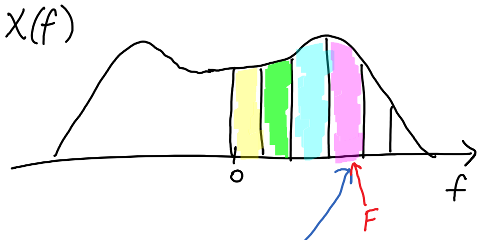
Lecture 18

Friday, April 18, 2014
10:38 AM

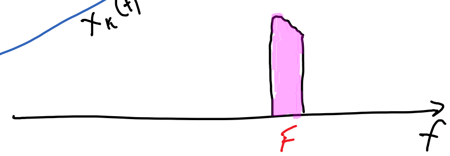
Group Delay:

Think of a signal as a sum of narrow-band components



A filter affects the mag. and phase.

Take one narrow-band part $x_k(t)$



Replace with Taylor expansion of mag. and phase

$$H_k(f) \approx |H(F)|\, e^{i\measuredangle H(F) + (f-F)\measuredangle H'(F)}$$

$$Y_k(f) = X_k(f)\, H_k(f) \approx \Big(X_k(f)\, \underbrace{H(F)\, e^{-i\measuredangle H'(F) F}}_{\text{Constant}}\Big)\, e^{i\measuredangle H'(F) f}$$

Delay by $\frac{-1}{2\pi}\measuredangle H'(F)$ Group delay

$\boxed{\text{Please interpret } \measuredangle H' \text{ as } (\measuredangle H)' = \frac{d}{df}\measuredangle H(f).}$

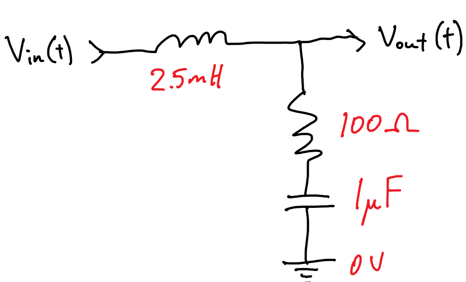
Example of Cont.-time filter:

Ideal Passive Circuit Elements:

Inductor: $v(t) = L \frac{d}{dt} i(t)$

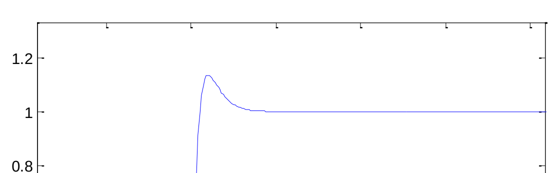
Capacitor: $C \frac{d}{dt} V(t) = i(t)$

Resistor: $v(t) = R\, i(t)$



LTI

Step response:

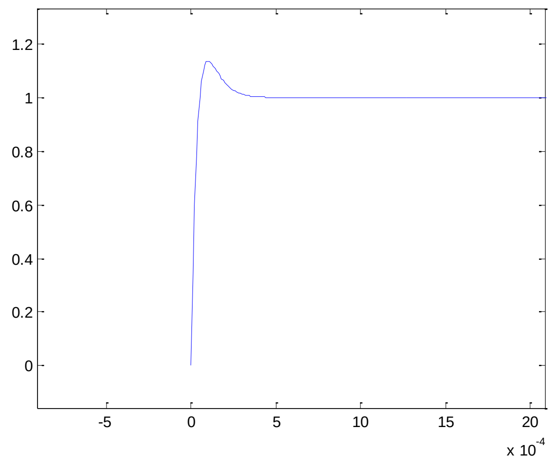


Step response:

$$1-(1-20{,}000t)\,e^{-20{,}000t}\,u(t)$$

$\downarrow$ derivative

$$h(t) = 40{,}000\,e^{-20{,}000t}\,u(t) - 400{,}000{,}000\,t\,e^{-20{,}000t}\,u(t)$$



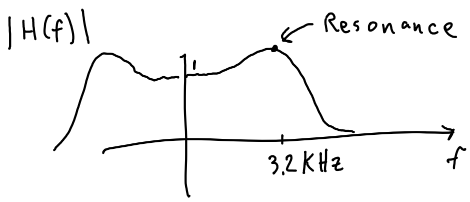
$$H(f) = \frac{40000}{20000+i2\pi f} - \frac{400{,}000{,}000}{(20000+i2\pi f)^2} = \boxed{\frac{1+i2\pi 10^{-4}f}{(1+i\pi 10^{-4}f)^2}}$$

p.329

$$|H(f)| = \frac{|1+i2\pi 10^{-4}f|}{|1+i\pi 10^{-4}f|^2} = \frac{\sqrt{1+(2\pi 10^{-4}f)^2}}{1+(\pi 10^{-4}f)^2}$$

