Final Exam Review

Thursday, May 08, 2014

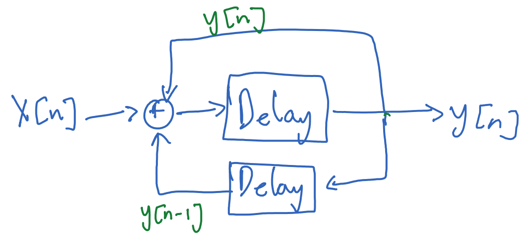
6:58 PM

Z - Transform

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$$

1) Fibonacci $0, 1, 1, 2, 3, 5, \ldots$

$$y[n] = y[n-1] + y[n-2] + x[n-1]$$



Input $x[n] = \delta[n]$

$y[0] = \underline{0}$

$y[1] = y[0] + x[0] = \underline{1}$

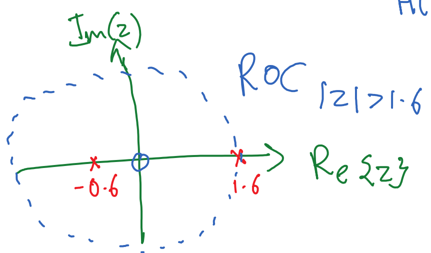
$y[2] = y[1] = \underline{1}$

$y[3] = y[2] + y[1] = \underline{2}$

$$y[n-k] \xrightarrow{\mathcal{Z}} z^{-k} Y(z)$$

$$Y(z) = z^{-1} Y(z) + z^{-2} Y(z) + z^{-1} X(z)$$

$$H(z) := \frac{Y(z)}{X(z)} = \frac{z^{-1}}{1 - z^{-1} - z^{-2}}$$



ROC $|z| > 1.6$

$$= \frac{z}{z^2 - z - 1} = \frac{z}{\Big(z - \underbrace{\tfrac{1+\sqrt{5}}{2}}_{1.6}\Big)\Big(z - \underbrace{\tfrac{1-\sqrt{5}}{2}}_{-0.6}\Big)}$$

$$\frac{z^{-1}}{1 - z^{-1} - z^{-2}} = \frac{1}{\sqrt{5}}\left(\frac{\sqrt{5}-1}{2}\right)\frac{1}{\left(\frac{\sqrt{5}-1}{2}\right) - z^{-1}} - \frac{1}{\sqrt{5}}\left(\frac{\sqrt{5}+1}{2}\right)\frac{1}{\left(\frac{\sqrt{5}+1}{2}\right) + z^{-1}}$$

$$= \frac{1}{\sqrt{5}}\,\frac{1}{1 - \left(\frac{2}{\sqrt{5}-1}\right) z^{-1}} - \frac{1}{\sqrt{5}}\,\frac{1}{1 - \left(\frac{-2}{\sqrt{5}+1}\right) z^{-1}}$$

$$\frac{1}{1 - a z^{-1}} \longrightarrow a^n u[n]$$

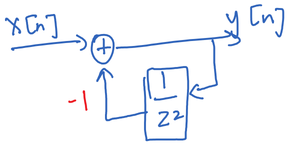
$|z| > a$

$$\Rightarrow h[n] = \frac{1}{\sqrt{5}}\left(\frac{2}{\sqrt{5}-1}\right)^n u[n] - \frac{1}{\sqrt{5}}\left(\frac{-2}{\sqrt{5}+1}\right)^n u[n]$$

$$= \frac{u[n]}{\sqrt{5}}\left[\underbrace{\left(\frac{1+\sqrt{5}}{2}\right)^n}_{1.6^n} - \underbrace{\left(\frac{-2}{\sqrt{5}+1}\right)^n}_{-0.6^n}\right]$$

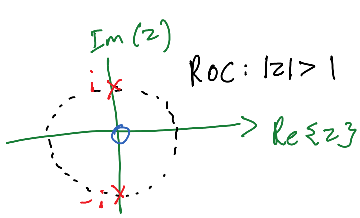
$$\xrightarrow{n \to \infty} \frac{\varphi^n}{\sqrt{5}} \qquad \varphi = \frac{1+\sqrt{5}}{2}$$

2) $y[n] = x[n] - y[n-2]$



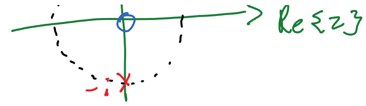
$$Y(z) = X(z) - z^{-2} Y(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 + z^{-2}} = \frac{z^2}{z^2 + 1}$$



