system with impulse response $h(t) = 5e^{-10t}u(t)$. What delay would a narrow-band signal of frequency 1 experience? What about frequency 1,000?