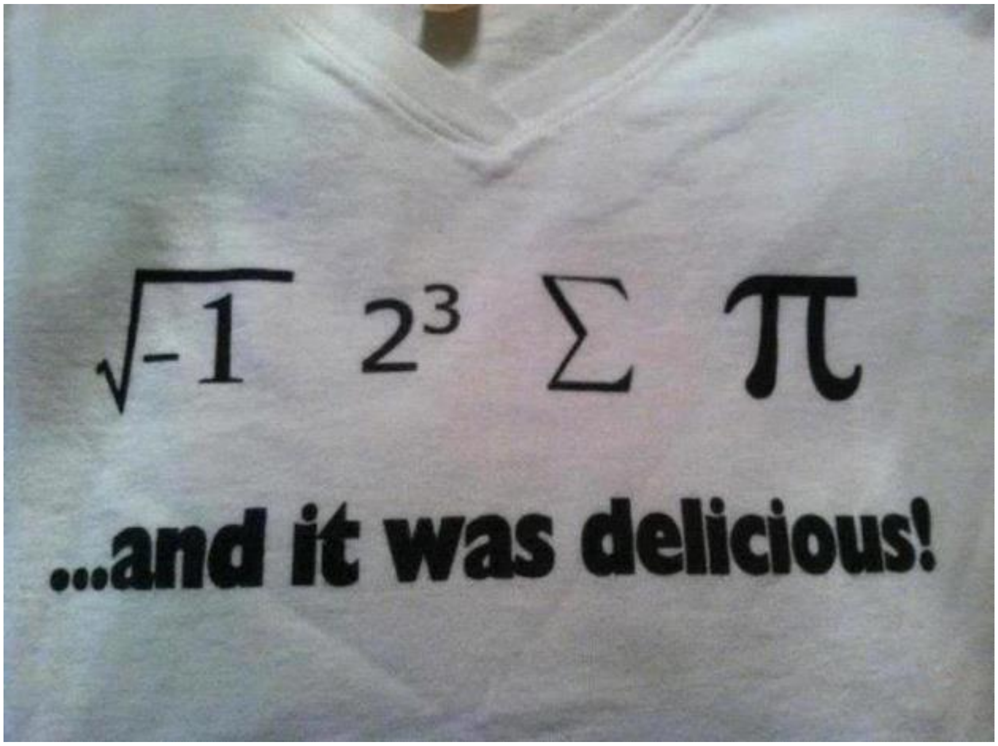
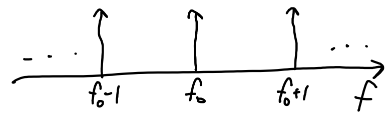
Lecture 11

Tuesday, March 25, 2014
11:32 PM



What is the DTFT of $e^{i2\pi f_0 n}$?



Check:

$$\int_{-1/2}^{1/2} (\ \ ) e^{i2\pi f n} \, df$$

$$= e^{i2\pi f' n} = e^{i2\pi f n}$$

where $f'-f$ is integer

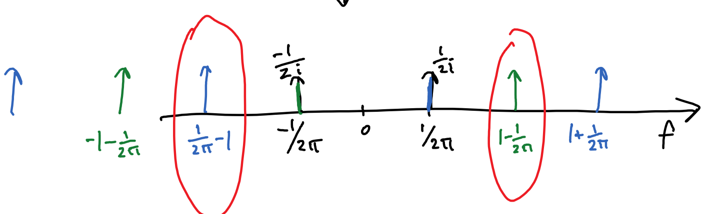
$f' \in [-1/2, 1/2]$

PS 3.5:

$$\sin[n] = \sin\left[\left(1-\frac{1}{2\pi}\right)n\right]$$

$$= \sin\left(2\pi\left(\frac{1}{2\pi}\right)n\right)$$

↓ Euler



$$\sin\left[\left(1-\frac{1}{2\pi}\right)n\right]$$

Self-similar FT pair

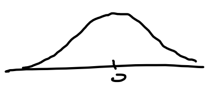
$$\sum_k \delta(t-k) \xrightarrow{\mathcal{F}} \sum_k \delta(f-k) \longleftarrow \text{Derive}$$

