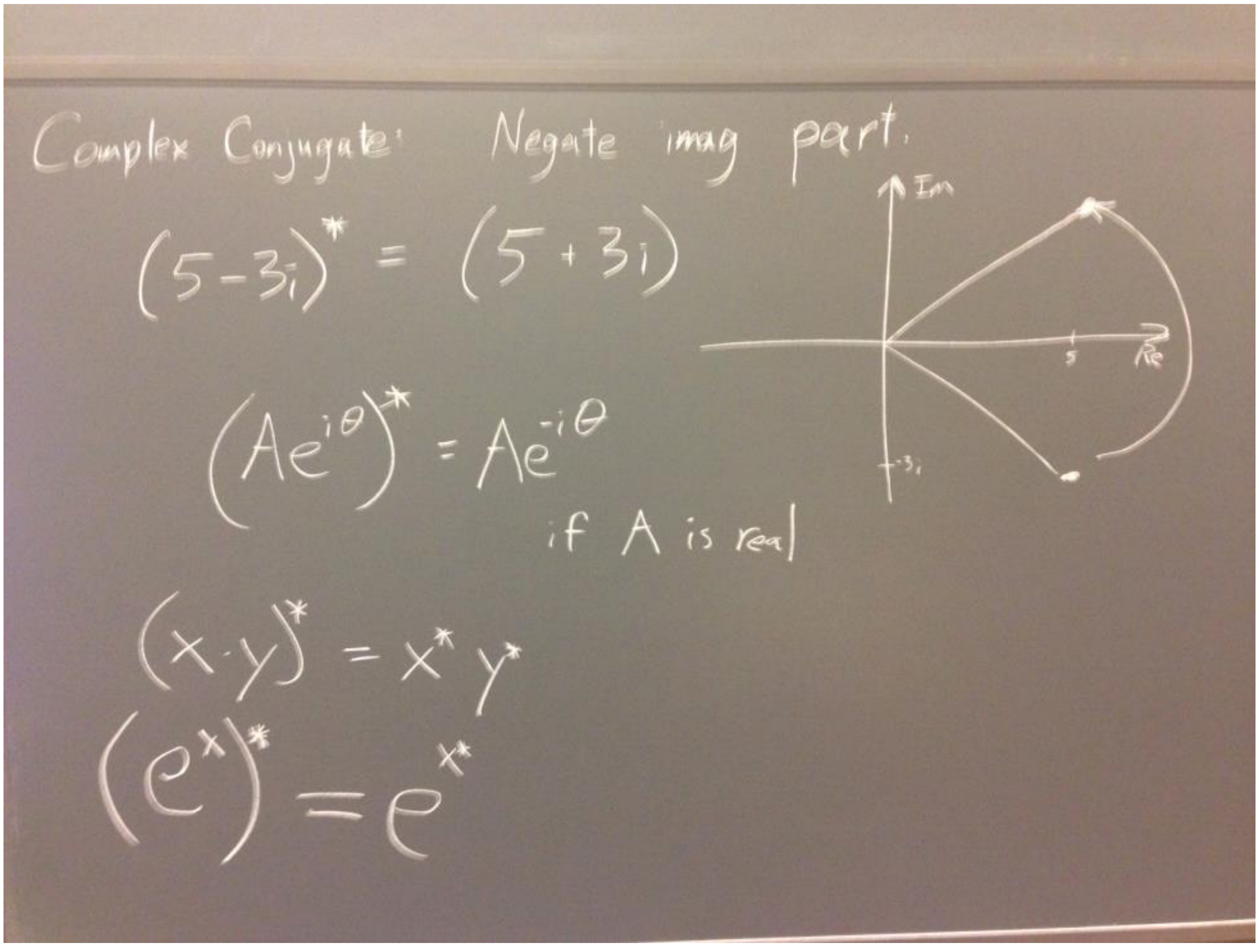
# Lecture 2

Tuesday, February 04, 2014
6:30 PM



Real part: $\mathrm{Re}(5-3i) = 5$

$\mathrm{Im}(5-3i) = -3$

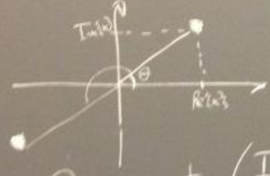
$$\mathrm{Re}(x) = \frac{x + x^*}{2} \qquad \mathrm{Im}(x) = \frac{x - x^*}{2i}$$

Example: $e^{it} = \underline{\cos(t)} + \underline{i\sin(t)}$

$$\cos(t) = \mathrm{Re}(e^{it}) = \frac{1}{2}\left(e^{it} + e^{-it}\right)$$

$$\sin(t) = \mathrm{Im}(e^{it}) = \frac{1}{2i}\left(e^{it} - e^{-it}\right)$$

Magnitude: $|x| = \sqrt{x \cdot x^*}$




Phase: $\theta = \arctan\left(\frac{\mathrm{Im}(x)}{\mathrm{Re}(x)}\right) + \pi$

Optional

Check: $a + ib$

$$|a+ib| = \sqrt{(a+ib)(a-ib)}$$

$$= \sqrt{a^2 - (ib)^2}$$

$$= \sqrt{a^2 + b^2}$$

Length of vector

From rectangular to Polar:

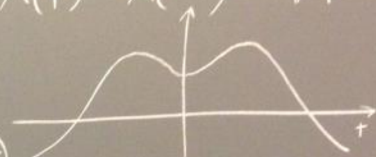
$$\mathrm{Re}(Ae^{i\theta}) = A\cos\theta, \quad \mathrm{Im}(Ae^{i\theta}) = A\sin\theta$$

$$x = Ae^{i\theta} = A\cos\theta + iA\sin\theta$$

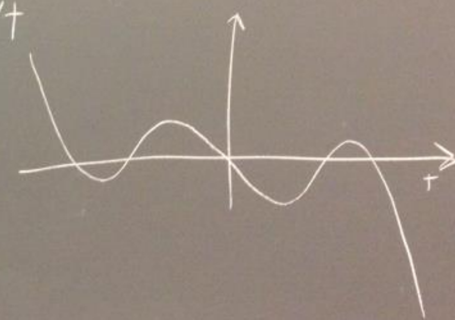
$X = Ae^{i\theta} = A\cos\theta + iA\,\mathrm{si}$

# Odd and Even Functions:

**Even:** $x(t)$ is even if $x(t) = x(-t)\ \forall t$



**Odd:** $x(t)$ is odd if $x(t) = -x(-t)$ $\forall t$



Amp 3
Phase .2

1 2 3

Kf
K
T

# Unique Decomposition:

$$x(t) = \underbrace{x_e(t)}_{\text{Even}} + \underbrace{x_o(t)}_{\text{Odd}}$$

$$x_e(t) = \frac{x(t) + x(-t)}{2} \qquad x_o(t) = \frac{x(t) - x(-t)}{2}$$

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**Periodic:** $x(t)$ is periodic with period $T$

if $x(t) = x(t+T)\ \forall t$

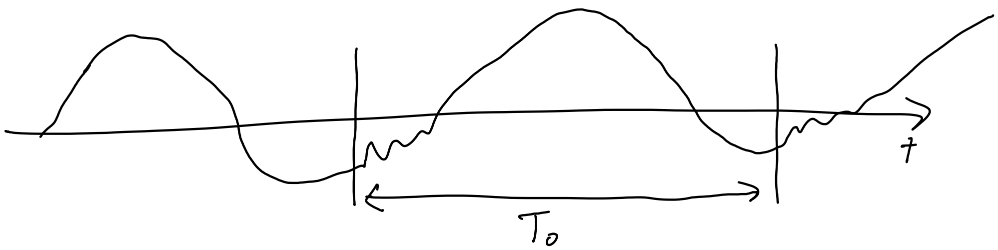
$\Downarrow$

$x(t) = x(t+kT)\ \forall t$ and integer $K$.

**Periodic:** $x(t)$ is periodic if there exists a $T>0$ such that $x(t) = x(t+T)$ $\forall t$

**Fundamental Period:** Let $T_0$ be the smallest period.

**Fundamental Freq.:** $f_0 = \frac{1}{T_0}$




**Sinusoids:** $x(t) = \cos(2\pi f T + \phi)$

$T_0 = \frac{1}{f}$

$\sin(2\pi f t + \phi)$ also periodic with period $T = \frac{1}{f}$

**Also:** $\cos(2\pi K f t)$ is also periodic with $T = \frac{1}{f}$

$\sum_{K=0}^{n} a_K \cos(2\pi K f t + \phi_K)$ is also periodic with period $\frac{1}{f}$

Surprise! "Any periodic signal" can be constructed this way by letting $n$ go to infinite.

See Mathematica and Matlab demos of representing a signal with sinusoids.

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**Continuous-time Signals:**

$x(t)$ where $t \in \mathbb{R}$

$x(t)$ where $t \in \mathbb{R}$ real numbers

Discrete-time Signals:

$x[n]$ where $n \in \mathbb{Z}$ integers

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Magnitude: $c_k$

Phase : $\phi_k$

Proper Conversion

$$a\cos(2\pi f t) + b\sin(2\pi f t) = \sqrt{a^2+b^2}\cos\left(2\pi f t - \arctan\left(\frac{b}{a}\right)\right)$$

$a_k$

$b_k$

$$x(t) = \sum_{k=0}^{\infty} a_k \cos(2\pi k f t) + b_k \sin(2\pi k f t)$$

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Use Complex Exponentials:

$$x(t) = \sum_k c_k e^{i 2\pi k f t}$$

Fourier Series:

Continuous-time: Let $x(t)$ be periodic with period $T$.

Forward: (Analysis) $$c_k = \frac{1}{T}\int_0^T x(\tau) e^{-i2\pi \frac{k}{T}\tau} d\tau$$

Backward: (Synthesis) $$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{i 2\pi \frac{k}{T} t}$$

<u>Discrete-time:</u> Let $x[n]$ be periodic with period $N$.

$$c_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-i2\pi \frac{k}{N} n}$$

$$x[n] = \sum_{k=0}^{N-1} a_k e^{i2\pi \frac{k}{N} n}$$

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## Inner Product:

Let $x$ and $y$ be column vectors:

$$\langle x, y \rangle = x^T y^*$$

Let $x(t)$ and $y(t)$ be signals:

$$\langle x, y \rangle = \int_{-\infty}^{\infty} x(t)\, y^*(t)\, dt$$

If $x(t)$ and $y(t)$ have period $T$.

$$\langle x, y \rangle_T = \frac{1}{T} \int_0^T x(t)\, y^*(t)\, dt$$

<u>Energy:</u> $E = \langle x, x \rangle = \int_{-\infty}^{\infty} |x(t)|^2 dt$

<u>Power:</u> For periodic signal:

$$P = \langle x, x \rangle_T = \frac{1}{T} \int_0^T |x(t)|^2 dt$$