Lecture 25

Monday, April 13, 2015 3:19 PM

Filtering

Designing LTI system to behave a certain way

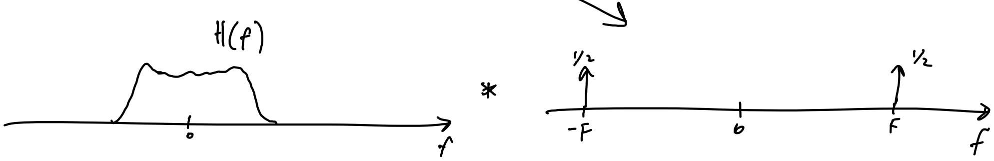
- Frequency response (i.e. Transfer function) ← Usually

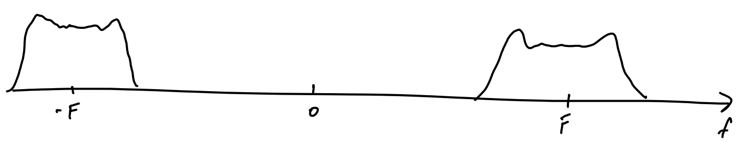
Consider LPF

Why the emphasis on LPF?

Build bandpass filter out of LPF.

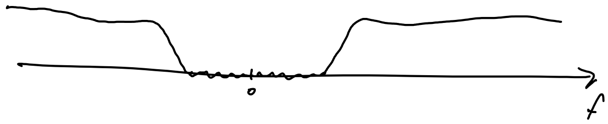
$h_{LPF}(t) \cdot \cos(2\pi F t)$ Convolution in frequency





HPF? $1 - H_{LPF}(f)$

Time domain: $\delta(t) - h_{LPF}(t)$



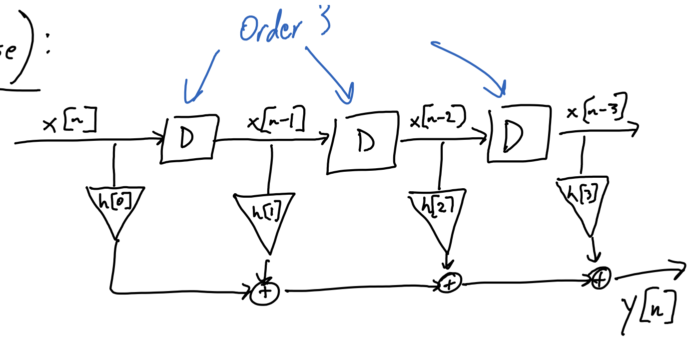
Digital Filter: (DSP)

FIR (finite impulse response): Order 3

FIR (finite impulse response):

$h[n]$ finite in length $(N+1)$

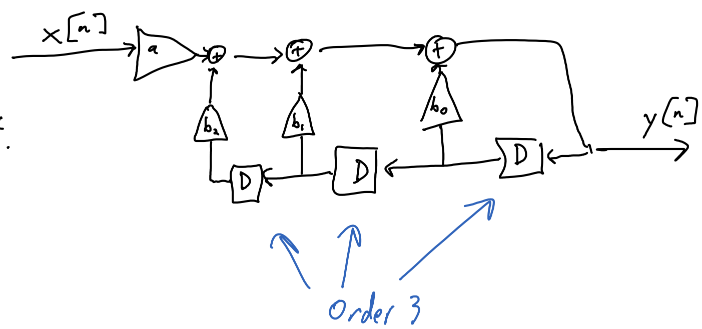
N memory elements for order N.



$$y[n] = x[n] * h[n]$$

IIR (infinite impulse response):

Use memory to store past output.



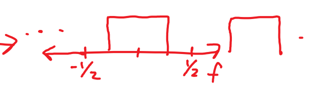
One FIR filter design method:

1.) Create ideal LPF

Discrete-time FT pairs

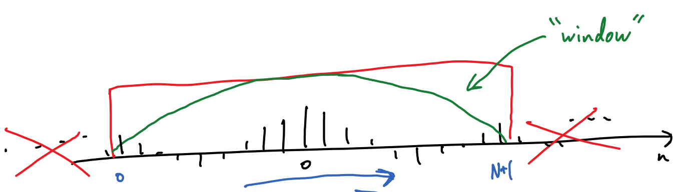
Assume $f_c < \frac{1}{2}$

$\text{sinc}[2f_c\, n] \xrightarrow{\mathcal{F}}$



Look in homework

$\text{rect}\left[\frac{n}{N}\right] \xrightarrow{\mathcal{F}} \dfrac{\sin(\quad f)}{\sin(\pi f)}$

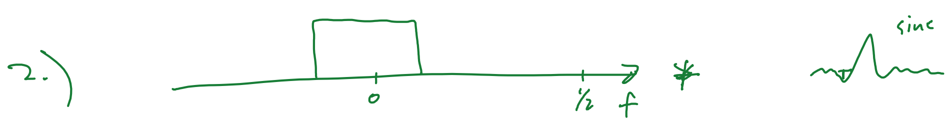


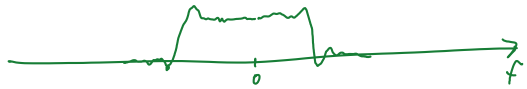
2.) Truncate

3.) Real-time ⇒ Causal

Make causal with delay

2.)





