Lecture 15

Monday, March 23, 2015 8:23 AM

Medical Imaging

(Applications of the Fourier Transform and Signal Processing)

2-D Fourier Transform:

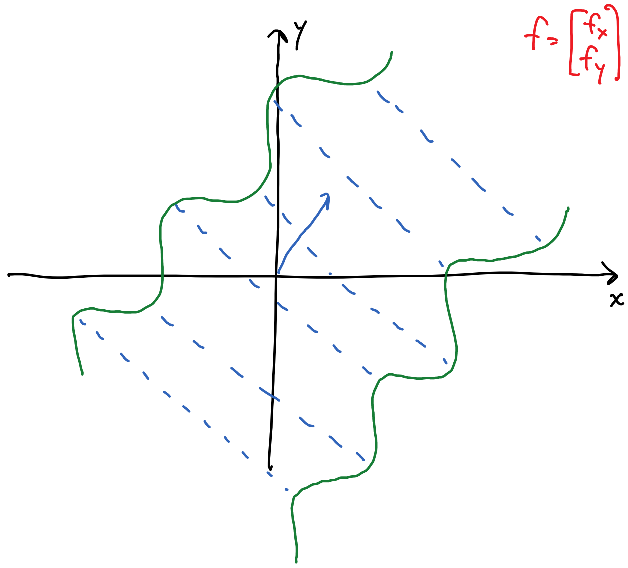
$s(x, y)$

$$S(f_x, f_y) = \iint s(x,y)\, e^{-i2\pi(f_x x + f_y y)}\, dx\, dy$$

$f^T p$

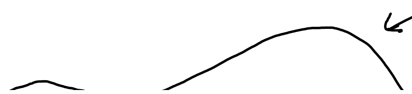
$$s(x,y) = \iint S(f_x, f_y)\, e^{i2\pi(f_x x + f_y y)}\, df_x\, df_y$$

$f = \begin{bmatrix} f_x \\ f_y \end{bmatrix}, \quad p = \begin{bmatrix} x \\ y \end{bmatrix}$

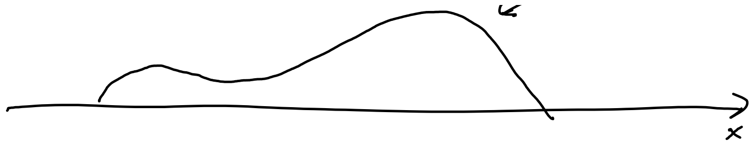


$$S(f_x, f_y) = \int \left(\int s(x,y)\, e^{-i2\pi f_x x}\, dx \right) e^{-i2\pi f_y y}\, dy$$

MRI in one-dimension:



$d(x)$



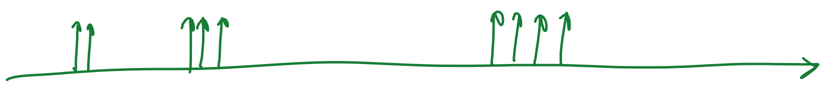
Magnetic field: $B_0 + gx$

$$s(t) = \int_{-\infty}^{\infty} d(x)\, e^{i2\pi\gamma(B_0+gx)t}\, dx = e^{i2\pi\gamma B_0 t} \int_{-\infty}^{\infty} d(x)\, e^{i2\pi\gamma g x t}\, dx$$

$$= e^{i2\pi\gamma B_0 t}\, \mathcal{F}^{-1}(d(x))(\gamma g t)$$

$$d(x) = \mathcal{F}\left(e^{-i2\pi\gamma B_0 t}\, s(t/\gamma g)\right)$$

Intuition:



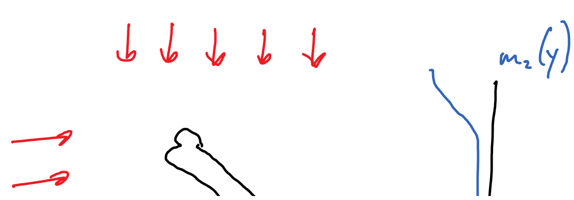
In reality: Time-varying gradient $g(t)$

