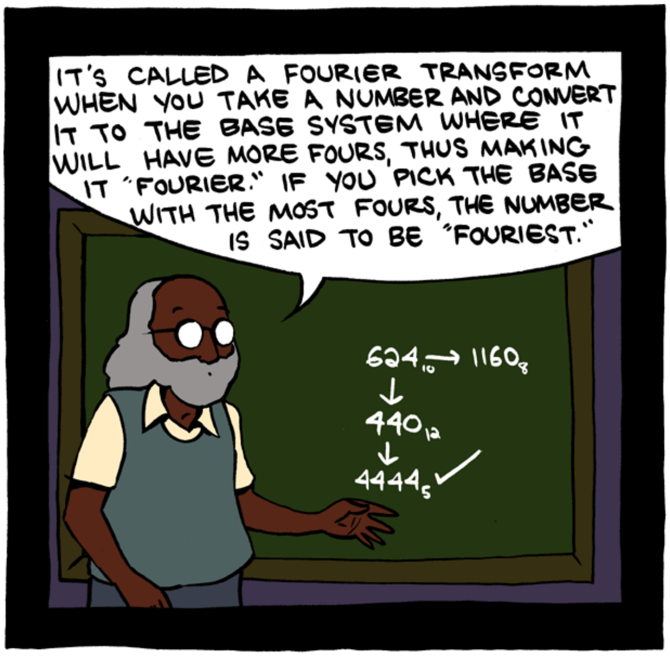
Lecture 10

Monday, March 24, 2014
4:48 PM



Teaching math was way more fun after tenure.

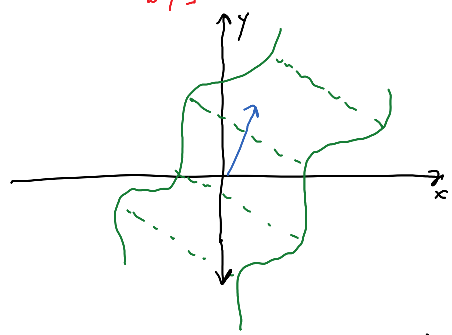
2-D Fourier Transform:

$s(x,y)$

$$S(f_x,f_y) = \iint s(x,y)\, e^{-i2\pi(f_x x + f_y y)}\, dx\, dy$$

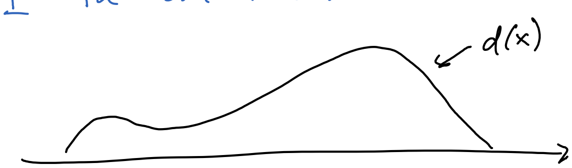
$$s(x,y) = \iint S(f_x,f_y)\, e^{i2\pi(f_x x + f_y y)}\, df_x\, df_y$$

$f = \begin{bmatrix} f_x \\ f_y \end{bmatrix}$, $p = \begin{bmatrix} x \\ y \end{bmatrix}$ $\quad f^T p$



$$S(f_x,f_y) = \int \left(\int s(x,y)\, e^{-i2\pi f_x x}\, dx \right) e^{-i2\pi f_y y}\, dy$$

MRI in one-dimension:



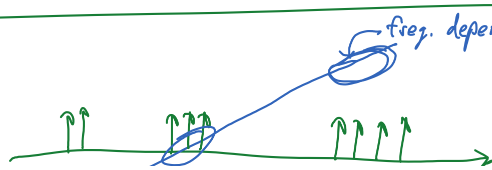
Magnetic Field: $B_0 + g x$

$$s(t) = \int_{-\infty}^{\infty} d(x)\, e^{i2\pi\gamma(B_0+gx)t}\, dx = e^{i2\pi\gamma B_0 t} \int_{-\infty}^{\infty} d(x)\, e^{i2\pi\gamma g x t}\, dx$$

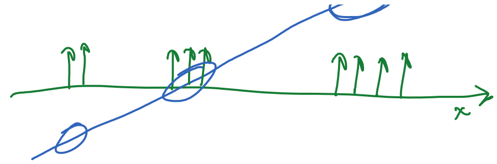
$$= e^{i2\pi\gamma B_0 t}\, \mathcal{F}^{-1}[d(x)](\gamma g t)$$

$$d(x) = \mathcal{F}\left(e^{-i2\pi\gamma B_0 t}\, s(t/\gamma g) \right)$$

Intuition:



