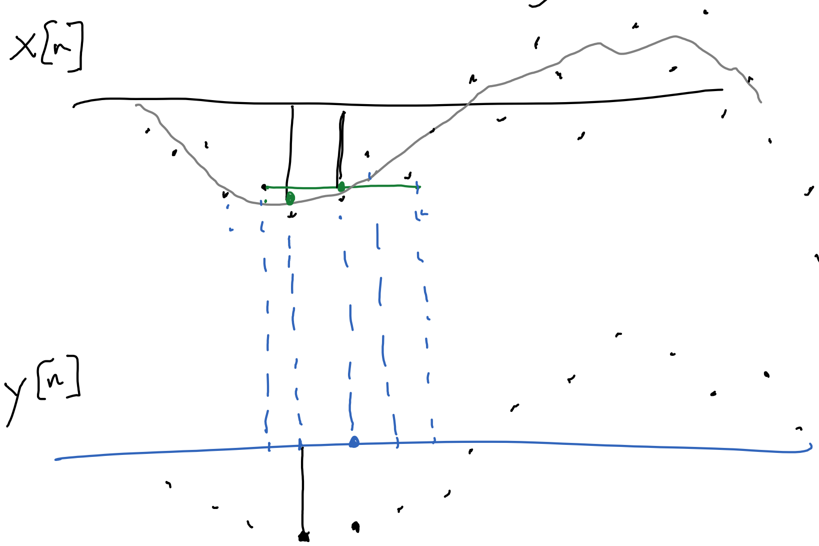
Lecture 8

Wednesday, March 05, 2014

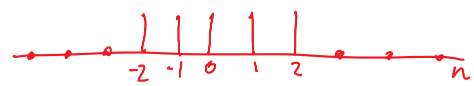
12:41 PM

Smooth using local averaging



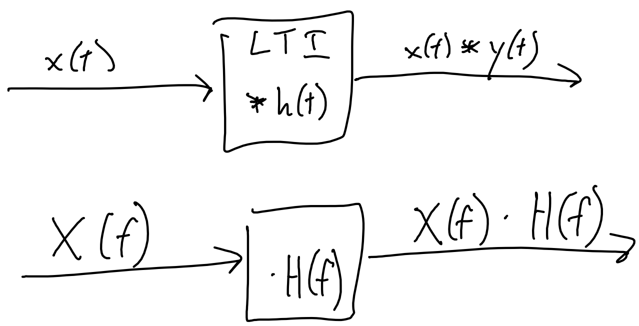
$y[n] = x[n] * ?$

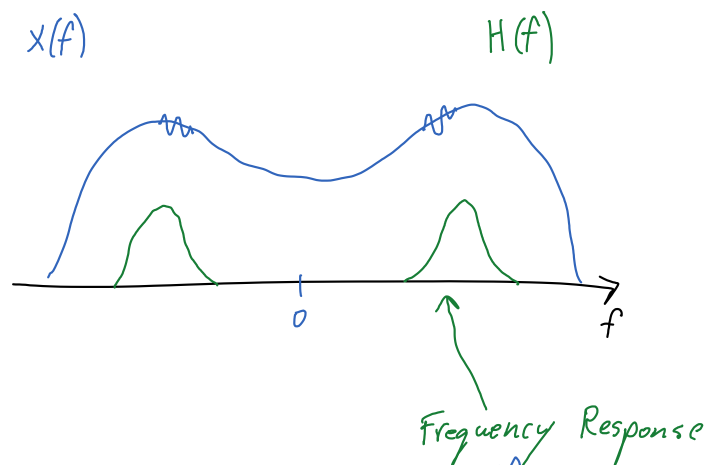
$\delta[n+2] + \delta[n+1] + \delta[n] + \delta[n-1] + \delta[n-2]$