3.) Adds linear phase.

Other methods include direct optimization of FIR and/or IIR parameters.

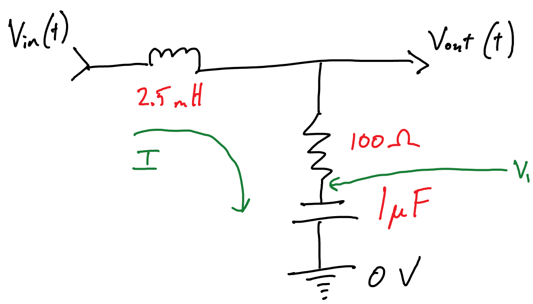
Example of CT Filter:

Ideal Passive Circuit Elements:

Inductor: $v(t) = L \frac{d}{dt} i(t)$

Capacitor: $C \frac{d}{dt} v(t) = i(t)$

Resistor: $v(t) = R\ i(t)$



LTI

Freq. Domain: $V(f) = L\ I(f) \cdot i2\pi f$

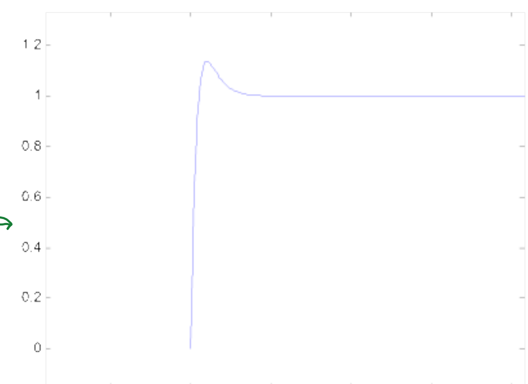
Inductor $\frac{V(f)}{I(f)} = i2\pi L f$

Capacitor $\frac{V(f)}{I(f)} = \frac{1}{i2\pi C f}$

Resistor $\frac{V(f)}{I(f)} = R$

Impedance

Solve Differential equation.



Step response: $1 - (1 - 20000t)e^{-20000t} u(t)$

derivative

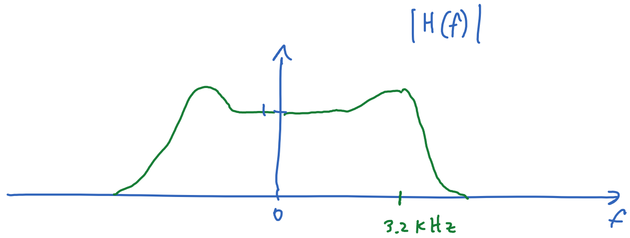
Impulse response: $h(t) = 40000\, e^{-20000t} u(t) - 400{,}000{,}000\, t\, e^{-20000t} u(t)$

$$H(f) = \frac{40000}{20000 + i2\pi f} - \frac{400{,}000{,}000}{(20000 + i2\pi f)^2} = \boxed{\frac{1 + i2\pi 10^{-4} f}{(1 + i\pi 10^{-4} f)^2}}$$

$$|H(f)| = \sqrt{1 + (2\pi 10^{-4} f)^2}$$

$$|H(f)| = \frac{\sqrt{1 + (2\pi 10^{-4} f)^2}}{1 + (\pi 10^{-4} f)^2}$$

$$\angle H(f) = \tan^{-1}(2\pi 10^{-4} f) - 2\tan^{-1}(\pi 10^{-4} f)$$



Fourier Transform method (we will use Laplace in general, next week):

$$V_{in}(f) = V_1(f) + (V_{out} - V_1(f)) + (V_{in}(f) - V_{out}(f))$$

$$= \frac{I(f)}{i2\pi Cf} + I(f)\cdot R + I(f)\cdot i2\pi f = \left(\frac{1}{i2\pi Cf} + R + i2\pi L\right) I(f)$$

$$V_{out}(f) = \left(\frac{1}{i2\pi Cf} + R\right) I(f)$$

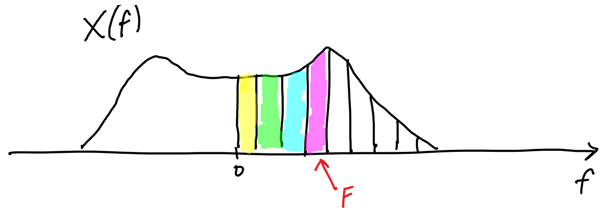
$$H(f) = \frac{V_{out}(f)}{V_{in}(f)} = \frac{\frac{1}{i2\pi CF} + R}{\frac{1}{i2\pi CF} + R + i2\pi Lf} = \text{above formula}$$