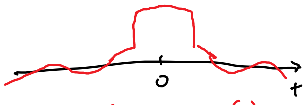
$$e^{-t^2} \xrightarrow{\mathcal{F}} \text{same}$$

Use DT

$x[n] = 1$

$e \longrightarrow$ same





$\xrightarrow{\mathcal{F}}$ same

sinc(t) + rect(t)

$x[n] = 1$

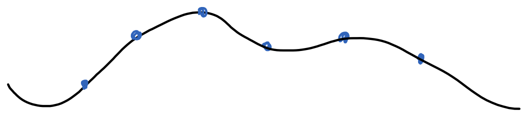
$X(f) = \sum_k \delta(f-k)$

CT Representation

$$x(t) = \sum_k x[k]\,\delta(t-k) = \sum_k \delta(t-k)$$

$$X(f) = \sum_k \delta(f-k)$$

Sampling:



$$\boxed{\sum_k \delta(t-kT_s) \xrightarrow{\mathcal{F}} F_s \sum_k \delta(f-kF_s) \quad \text{where } T_s = 1/F_s}$$

Time Scaling:

$$\sum \delta(t-k) \xrightarrow{\mathcal{F}} \sum \delta(f-k)$$

$$\sum_k \delta(t/T_s - k) \qquad\qquad T_s \sum \delta(T_s f - k)$$

$$= \qquad\qquad =$$

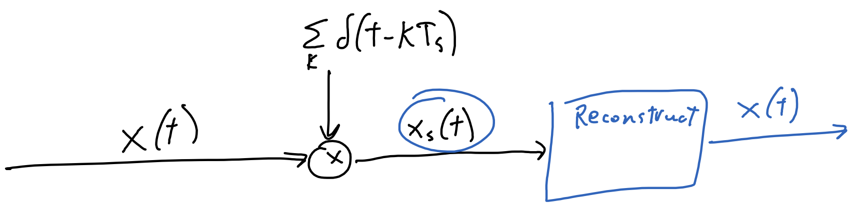
$$\sum_k \delta\left(\tfrac{1}{T_s}(t-kT_s)\right) \qquad\qquad T_s \sum_k \delta\left(T_s(f - k/T_s)\right)$$

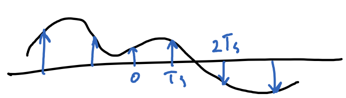
$$T_s \sum_k \delta(t-kT_s) \qquad\qquad = \sum_k \delta(f - k/T_s)$$

$$\sum_k \delta(t-kT_s) \xrightarrow{\mathcal{F}} F_s \sum_k \delta(f-kF_s)$$

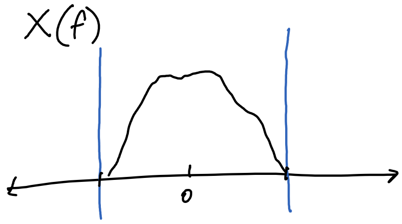
Sampling:

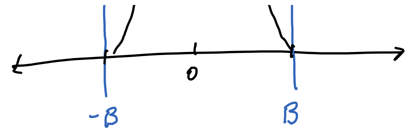
$x[n] = x(nT_s) \quad \forall n$





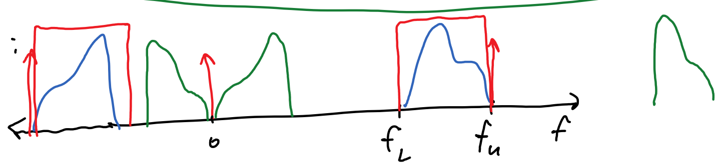
X(t) is band limited:





Bandwidth B

More generally:
Upper bound f_u
Lower bound f_L



Bandwidth $= f_u - f_L$

Landau rate

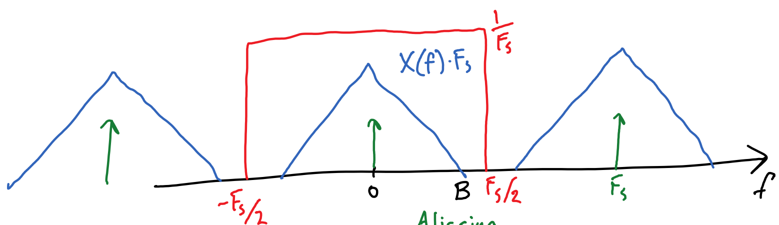
$2(f_u - f_L)$

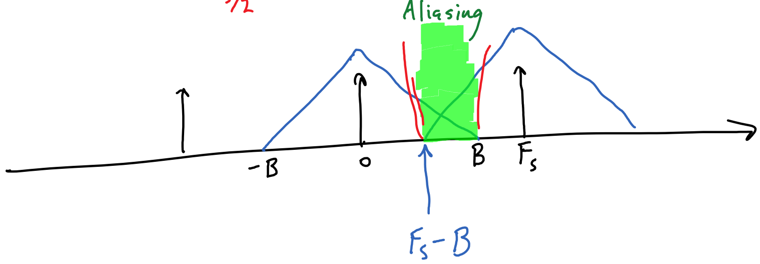
Suppose $f_L > \frac{1}{2} f_u$

Let $F_s = f_u$

$$x(t)\left(\sum_k \delta(t-kT_s)\right) \xrightarrow{\mathcal{F}} F_s\, X(f) * \underline{\left(\sum_k \delta(f-kF_s)\right)}$$

We can recover

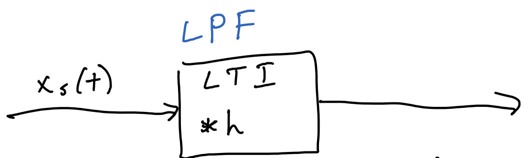




No aliasing if $F_s - B > B \Rightarrow \underline{F_s > 2B}$

Nyquist rate

Filter:



$H(f) = \frac{1}{F_s} \text{rect}(f/F_s)$

$h(t) = \text{sinc}(F_s t) = \text{sinc}(t/T_s)$

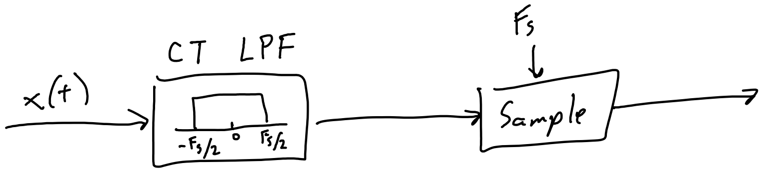
Sample rate constrained at F_s

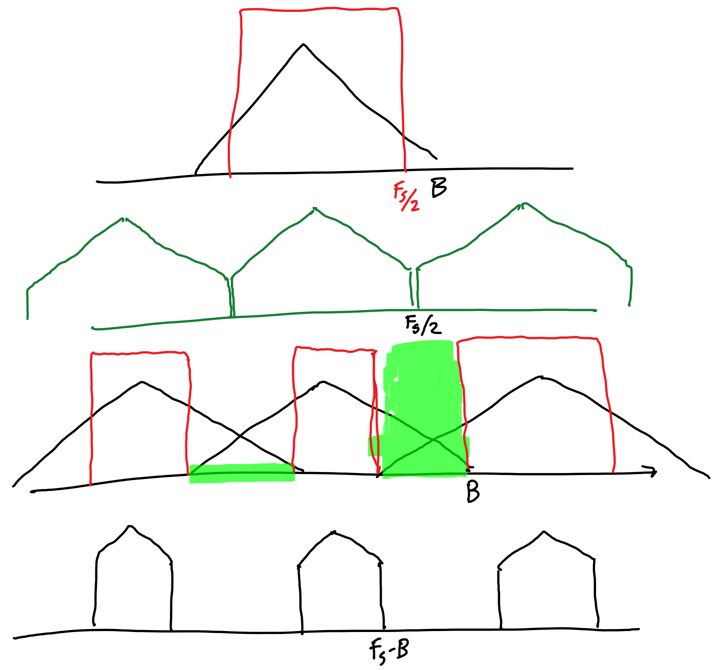
Signal Bandwidth B

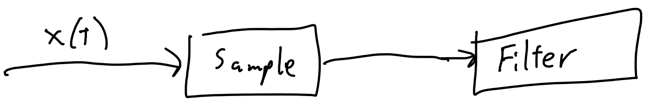
$2B > F_s$

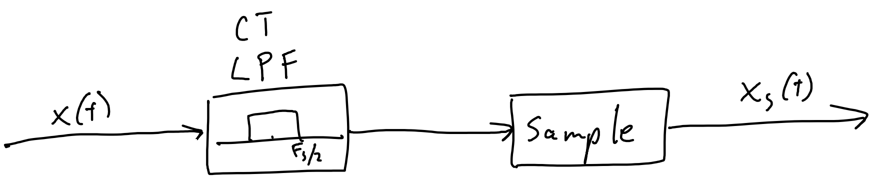
$2B > F_s$

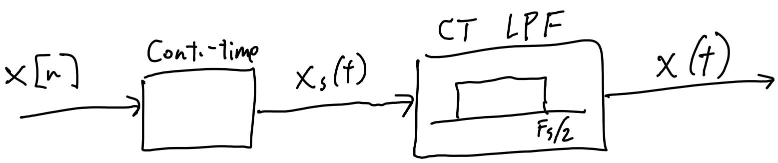
Typical Sampler:











$x(t)$ even, real $\xrightarrow{\mathcal{F}}$ even real

Phase? $\angle X(f)$

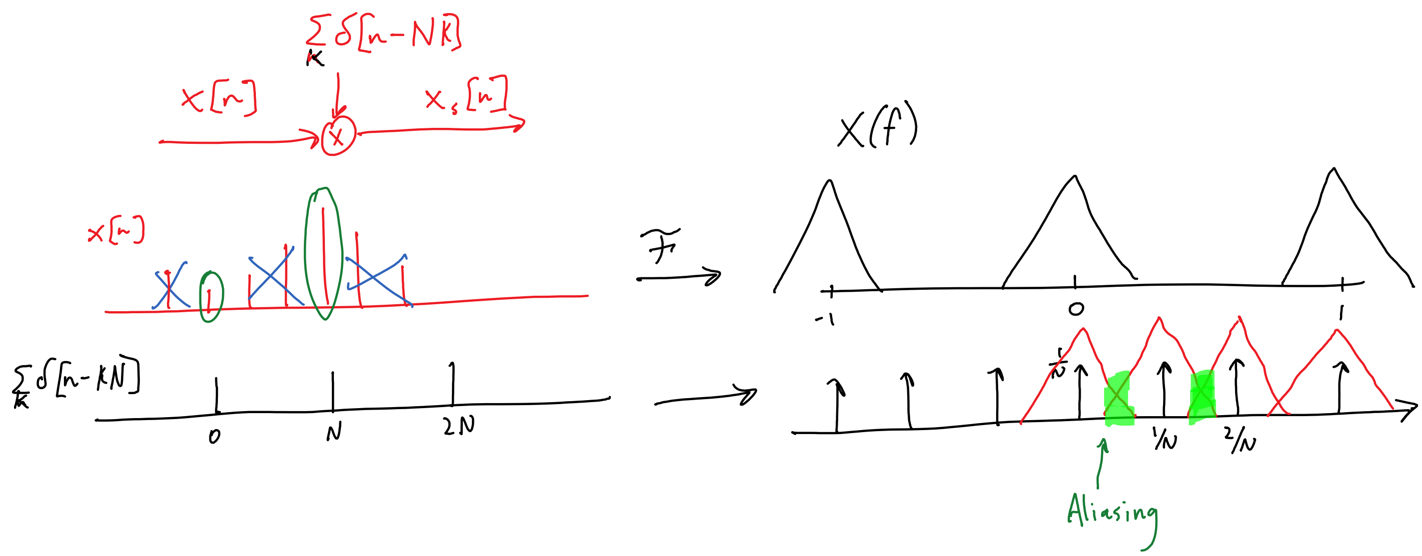
Zero-phase

$x(t-T) \xrightarrow{\mathcal{F}} e^{-j2\pi f T} X(f)$

Phase $= -2\pi T f$ "Linear-phase"

Discrete-time Downsampling:

Just like CT Sampling



∑ δ[n−Nk]
k
x[n]
x_s[n]
X(f)
x[n]
-1
0
1
∑ δ[n−kN]
k
0
N
2N
1/N
1/N
2/N
Aliasing