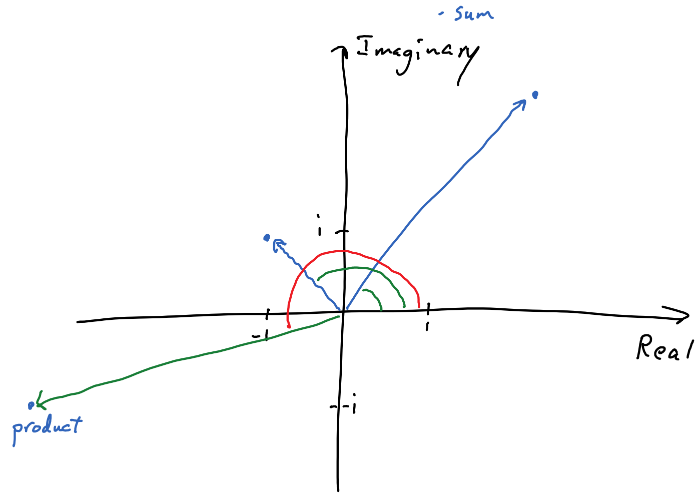
Lecture 2

Monday, February 09, 2015
8:40 AM

Complex Numbers:

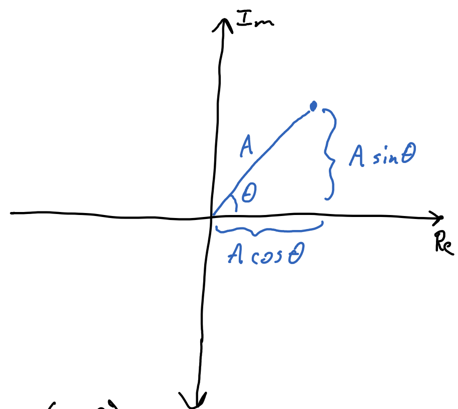


$$(2+3i) + (-1+i) = 1+4i$$

$$(2+3i)(-1+i) = -5-i$$

Polar Coordinates from Euler's identity:

$$\underset{\uparrow\ \text{magnitude}}{A}\, e^{i\overset{\swarrow\ \text{phase}}{\theta}} = A\cos\theta + iA\sin\theta$$



Multiplication:

$$A_1 e^{i\theta_1} \cdot A_2 e^{i\theta_2} = (A_1 A_2)\, e^{i(\theta_1+\theta_2)}$$

$$A_1 e^{i\theta_1} \big/ A_2 e^{i\theta_2} = \left(\frac{A_1}{A_2}\right) e^{i(\theta_1-\theta_2)}$$

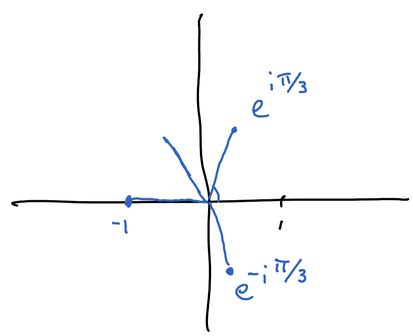
Example: $\sqrt[3]{-1} = -1$

$$-1 = 1e^{i\pi}$$

$$\sqrt[3]{-1} = \left(1e^{i\pi}\right)^{1/3} = 1e^{i\pi/3}$$

$$-1 = e^{i3\pi} \Rightarrow \sqrt[3]{-1} = e^{i\pi}$$

$$-1 = e^{i5\pi} \Rightarrow \sqrt[3]{-1} = e^{i\frac{5}{3}\pi}$$



Complex Conjugate: Negate imaginary part

$$(5-3i)^* = 5+3i$$

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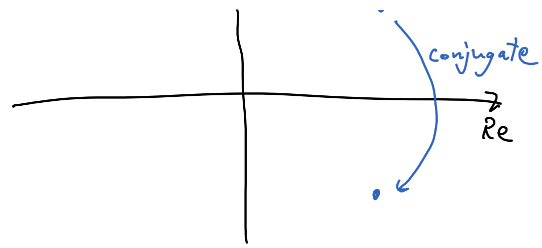

$$\left(Ae^{i\theta}\right)^* = Ae^{-i\theta}$$

$$(x \cdot y)^* = x^* y^*$$

$$\left(e^{x}\right)^* = e^{x^*}$$

Real part : $\mathrm{Re}(5-3i) = 5$

Imaginary part: $\mathrm{Im}(5-3i) = -3$

$$\mathrm{Re}(x) = \frac{x + x^*}{2} \qquad \mathrm{Im}(x) = \frac{x - x^*}{2i}$$

