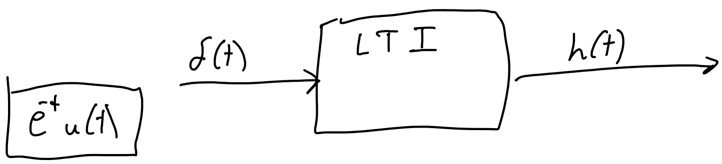
Lecture 17

Tuesday, April 15, 2014
5:45 PM

Probe an LTI system:



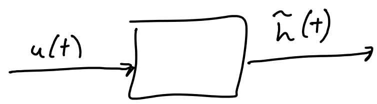
Does this measure everything about the system?

F.T. has zero-regions, so not a sufficient probe.

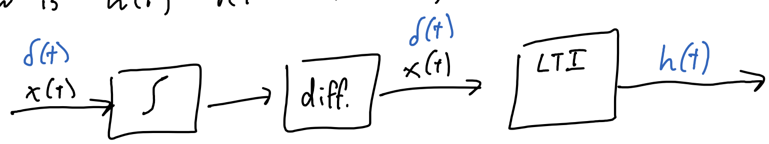
Output

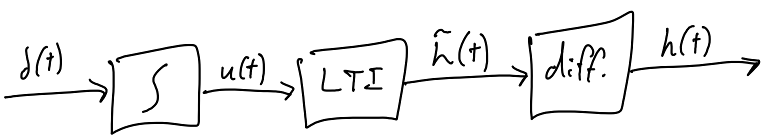
$h(t)$: $H(f) = \dfrac{Y(f)}{X(f)}$ should not be zero $\forall f$

Unit step response:



How is $\tilde{h}(t)$ related to $h(t)$?





$$h(t) = \frac{d}{dt}\tilde{h}(t).$$

Modulation:

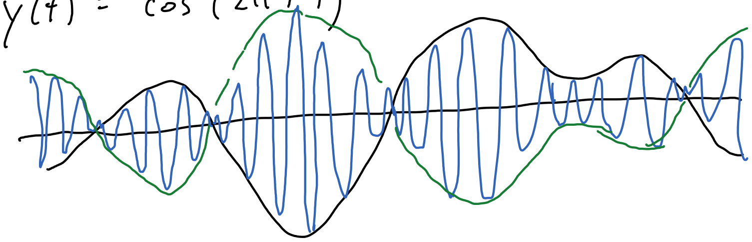
AM Radio: (Amplitude Modulation).

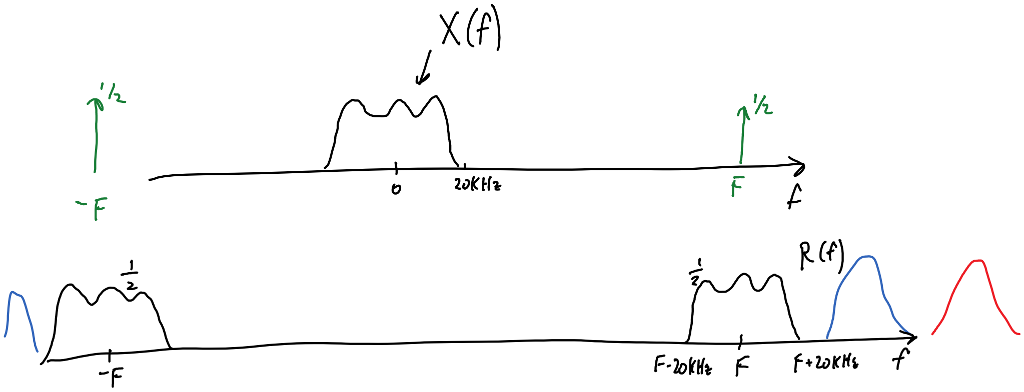
$x(t)$ is an audio signal (bandlimited)

Let F be a carrier frequency (800 KHz)