Intuition:



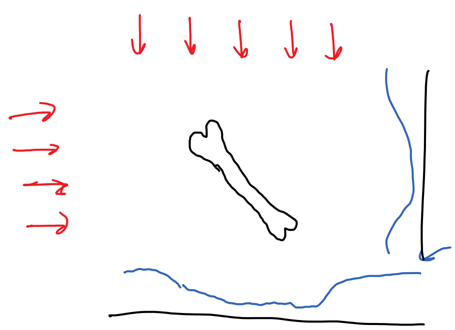
In reality: $-$Time-varying gradient. $g(t)$

$$\int_{-\infty}^{\infty} d(x)\, e^{i2\pi\gamma \int (B_0 + x g(t))\,dt}$$

$-$ $s(t)$ is sampled. (discrete-time)

CT: Computed Tomography

- X-ray — Shadow. eg. Bone is more absorbant.
- 3D-imaging



absorbtion coefficient

$$m_1(x) = \cancel{I_0} - \int d(x,y)\, dy \quad \leftarrow \text{Projection}$$

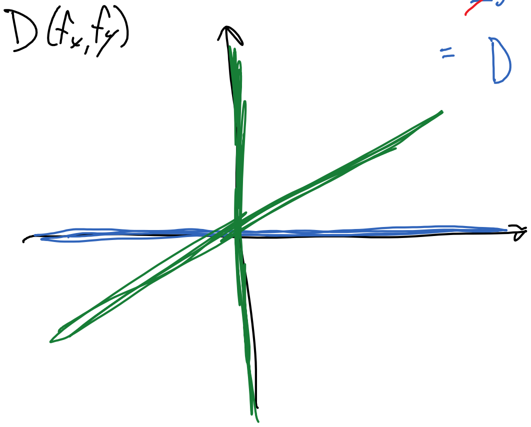
$$\int_{-\infty}^{\infty} d(x,y)\, e^{-i2\pi(0)y}\, dy$$

$$M_1(f_x) = \mathcal{F}(m_1(x)) = \int_{-\infty}^{\infty} m_1(x)\, e^{-i2\pi f_x x}\, dx$$

$$= \cancel{I_0 \delta(f_x)} - \iint d(x,y)\, e^{-i2\pi(0y + f_x x)}\, dx\, dy$$

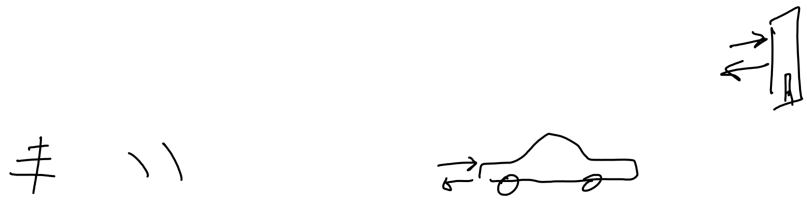
$$= D(f_x, 0)$$

$D(f_x, f_y)$



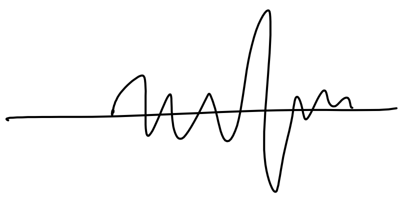
Ultrasound:

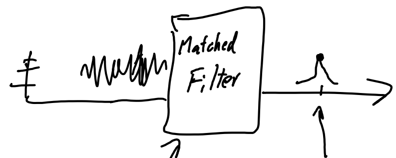
Radar



Pulse signal:

$s(t)$





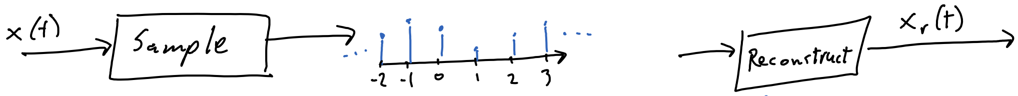
LTI

$h(t) = s^*(-t)$

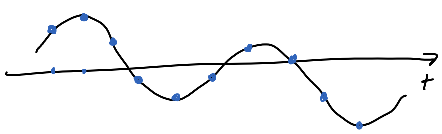
Peaks locate $s(t)$

Sampling:

Discrete-time

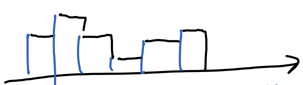


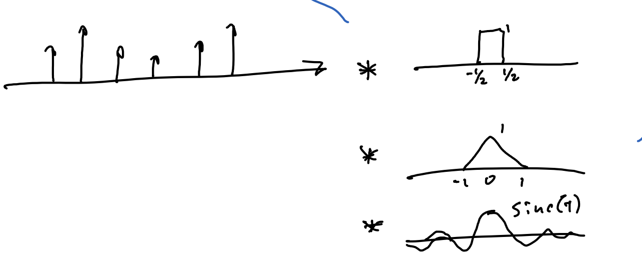
How do we do this so that $x_r(t) = x(t)$?



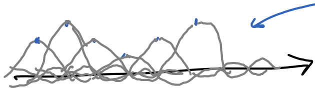
One attempt:

Fail



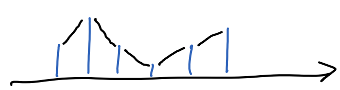


Pass

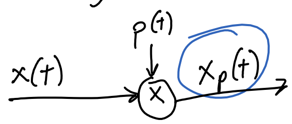


Second attempt:

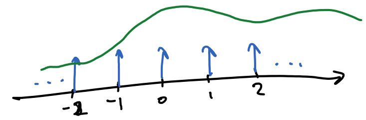
Fail



Sampling Process:



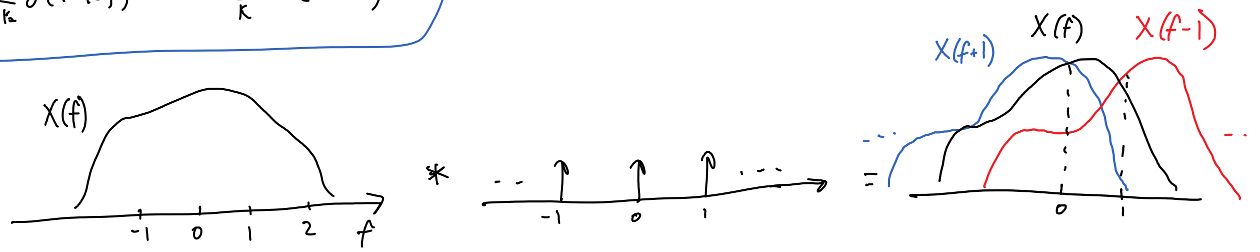
$$p(t) = \sum_k \delta(t-k)$$



$$x(t)\,p(t) = \sum_k x(t)\,\delta(t-k)$$
$$= \sum_k x(k)\,\delta(t-k)$$

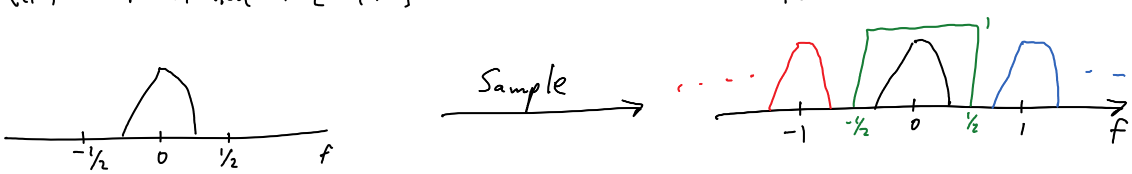
$x_p(t)$ is the CT representation of $x[n]$

$$\boxed{\mathcal{F}\left(\sum_k \delta(t-k)\right) = \sum_k \delta(f-k)}$$

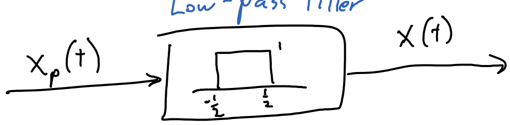


$X(f)$ band limited to $[-1/2, 1/2]$

$X_p(f)$



Low-pass filter



$$\sum_k \delta(t - k/F_s) \xrightarrow{\mathcal{F}} F_s \sum_k \delta(f - F_s k)$$