ROC: $|z| > 1$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1+z^{-2}} = \frac{z}{z^2+1}$$

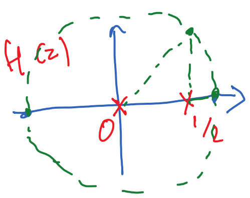


$$h[n] = \begin{cases} (-1)^k, & \text{if } n = 2k \\ 0, & \text{otherwise} \end{cases}$$

3) Is $H(z) = \frac{z^2+1}{3z}$ (ROC: $|z|>0$) causal?

$$= \frac{z}{3} + \frac{1}{3z} \longrightarrow \frac{1}{3}\delta[n+1] + \frac{1}{3}\delta[n-1]$$

4)

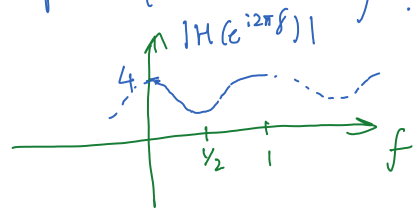


$H(1) = 1$

Plot the frequency response $(|H(e^{i2\pi f})|)$ and its phase $(\angle H(e^{i2\pi f}))$.

$$H(z) = \frac{K}{z(z-1/2)} = \frac{2}{z(z-\frac{1}{2})}$$

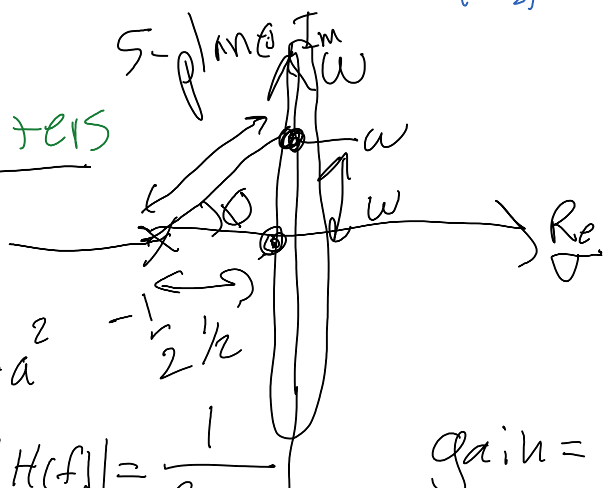
$$H(1) = \frac{K}{1(\frac{1}{2})} = 1 \Rightarrow K = 2$$



$$H(e^0) = \frac{2}{1(1-\frac{1}{2})} = 4$$

Recall: $\angle H(z) = \Sigma$ phase of zeros $- \Sigma$ phases of poles

filters



$$S = \sigma + j\omega$$

$$\sigma = 0$$

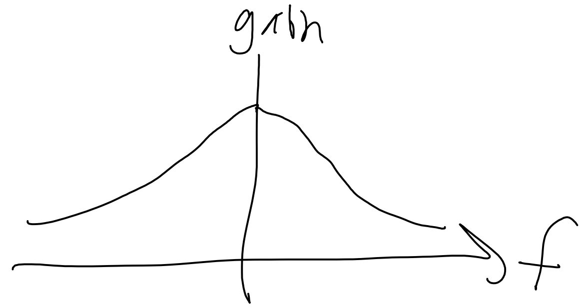
$h^2 = o^2 + a^2$

pole $|H(f)| = \frac{1}{\text{distance}}$

gain $= |H(f)|$

zero $|H(f)| = \text{distance}$

$\measuredangle H(f) = -\theta$ for pole



$\measuredangle H(f) = \theta$ for zero

$$|H(f)|^2 = \frac{1}{\ldots}$$

$$\frac{1}{\ldots} \rightarrow s \rightarrow i\omega$$

$$|H(f)|^2 = \frac{1}{\left(\frac{1}{2}\right)^2 + \omega^2} \qquad \frac{1}{s - \frac{1}{2}} \to s \to j\omega$$

$$\theta = \tan^{-1}\left(\frac{\omega}{-1/2}\right) \to \measuredangle H(f) = -\theta$$

