Problem Set #2
Due: Friday, Feb. 28, 2014

1. The signal $x(t)$ is periodic with period 2 and takes the values $x(t) = e^t$ for $|t| < 1$. What is the Fourier series representation?

2. Use the Fourier transform analysis equation to calculate the Fourier transform of the following signals:

   (a) (Continuous-time) $e^{-5|t-1|}$.

   (b) (Discrete-time) $a^n(u[n] - u[n-m])$, where m is a positive integer. (Hint: Express as a finite sum b^n for some b and use the geometric series.)

3. *Shift and Scale.* Let

$$f(t) = \begin{cases} t & 0 \leq t < 2, \\ 6 - 2t & 2 \leq t < 3, \\ 0 & \text{else.} \end{cases}$$

   Sketch each of the following signals. All sketches must have clearly labeled axes.

   (a) $f(t)$

   (b) $f(t) + 1$

   (c) $f(\frac{t}{2})$

   (d) $f(t+2)$

   (e) $f(-t-1)$

4. *Fourier Transform Properties.* In this exercise, you will derive Fourier properties as we did in class. Since these properties are all easily searchable, you must show work to get credit for this exercise. These properties are important for future use, and they will also help illuminate the similarities and differences of the different versions of the Fourier transform (i.e. CTFT, CTFS, DTFT, DTFS).

   (a) *Time-shifting.*
   The time-shifting property we derived in lecture for the CTFT was

$$x(t-T) \quad \longrightarrow^{\mathcal{F}} \quad e^{-i2\pi Tf}X(f).$$

   This property also holds for the DTFT; however, it only makes sense to talk about shifts in discrete-time if the shift is an integer.
   As an exercise, show that the property holds for the Fourier series as well, in particular the DTFS. That is, derive the DTFS for x[n-T], where T is an integer. Remember that the Fourier series is always defined by both the sequence of coefficients and the period.

(b) *Time-scaling.*

We derived the time-scaling property for the CTFT as

$$x(at) \quad \stackrel{\mathcal{F}}{\longrightarrow} \quad \frac{1}{|a|}X(f/a).$$

We now want to generalize this to the Fourier series and to discrete-time.

i. Suppose $x(t)$ is a continuous-time signal with period T and Fourier series coefficients $\{a_k\}$. What is the Fourier series of $x(at)$? It may help to separate the cases $a > 0$ and $a < 0$ ($a = 0$ is not interesting).

ii. The time-scaling property for the Fourier series that you derived may appear very different than the time-scaling property of the Fourier transform. Aside from the normalization $1/|a|$, these are actually consistent if you interpret the frequency domain properly. Describe how to interpret the Fouries series coefficients as a frequency domain representation and how this relates to the time-scaling property of the CTFT.

(c) *Time-expansion in discrete-time.*

Time-scaling is not well defined for discrete-time. But we can attempt to make an appropriate definition. Let m be an integer and define the time-expansion function $x_{(m)}[n]$ as,

$$x_{(m)}[n] = \begin{cases} x[n/m], & n \text{ a multiple of } m, \\ 0, & \text{else.} \end{cases}$$

i. Draw an example of $x[n]$ and $x_{(2)}[n]$.

ii. Derive the DTFT of $x_{(m)}[n]$ in terms of $X(f)$.

iii. For discrete-time signals, $X(f)$ is always periodic with period one. After time-expansion by a factor m it has an even shorter period. Find a (positive) period less than one.

5. *Discrete-time convolution.* Compute and plot $y[n] = x[n] * h[n]$, where

$$x[n] = \delta[n-2] + 2\delta[n-3] - 2\delta[n-4] - \delta[n-5]$$

$$h[n] = \begin{cases} 1 & \text{if } 3 \le n \le 7, \\ 0 & \text{otherwise.} \end{cases}$$

6. 3.21 from text

7. 3.24 from text