Carrier wave: $y(t) = \cos(2\pi F t)$

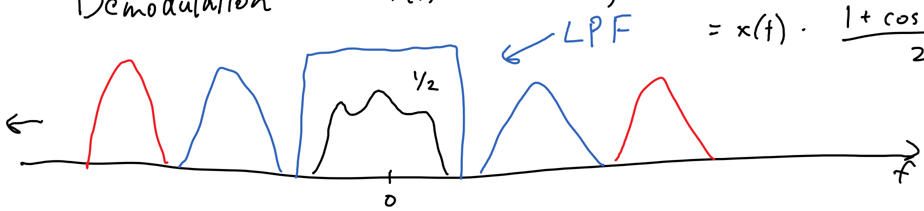
$r(t) = x(t)\cdot \underline{y(t)}$

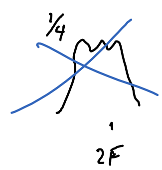




Demodulation: $r(t)\cdot\cos(2\pi F t) = x(t)\cos^2(2\pi F t)$

$= x(t)\cdot\dfrac{1+\cos(2\pi\, 2Ft)}{2}$





Circular Convolution:
(Periodic Convolution)

Convolution for periodic signals:

Let $x(t)$ and $y(t)$ have period T.

$$x(t) \circledast_T y(t) = \int_{\langle T\rangle} x(\tau)\,y(t-\tau)\,d\tau$$

This means integrate over one period.

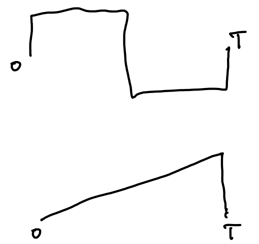
Or (equivalently)

Let $\tilde{y}(t)$ be one period of $x(t)$.

$$x(t) \circledast y(t) = x(t) * \tilde{y}(t)$$ ← Result is periodic.

Two finite-duration signals:

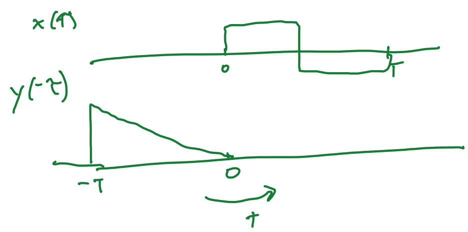
Duration T for both.



Regular convolution?

Assume zero elsewhere. (Matlab would do this)

Flip-and-shift



Overlap for $t \in (0, 2T)$

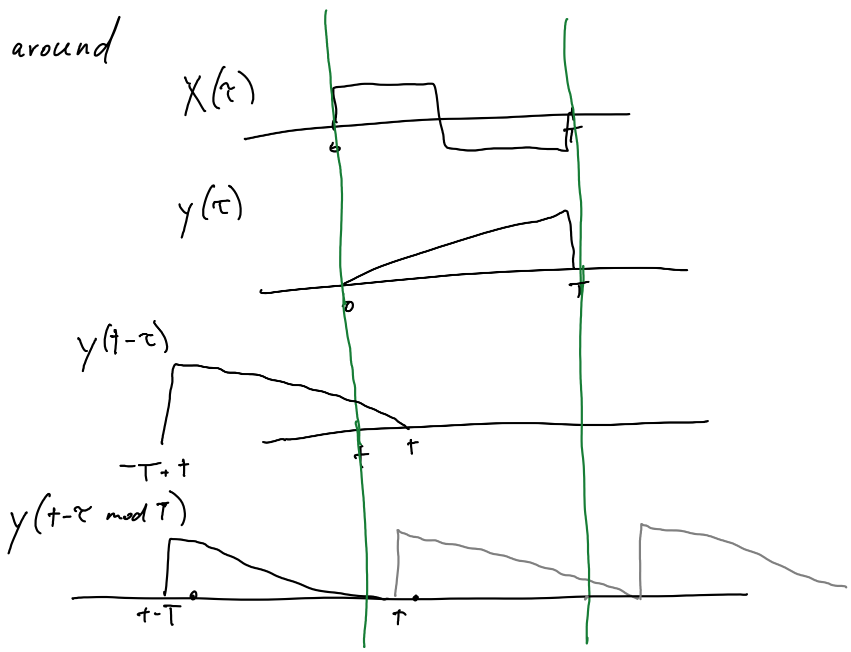
($2N-1$ in discrete-time if signals from 0 to $N-1$)