$$(1+i2\pi 10^{-4}f)(1-i2\pi 10^{-4}f) = 1-(i2\pi 10^{-4}f)^2$$

$$\measuredangle H(f) = \tan^{-1}(2\pi 10^{-4}f) - 2\tan^{-1}(\pi 10^{-4}f)$$



FYI: Bode Plot: (C-T) Plot $\log|H(f)|$ vs. $\log f$

phase v.s. $\log f$

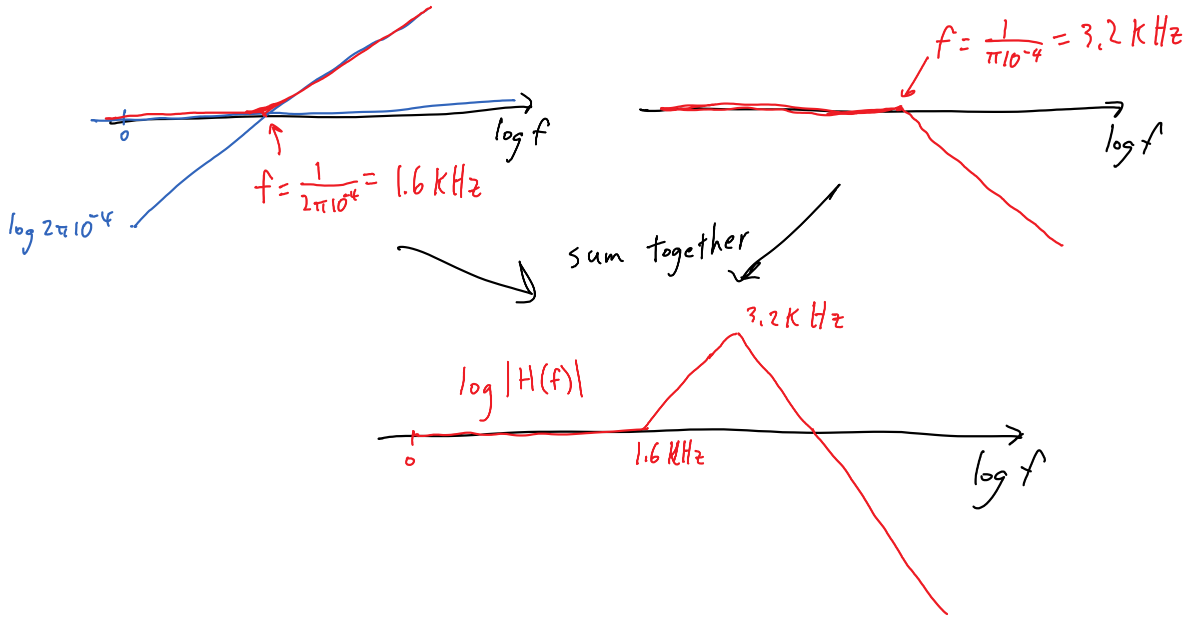
$$\log|H(f)| = \tfrac{1}{2}\log\left(1+(2\pi 10^{-4}f)^2\right) - \log\left(1+(\pi 10^{-4}f)^2\right)$$

$$\approx \tfrac{1}{2}\log \max\left(1, (2\pi 10^{-4}f)^2\right)$$

$$= \tfrac{1}{2}\max\{0,\ 2\log 2\pi 10^{-4}f\}$$

$$= \max\{0,\ \log f + \log 2\pi 10^{-4}\}$$

same steps

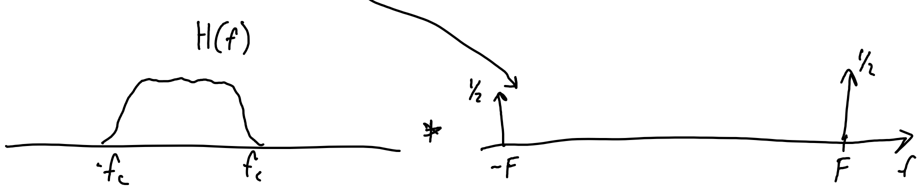


Low-pass filter design:

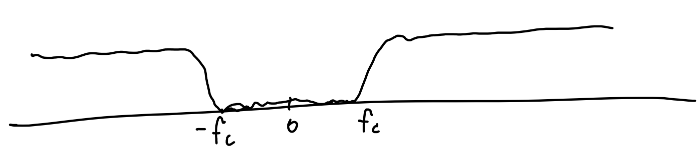
Why the emphasis on LPF?

Build bandpass out of LPF.

$h_{LPF}(t) \cdot \cos(2\pi F t)$ Convolution in frequency.



HPF? $1 - H_{LPF}(f) = H_{HPF}(f)$ Time-domain: $\delta(t) - h_{LPF}(t)$



Digital Filter: (DSP)

FIR (finite-impulse response)

$h[n]$ finite in length (length N)

N memory elements, weighted sum.