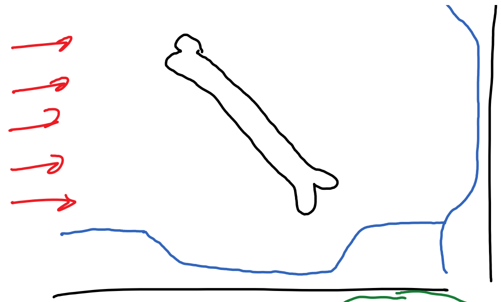
$$\int_{-\infty}^{\infty} d(x)\, e^{i2\pi\gamma \int_{-\infty}^{t} (B_0 + x g(\tau))\, d\tau}\, dx$$

- $s(t)$ is sampled (discrete-time)

CT: Computed Tomography

- X-ray — Shadow. e.g. Bone is more absorbant.
- 3D imaging





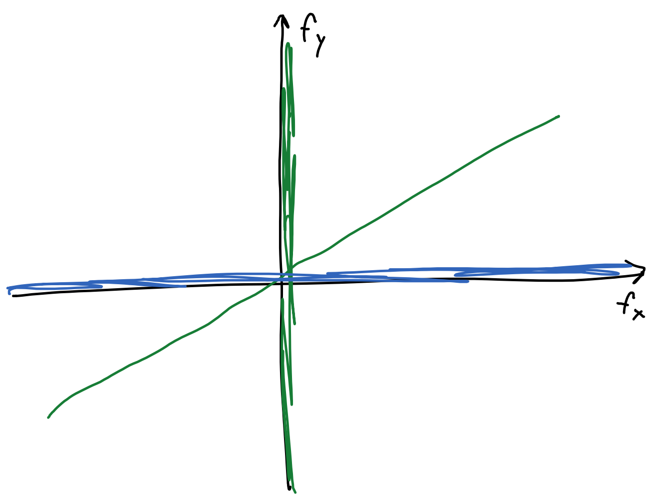
$$m_1(x) = I_0 - \int d(x,y)\,dy$$

$$\int_{-\infty}^{\infty} d(x,y)\, e^{-i2\pi(0)}\, dy$$

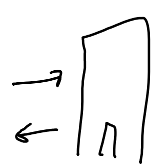
$$M_1(f_x) = \mathcal{F}(m_1(x)) = \int_{-\infty}^{\infty} m_1(x)\, e^{-i2\pi f_x x}\, dx$$

$$= \cancel{I_0\delta(f_x)} - \iint d(x,y)\, e^{-i2\pi(0y + f_x x)}\, dx\, dy$$

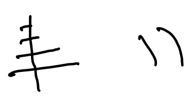
$$= D(f_x, 0)$$



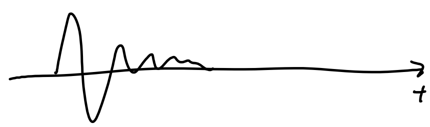
Utrasound:

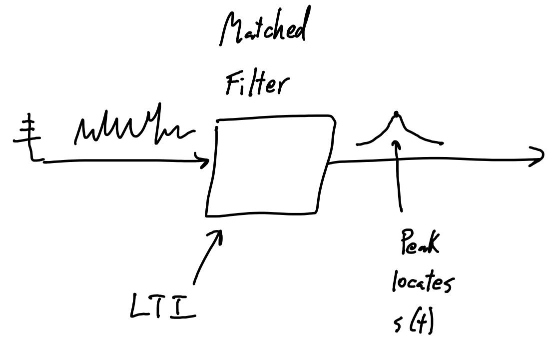


Radar:



Pulse signal: $s(t)$





$$h(t) = s^*(-t)$$