Circular convolution results in a length T signal.

$$x(t) \circledast_T y(t) = \int_0^T x(\tau)\, y(t-\tau \bmod T)\, d\tau$$

This means add multiples of T such that $t-\tau+kT \in (0,T)$

Flip and shift wraps around



Equivalently: Let $\hat{y}(t)$ be the periodic extension of $y(t)$, with period T.

$$x(t) \circledast_T y(t) = \big(x(t) * \hat{y}(t)\big) \leftarrow \text{Only Keep to first period, } t \in (0,T)$$

Why Circular convolution?

Multiplication rule in D.T.

$$x[n]\cdot y[n] \xrightarrow{\mathcal{F}} X(f) \circledast_1 Y(f)$$

Justification: $\downarrow$ c.t.

$$\sum_n x[n]\,\delta(t-n) \cdot y(t)$$

interpolate

one period in freq. domain

Period T

Dual property: CTFS

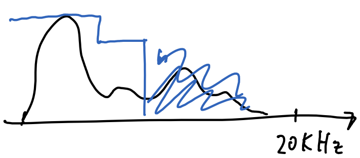
$$x(t) \xrightarrow{\mathcal{F}} \{a_k\}$$

$$y(t) \xrightarrow{\mathcal{F}} \{b_k\}$$

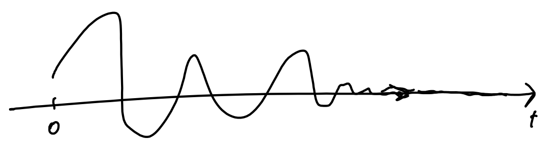
$$x(t) \circledast_T y(t) \xrightarrow{\mathcal{F}} \frac{1}{T}\{a_k b_k\}$$

DFT: Multiplication in frequency is Circular-convolution in time.

Illustration of freq. domain filtering

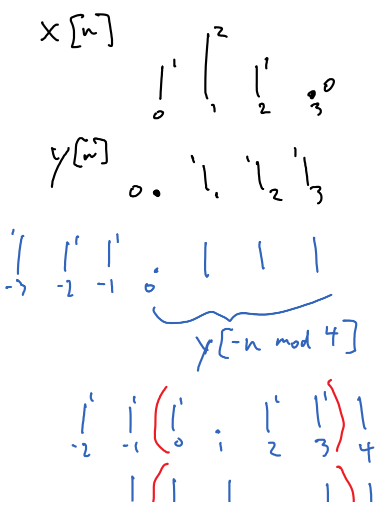


$\xrightarrow[\text{of filter}]{\text{IDFT}}$



If Relatively short duration compared to signal then Convolution and circ. conv. are very similar

Example of circ. conv.:



$x[n] = (1, 2, 1, 0)$

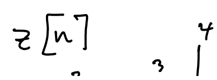
$y[n] = (0, 1, 1, 1)$

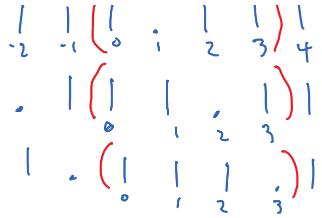
$z[n] = x[n] \circledast y[n]$

$z[0] = 3$

$z[1] = 2$

$z[2] = 3$

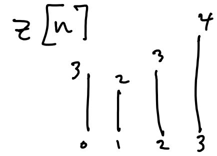




$z[1] = 2$

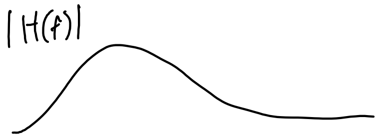
$z[2] = 3$

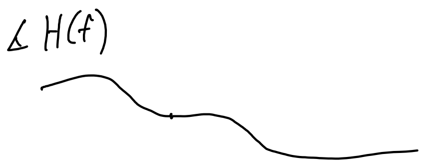
$z[3] = 4$



How does phase effect a signal?

Group delay:





$$\text{Group delay} = \frac{-1}{2\pi}\frac{d}{df}\angle H(f)$$

This is more meaningful information about a filter.