Problem Set #1
Due: Monday, Feb. 14, 2014

1. *Complex numbers - polar.* Compute the magnitude and phase of the complex numbers:

   (a) $3 + 2i$ (b) $1 - i$ (c) $e^{2-i\pi}$

2. *Complex numbers - rectangular.* Compute the real and imaginary parts of the complex numbers:

   (a) e^i
   (b) $e^{it}(\cos(3t) + sin(2t))$ (t is real)
   (c) $1/(1 + i)$

3. *Imaginary square root.* Express the square root of $(1 + i)$ in rectangular coordinates.

4. (a) Give examples of signals that have the listed properties:
      i. A signal with one independent variable that is something other than time,
      ii. A signal (other than image or video) where there is more than one independent variable,
      iii. A signal that is most naturally modeled as analog,
      iv. A signal that is most naturally modeled as digital.

   (b) Determine whether the following signals (a) are continuous-time or discrete-time; (b) take on a continuous or discrete set of values.
      i. Gear of a car in motion (i.e. 2nd gear, 3rd gear, etc.),
      ii. Speed of a car in motion,
      iii. The Hi and Low temperature everyday in the past 10 days.

5. Let us define the *indicator function* of the set A to be

$$1_A(x) = \begin{cases} 1, & \text{if } x \in A \\ 0, & \text{otherwise} \end{cases}.$$

   Are the following signals orthogonal? Are they orthonormal?

   (a) $e^{i2\pi t} \cdot 1_{[-\pi,\pi]}(t)$ and $1_{[0,1]}(t)$.
   (b) $\left\{1_{[j,j+1]}(t)\right\}_{j\in\mathbb{Z}}$, where $\mathbb{Z}$ is the set of integers.
   (c) $\left\{e^{(t-a)/2} \cdot 1_{(-\infty,a]}(t)\right\}_{a\in\mathbb{R}}$.

6. Assume that the signal $x(t)$ is periodic with period T_0, and that $x(t)$ is odd (*i.e.* $x(t) = -x(-t)$). What is the value of $x(T_0)$?

7. Assume that $y(t)$ is an arbitrary periodic signal with fundamental period T_0. Must $x_1(t)$ and $x_2(t)$ both be periodic if:

   (a) $y(t) = x_1(t) + x_2(t)$
   (b) $y(t) = x_1(t) \times x_2(t)$

8. What is the fundamental period of $cos(2\pi t/T_1) + cos(2\pi t/T_2)$ if $T_1 = 8$ and $T_2 = 10$? What about if $T_1 = 3$ and $T_2 = \pi$?

9. *Energy.* What are the energies of these signals (where t is the independent variable)?

   (a) $x(t) = Ae^{-at}u(t)$ with $a > 0$ (Note: $u(t)$ is the unit step function defined as $1_{[0,\infty)}(t)$).
   (b) The unit area rectangular pulse of width a, $\Delta_a(t)$.

10. *Power.* What are the powers of these signals?

   (a) $x(t) = A_1 e^{i\omega t} + A_2 e^{-i\omega t}$.
   (b) $x(t) = \sum_{k=-N}^{N} A_k e^{i\omega_0 kt}$.

11. *Fourier Series.*

   (a) State the fundamental period and Fourier series coefficients of the signal

$$x(t) = e^{-i\pi t} + e^{2(1+i\pi t)}.$$

   (b) What signal with fundamental period $T_0 = 1$ corresponds to the Fourier series coefficients

$$c_k = \begin{cases} \frac{1}{2i}, & k = 1 \\ \frac{-1}{2i}, & k = -1 \\ 0, & \text{otherwise} \end{cases},$$

   where c_k is the coefficient of the basis element $e^{i\frac{2\pi}{T_0}kt}$ (please simplify with Euler's formula)?

12. *Wavelet Transform.* Given the transform pairs

$$\begin{aligned} f(t) &\rightarrow F(a,b) \\ g(t) &\rightarrow G(a,b), \end{aligned}$$

   what is the transform of the function $h(t) = 3f(t) - 2g(t)$?

Bonus Question: This question is worth up to 5% but can only replace missed points.

The transforms we've discussed are linear. As linear operations with the same domain and range, we can search for eigenfunctions. Eigenfunctions are like eigenvectors in finite dimensional spaces. They have the property that when the linear operation is applied to them it produces an output that is just the eigenfunction times a constant. The Fourier transform has infinitely many of these, but at least two of them are well known and appear in Fourier transform tables. Search online for Fourier transform tables, and find two eigenfunctions (you may have to look through several tables to find them). You may also need to use the time-scaling property of the Fourier transform to get the constants just right. If you search for something related to "eigenfunctions of the Fourier transform" and find the answer directly, go ahead and report what you learned (and let us know that this is what you did), and receive a maximum of 2% of bonus points.