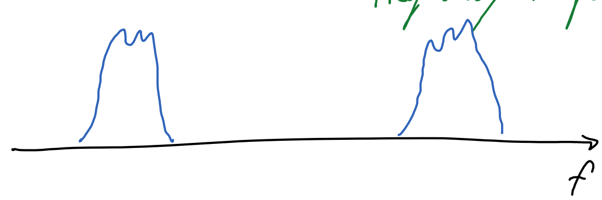
Convolution and the Fourier Transform:

$$\mathcal{F}\big(x(t) * y(t)\big) = \underbrace{\mathcal{F}\big(x(t)\big)}_{X(f)} \cdot \underbrace{\mathcal{F}\big(y(t)\big)}_{Y(f)}$$





Output in frequency domain.



Consider an averaging system: $h(t)$

$$x(t) * \frac{1}{T}\mathrm{rect}(t/T)$$

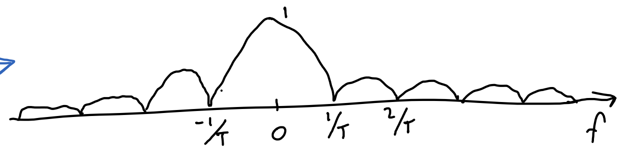
$$\mathcal{F}\left(\frac{1}{T}\mathrm{rect}(t/T)\right) = \frac{1}{T}\mathcal{F}\left(\mathrm{rect}(t/T)\right)$$

$$= \frac{1}{T}\cdot T\,\mathrm{sinc}(Tf) = \mathrm{sinc}(Tf) = H(f)$$

Frequency response

$$|H(f)| = |\mathrm{sinc}(Tf)|$$

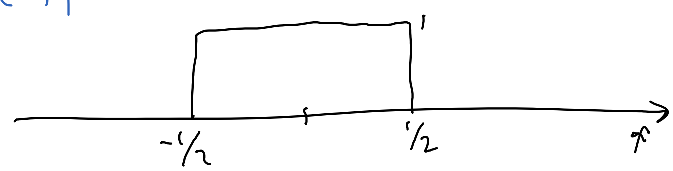
"Frequency Response"



If you want a sharp low-pass filter:

Ideal low-pass filter

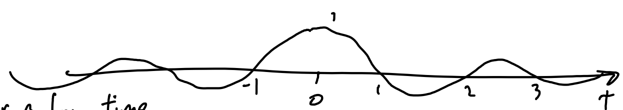
$|H(f)|$



$$H(f) = \mathrm{rect}(f) \Rightarrow h(t) = \mathrm{sinc}(t)$$

Averaging weights are sinc(t)

- May cause smoothing artifacts that extend for a long time.



Modulation Property:

Multiplication in time (modulation) is convolution in frequency:

$$\mathcal{F}\left(x(t)y(t)\right) = X(f) * Y(f)$$

Property results from duality:

$$X(t) * Y(t) \xrightarrow{\mathcal{F}} \mathcal{F}(X(t))\,\mathcal{F}(Y(t))$$

$$x(-f)\,y(-f)$$

Duality $\rightarrow$ $x(-t)\,y(-t) \xrightarrow{\mathcal{F}} \big(X(f) * Y(f)\big)$ $\leftarrow$ reverse in time

$\Rightarrow x(t)\,y(t) \xrightarrow{\mathcal{F}} X(f) * Y(f)$

Time Reversal Property:

$\mathcal{F}(x(-t)) = X(-f)$ Time Scaling property with $a=-1$.