(substitute R, C, and L)

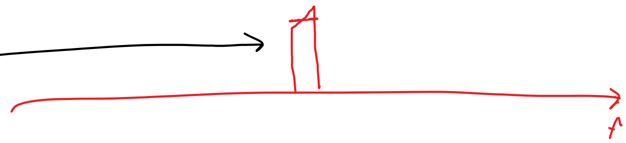
Group Delay:

A filter affects the mag. and phase

$$\text{Group delay} = -\frac{1}{2\pi}\frac{d}{df}\angle H(f).$$



Replace with Taylor expansion



$$H_K(f) \approx |H(F)|\, e^{i\angle H(F) + (f-F)\underbrace{\frac{d}{df}\angle H(f)\Big|_{f=F}}_{d(F)}}$$

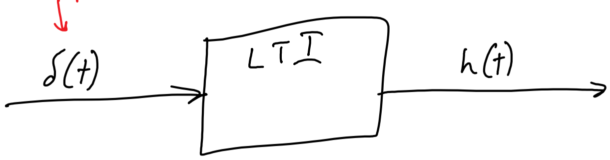
$$Y_K(f) = X_K(f)\cdot H_K(f) \approx X_K(f)\cdot \underbrace{|H(f)|\, e^{i\angle H(f)}}_{H(f)}\, e^{-d(F)\cdot F}\, e^{i\, d(F)\cdot f}$$

$$= \left(X_K(f)\cdot \underline{H(f)\cdot e^{-d(F)\cdot F}}\right)\cdot e^{i\, d(F)\cdot f}$$

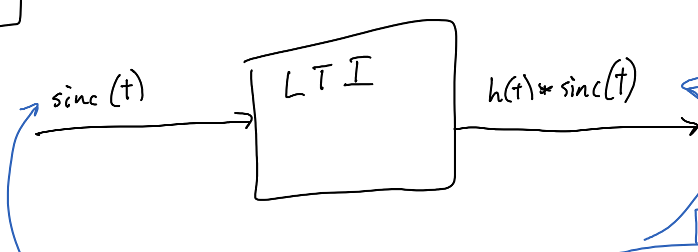
Delay by $\frac{-1}{2\pi} d(F)$.

Probe an LTI system: How do you measure $h(t)$?

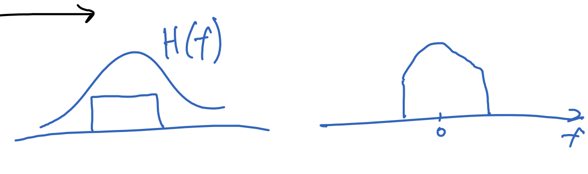
Not practical



$\boxed{e^{-t}u(t)}$ OK



Does not measure all frequencies.



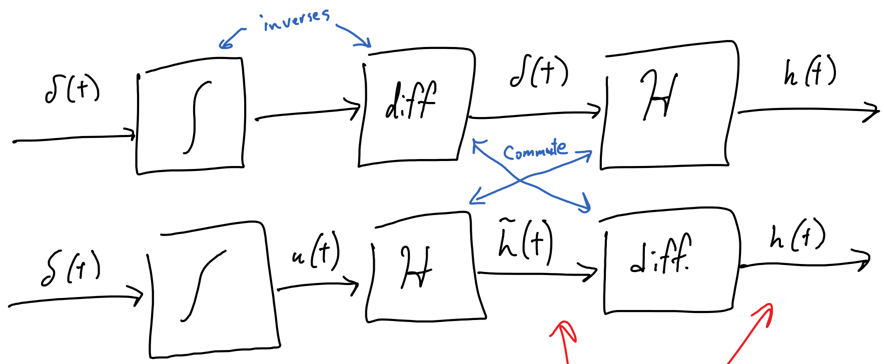
F.T. has zero regions.

$$h(t) = \mathcal{F}^{-1}\left(H(f)\right) = \mathcal{F}^{-1}\left(\frac{Y(f)}{X(f)}\right)$$

Should not be zero.