6.24 from Book

$$|H(j\omega)| = \begin{cases} 1 & |\omega| \le 200\pi \\ 0 & \text{otherwise} \end{cases} \qquad \text{group delay} = 5s$$

what is $h[t]$ when $h(t)$ is real

$$\measuredangle H(j\omega) = \sigma + -\omega\alpha \qquad |H(j\omega)| e^{i \measuredangle H(j\omega)} = H(j\omega)$$

$$x(t - t_0) \to e^{-j\omega t_0} X(j\omega) \qquad t_0 = 5 = \alpha$$

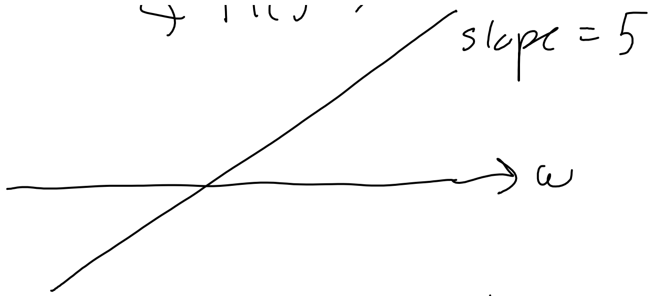
$$|H(j\omega)| e^{-i\sigma} e^{-i\omega\alpha} \cdot X(j\omega) = Y(j\omega)$$

$$h(t) = \cancel{e^{-i\sigma}}^{\,1}\ 200 \operatorname{sinc}\left(200\pi(t-5)\right)$$

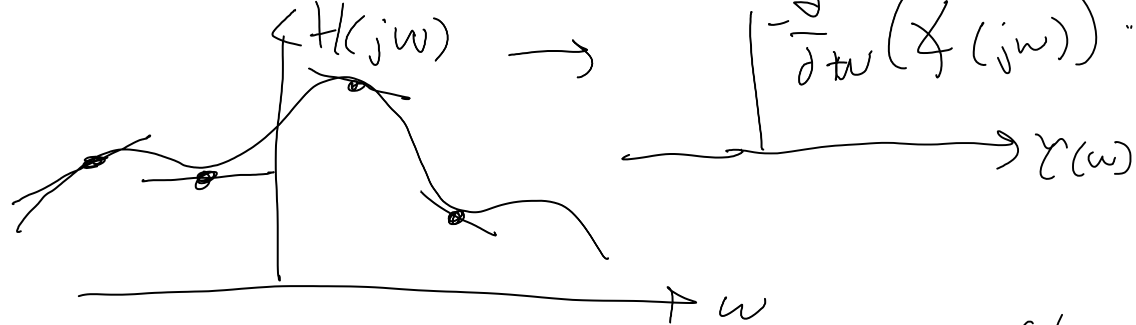
$$h(t) = \text{real} \to = 1 \to \sigma = 0$$

$$\measuredangle H(j\omega) = 5\omega$$

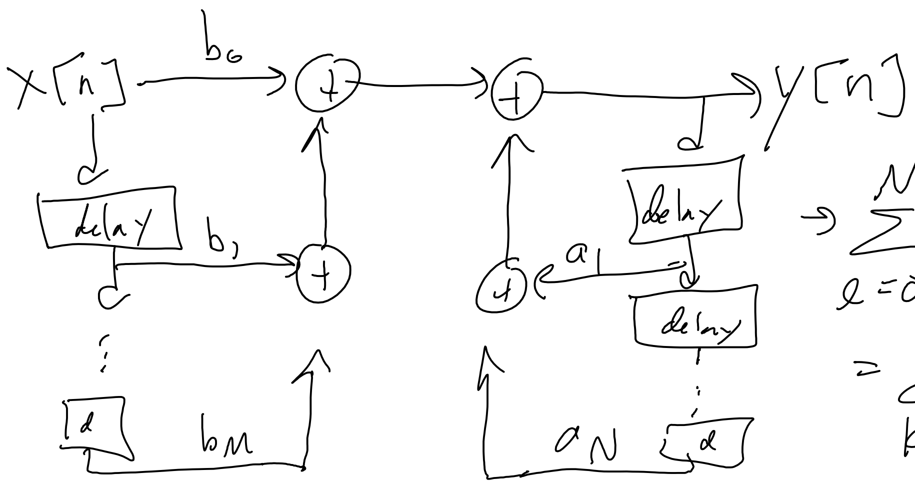
slope = 5



$$\tau(\omega) = -\frac{1}{2\pi}\frac{d}{df}H(f) = -\frac{d}{d\omega}H(\omega) = -\frac{d}{d\omega}H(j\omega)$$



