Problem Set #6
Due: Friday, May 2, 2014

1. **Group Delay:**

   Consider an LTI system with impulse response $h(t) = 5e^{-10t}u(t)$. What delay would a narrow-band signal of frequency 1 experience? What about frequency 1,000?

2. **Laplace Transform and Z-transform:**

   (a) Use the Laplace transform integral to calculate the Laplace transform of $x(t) = 5te^{-2t}u(t) - 3e^{-t}u(2-t)$.

   (b) Calculate the z-transform of $x[n] = 4^n u[n-5]$.

   (c) Calculate the z-transform of

   $$x[n] = \begin{cases} 3, & n = 2, \\ -5, & n = 7, \\ 1, & n = 10, \\ 0, & \text{else.} \end{cases}$$

3. **Stability:** Consider the following list of poles and zeros:

   - (Three) Poles: $(.5 + .5i), \quad (.5 - .5i), \quad (-.25)$
   - (Two) Zeros: $(2), \quad (-1.5)$

   (a) Plot the poles and zeros in the complex plane.

   (b) If the Laplace transform of the impulse response of a *causal* LTI system is rational and has the above poles and zeros, is it stable? What is the region of convergence for the Laplace transform of a system that is stable but perhaps not causal and has the same poles and zeros. Is the causal inverse to this system stable?

   (c) Answer the same questions as above for a discrete-time system, where the stated poles and zeros correspond to a rational Z-transform. Also, considering this system as a filter, approximately which frequency is amplified the most?

4. **Inverse Z-transform:**

A discrete-time causal LTI system is described by the block diagram below (the numbers next to the vertical arrows mean that the signal is multiplied by that number). Locate the poles and zeros of the Z-transform of the impulse response of this system. (Hint: Write the system as a difference equation.) Use the Z-transform to derive the impulse response.
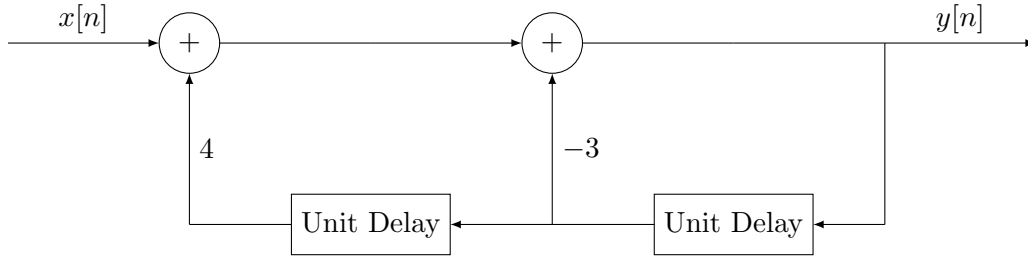
5. **Control:**

(a) Notice that the causal inverse to the system in question 4 is stable. What is the impulse response?

(b) **Open Loop Control:** Imagine the entire unstable system of question 4, with input $x[n]$ and output $y[n]$, as one block. Suppose we would like to make it stable, so that the output of the block closely tracks some control signal. If we use the inverse system (derived in part a) as a controller in series with the system block, as shown in the below figure, will the result be stable? Next, assume there is some error in your implementation of the inverse system, due to imperfect modeling of the system or component tolerances. Incorporate this assumption by adding 0.01 to all non-zero values in the impulse response of the controller. Is the controlled system still stable?

(c) **Closed Loop Control:** Consider the feedback controller in the below diagram, where the "system" block contains the system from question 4. Find the poles to verify that this causal system is stable. A general feature of feedback control is that it is more robust than open loop control—even if the system isn't exactly as described in question 4, the feedback-controlled system would still remain stable. What is the gain of this system at low frequencies (check the Fourier transform at $f = 0$)?