Example: $e^{ix} = \cos(x) + i\sin(x)$

$$\cos(x) = \mathrm{Re}(e^{ix}) = \frac{1}{2}\left(e^{ix} + e^{-ix}\right)$$

$$\sin(x) = \mathrm{Im}(e^{ix}) = \frac{1}{2i}\left(e^{ix} - e^{-ix}\right)$$

Magnitude (absolute value)

$$|x| = \sqrt{x\,x^*}$$

Check:

$$x = Ae^{i\theta}$$

$$x\,x^* = \left(Ae^{i\theta}\right)\left(Ae^{-i\theta}\right)$$

$$= A^2 \cancel{e^{i(\theta-\theta)}}$$

$$\sqrt{x\,x^*} = A$$

Rectangular : $x = a + ib$

$$|x| = \sqrt{(a+ib)(a-ib)} = \sqrt{a^2 - (ib)^2} = \sqrt{a^2+b^2}$$

Phase : $\theta = \arctan\left(\frac{\mathrm{Im}(x)}{\mathrm{Re}(x)}\right) + \pi\, I_{\{\mathrm{Re}(x)<0\}}$

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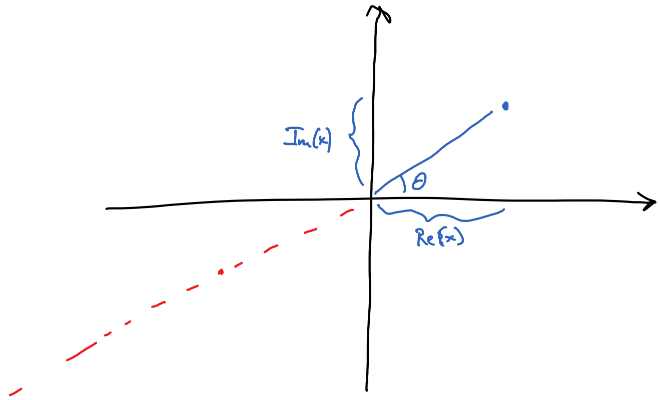
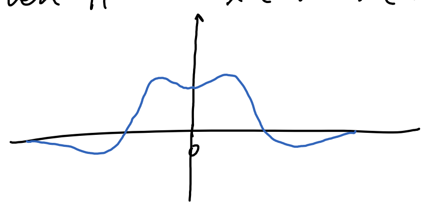
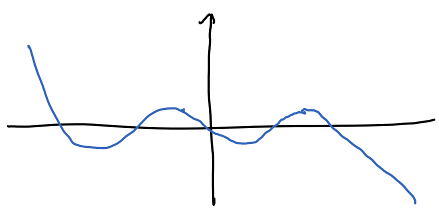


Signal (function):

Even: $x(t)$ is even if $x(t) = x(-t) \quad \forall t$



Odd: $x(t)$ is odd if $x(t) = -x(-t) \quad \forall t$

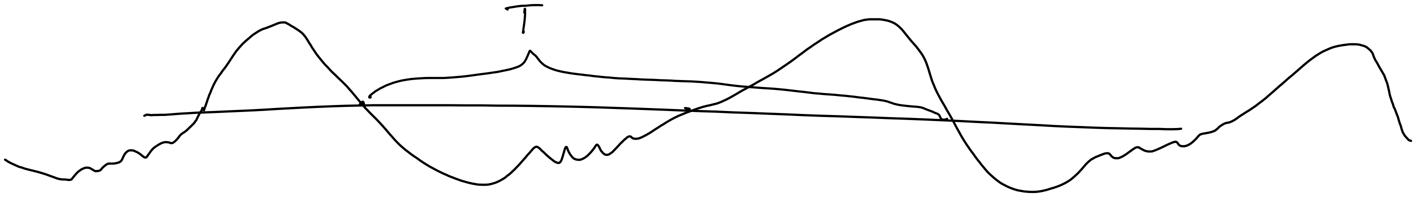


Unique Decomposition:

$$x(t) = \underbrace{x_e(t)}_{\text{Even part}} + \underbrace{x_o(t)}_{\text{odd part}}$$

$$x_e(t) = \frac{x(t)+x(-t)}{2} \qquad x_o(t) = \frac{x(t)-x(-t)}{2}$$

Periodic: $x(t)$ is periodic if there exists a $T>0$ such that $x(t) = x(t+T) \quad \forall t$ "with period T"



Fundamental Period : Smallest period T_0

Fundamental Frequency : $f_0 = \frac{1}{T_0}$

Example: $\cos(2\pi f t + \phi)$ Sinusoidal

Period : $\frac{1}{f}$

Fundamental period.

Continuous-time Signal : $x(t)$ where $t \in \mathbb{R}$

Discrete-time signal : $x[n]$ where $n \in \mathbb{Z}$

These are statements about domain.

Example signal :

Grades in H.S. math

Discrete-time.

Discrete range.