Unit step response: $u(t) \rightarrow \boxed{} \xrightarrow{\tilde{h}(t)}$ $\qquad \tilde{h}(t) = \mathcal{H}\left(u(t)\right)$

How is $\tilde{h}(t)$ related to $h(t)$?



$$h(t) = \frac{d}{dt}\tilde{h}(t)$$

FYI: Bode Plot: (CT only)

Plot $\log|H(f)|$ vs. $\log f$

Phase v.s $\log f$

Circuit example above.

$$H(f) = \frac{1 + i2\pi 10^{-4} f}{\left(1 + i2\pi 10^{-4} f\right)^2} \Rightarrow |H(f)| = \frac{\sqrt{1 + (2\pi 10^{-4} f)^2}}{1 + (\pi 10^{-4} f)^2}$$

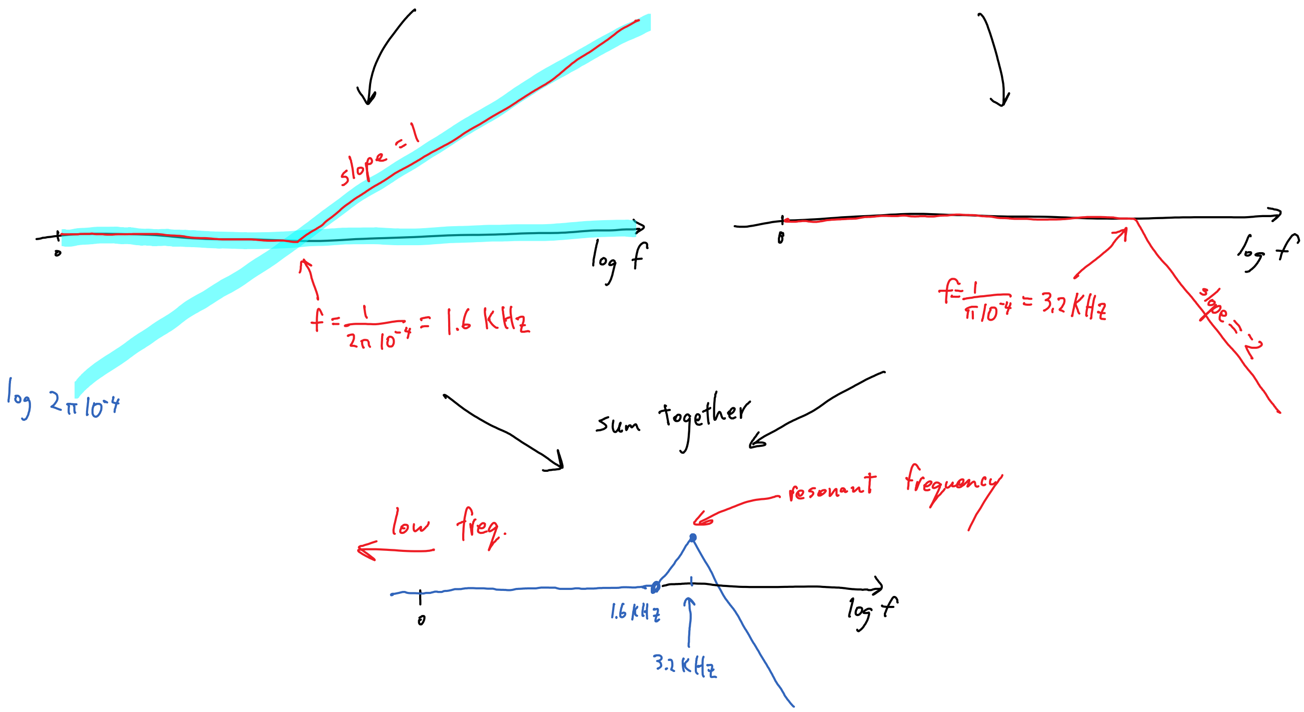
$$\log|H(f)| = \frac{1}{2}\log\left(1 + (2\pi 10^{-4} f)^2\right) - \log\left(1 + (\pi 10^{-4} f)^2\right)$$

$$\approx \frac{1}{2}\log\left(\max\left(1, (2\pi 10^{-4} f)^2\right)\right) - \log\left(\max(\cdot,\cdot)\right)$$

$$= \frac{1}{2}\max\left(0, 2\log(2\pi 10^{-4} f)\right) - \max\left(0, 2\log(\pi 10^{-4} f)\right)$$

$$= \frac{1}{2}\max\left(0,\ 2\log\left(2\pi 10^{-4} f\right)\right) - \max\left(0,\ 2\log\left(\pi 10^{-4} f\right)\right)$$

$$= \frac{1}{2}\max\left(0,\ 2\log f + 2\log\left(2\pi 10^{-4}\right)\right) - \max\left(0,\ 2\log f + 2\log\left(\pi 10^{-4}\right)\right)$$



Also simplifications for plotting phase v.s. log f.