IIR (infinite impulse response)

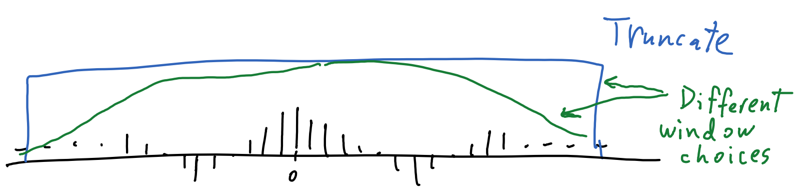
Use memory to store past output instead of inputs (or in addition to)

One FIR filter design method:

1.) Create ideal filter:

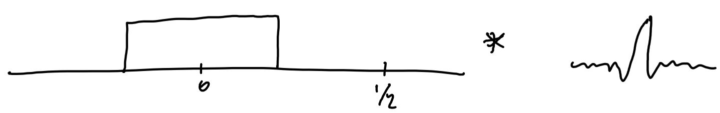
2.) Truncate

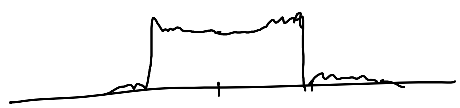
(and delay if real-time)



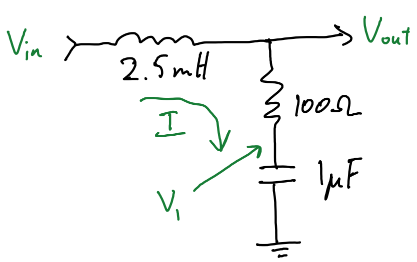
Make causal with delay (linear phase: Same group delay for all freq.)

In freq. domain:





Revisit Circuit Example



Inductor: $v(t) = L \frac{d}{dt} i(t)$ $\Rightarrow \frac{V(f)}{I(f)} = i2\pi Lf$

$V(f) = L\, i2\pi f\, I(f)$

Cap: $i2\pi f C\, V(f) = I(f)$ $\Rightarrow \frac{V(f)}{I(f)} = \frac{1}{i2\pi C f}$

Resistor: $V(f) = R\, I(f)$ $\Rightarrow \frac{V(f)}{I(f)} = R$

– All are like (freq. dependent) resistors.

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$$V_{in}(f) = V_1(f) + \left(V_{out}(f) - V_1(f)\right) + \left(V_{in}(f) - V_{out}(f)\right)$$
$$= \left(\frac{1}{i2\pi Cf} + R + i2\pi Lf\right) I(f)$$

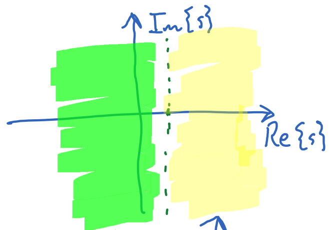
$$V_{out}(f) = \left(\frac{1}{i2\pi Cf} + R\right) I(f)$$

$$H(f) = \frac{V_{out}(f)}{V_{in}(f)} = \frac{\frac{1}{i2\pi Cf} + R}{\frac{1}{i2\pi Cf} + R + i2\pi Lf}$$

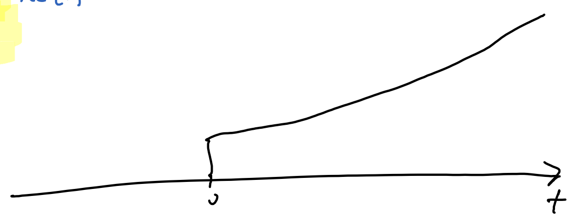
Laplace Transform:

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st}\, dt \qquad s \text{ is complex.}$$

The domain of $X(s)$ is the complex plane.



Example : $x(t) = e^t u(t)$



$$X(s) = \int_{-\infty}^{\infty} e^t u(t) e^{-st}\, dt$$
$$= \int_0^{\infty} e^{(1-s)t}\, dt = \frac{-1}{1-s} = \frac{1}{-1+s}$$

Converge only if

$Re\{1-s\} < 0$

$Re\{s\} > 1$

Region of convergence

$$x(t) = -e^t u(-t) \qquad X(s) = -\int_{-\infty}^{0} e^{(1-s)t}\, dt = \frac{-1}{1-s} = \frac{1}{-1+s}$$

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$X(s) = -\int_{-\infty} e^{(1-s)t}\, dt = \frac{-1}{1-s} = \frac{1}{-1+s}$

Converges: $Re\{1-s\} > 0$

$Re\{s\} < 1$

Let $s = \sigma + i2\pi f$

$$X(\sigma + i2\pi f) = \int_{-\infty}^{\infty} x(t)\, e^{-\sigma t}\, e^{-i2\pi f t}\, dt = \mathcal{F}\{x(t) e^{-\sigma t}\}$$