IIR filter

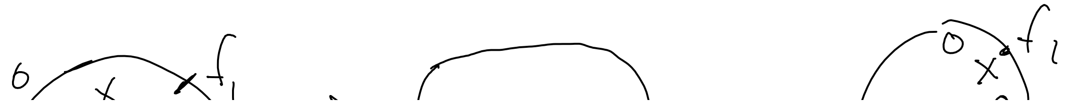


$$\sum_{l=0}^{N} a_l\, y(n-l) = \sum_{k=0}^{M} b_k\, x[n-k]$$

$$H(z) = \frac{b_0 + b_1 z^{-1} + \cdots b_M z^{-M}}{a_0 + a_1 z^{-1} + a_2 z^{-2} \cdots b_N z^{-N}}$$

FIR just have top

$$\rightarrow \frac{(z-z_1)(z-z_2)\cdots(z-z_M)\, z^{N-M}}{(z-\gamma_1)(z-\gamma_2)\cdots(z-\gamma_N)}$$



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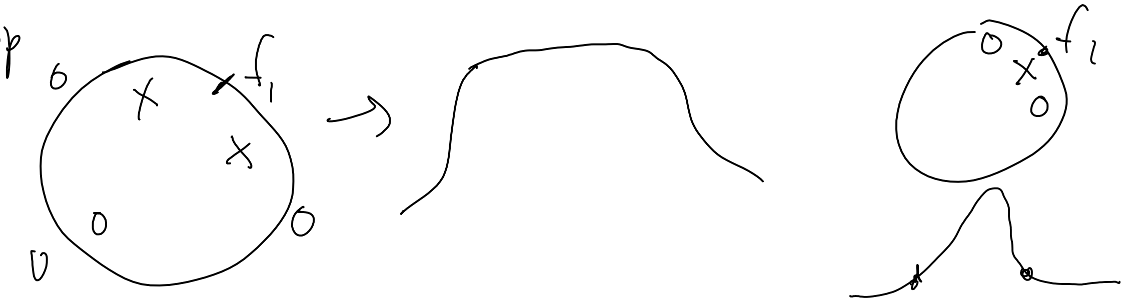
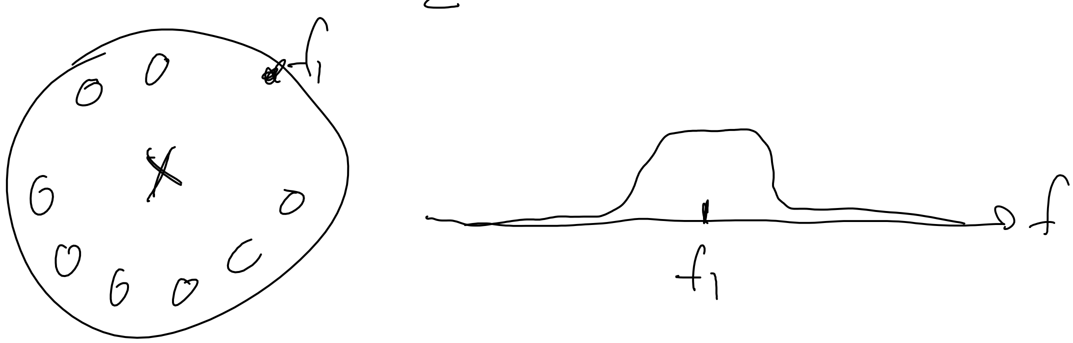


FIR filter all $a_s = 0$

$$\frac{b_0 z^M + b_1 z^{M-1} \cdots b_M}{z^M}$$



$H < E(L) < H + 1$ if optimal

not iff optimal

$$z^2 + az - b \,\big)\, \overline{z^2 + cz}$$

$$c_1 \delta[n] + c_2 \delta[n-1] + c_3 \delta[n-2] \cdots$$

$$-b + az + z^2 \,\big)\, \overline{cz + z^2}$$

$$= c_1 \delta[n] + c_2 \delta[n+1] \cdots$$

$-\alpha^n u[-n-1]$ or $\alpha^n u[n]$

$$H(z) = \frac{1}{1-az^{-1}}$$ not enough information