Modulation Example:

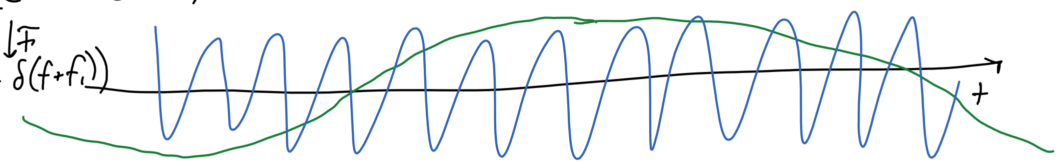
$\cos(2\pi f_1 t)$ where f_1 is large. (e.g. 400)

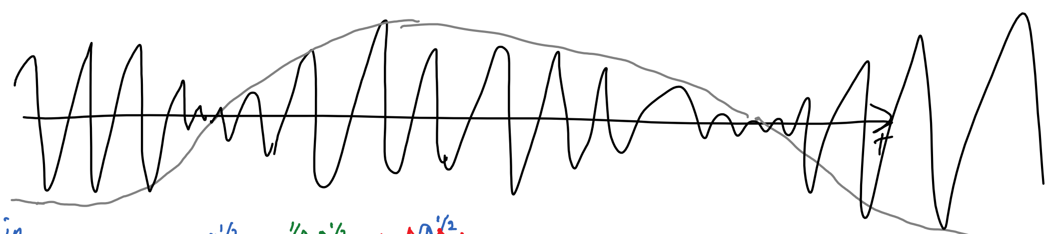
$\cos(2\pi f_1 t)\cos(2\pi f_2 t)$ where f_2 is small (e.g. 10)

$\frac{1}{2}\left(e^{i2\pi f_1 t} + e^{-i2\pi f_1 t}\right)$

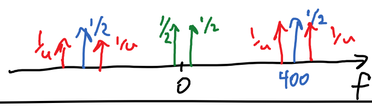
$\downarrow \mathcal{F}$

$\frac{1}{2}\left(\delta(f-f_1) + \delta(f+f_1)\right)$





In frequency Domain

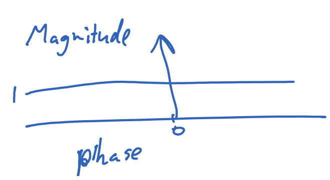


Convolution Commutes:

$$x(t) * y(t) \xrightarrow{\mathcal{F}} X(f)Y(f) = Y(f)X(f) \xrightarrow{\mathcal{F}^{-1}} y(t) * x(t)$$

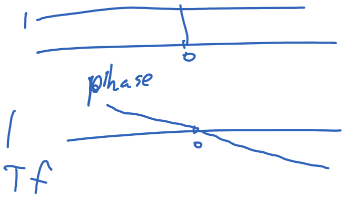
Delta functions:

$$\mathcal{F}(\delta(t-T)) = e^{-i2\pi Tf}$$



$$\mathcal{F}(\delta(t-T)) = e^{-i2\pi T f}$$

magnitude $= 1$
phase $= -2\pi T f$



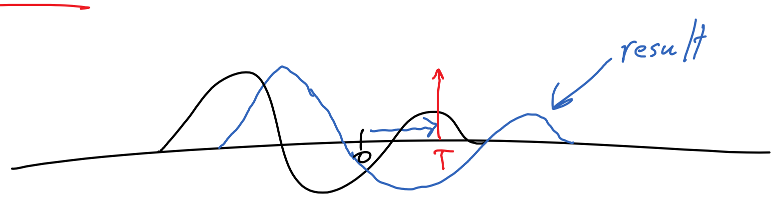
$$\delta(t) \xrightarrow{\mathcal{F}} 1$$

By duality: $e^{i2\pi F t} \xrightarrow{\mathcal{F}} \delta(f-F)$

Delta functions allow us to cheat the Fourier transform.

Convolution with delta functions:

$$\underline{\delta(t-T)} * x(t) = x(t-T)$$



$$\left(\delta(t-T_1) + 3\delta(t-T_2)\right) * x(t) = x(t-T_1) + 3x(t-T_2)$$

Even/odd Real/Imag Parts

Split a signal into four parts uniquely

Time		Frequency
Real, even	⟷	real, even
real, odd		real, odd
imag, even	⟷	imag, even
imag, odd		imag, odd

x(t) is real

What does this imply about mag. phase.

Conjugate symmetric

Conjugate symmetric

$X(f) = X^*(-f)$

Redundant for real signals