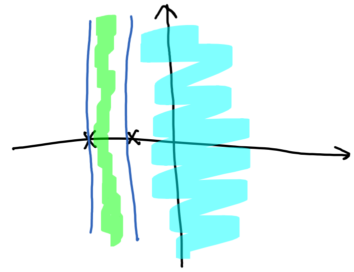
Partial Fraction Expansion:

C.T.:

$$H(s) = \frac{3s^2+8s+3}{s^2+3s+2} = 3 + \frac{-s-3}{s^2+3s+2}$$



$$= 3 + \frac{-s-3}{(s+2)(s+1)}$$

Poles in left half-plane $\Rightarrow$ stable (assuming causal)

$$= 3 + \frac{A}{s+2} + \frac{B}{s+1}$$

$$\frac{A(s+1) + B(s+2)}{(s+1)(s+2)}$$

$$(A+B)s + A+2B = -s-3$$

$$A+B = -1, \quad A+2B = -3$$

Cover-up Method (distinct poles)

$$\frac{-s-3}{(s+2)(s+1)} = \frac{A}{s+2} + \frac{B}{s+1}$$

$$A = 1, \quad B = -2$$

$$H(s) = 3 + \frac{1}{s+2} - \frac{2}{s+1}$$

$$h(t) = 3\delta(t) + e^{-2t}u(t) - 2e^{-t}u(t)$$

High-order poles:

$$\frac{1}{(s+2)^2(s+1)} = \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{C}{s+1}$$

Can't use cover-up method.

DT: (z-transform)

Same method except:

How do we invert $\frac{1}{z-a}$?

Table: $\frac{1}{1-az^{-1}} \longrightarrow a^n u[n]$

$$\frac{z^{-1}}{1-az^{-1}} \longrightarrow a^{n-1}u[n-1]$$

More common way (often simpler formulas),

Do partial fraction expansion w.r.t. z^{-1}:

Example:

$$H(z) = \frac{(z+2)z}{4(z-3)(z+\frac{1}{4})} = \frac{(1+2z^{-1})}{4(1-3z^{-1})(1+\frac{1}{4}z^{-1})}$$

$$= \frac{A}{1-3z^{-1}} + \frac{B}{1+\frac{1}{4}z^{-1}}$$

$$\Rightarrow A = \frac{\frac{5}{3}}{4(\frac{13}{12})}$$

$$B = \frac{-7}{4 \cdot 13}$$

$$\Rightarrow h[n] = A\, 3^n u[n] + B \left(\tfrac{-1}{4}\right)^n u[n]$$

From rational LT or ZT,

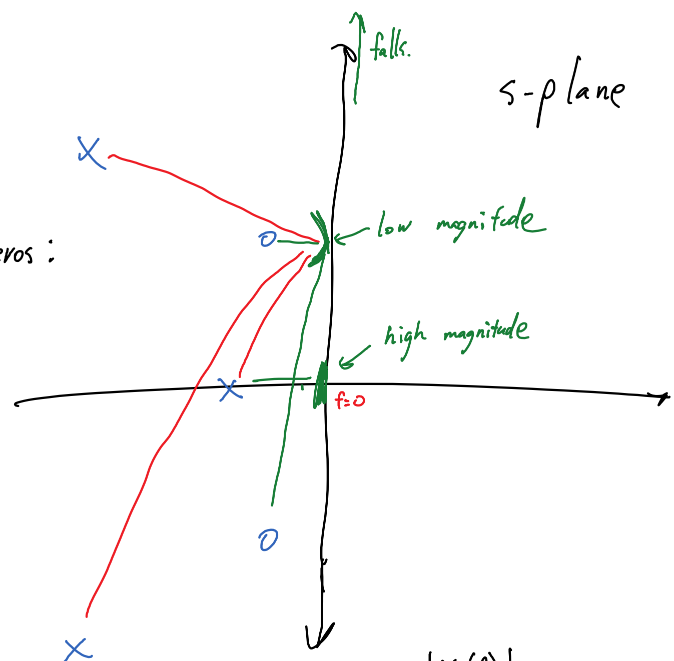
1.) Interpret frequency domain:

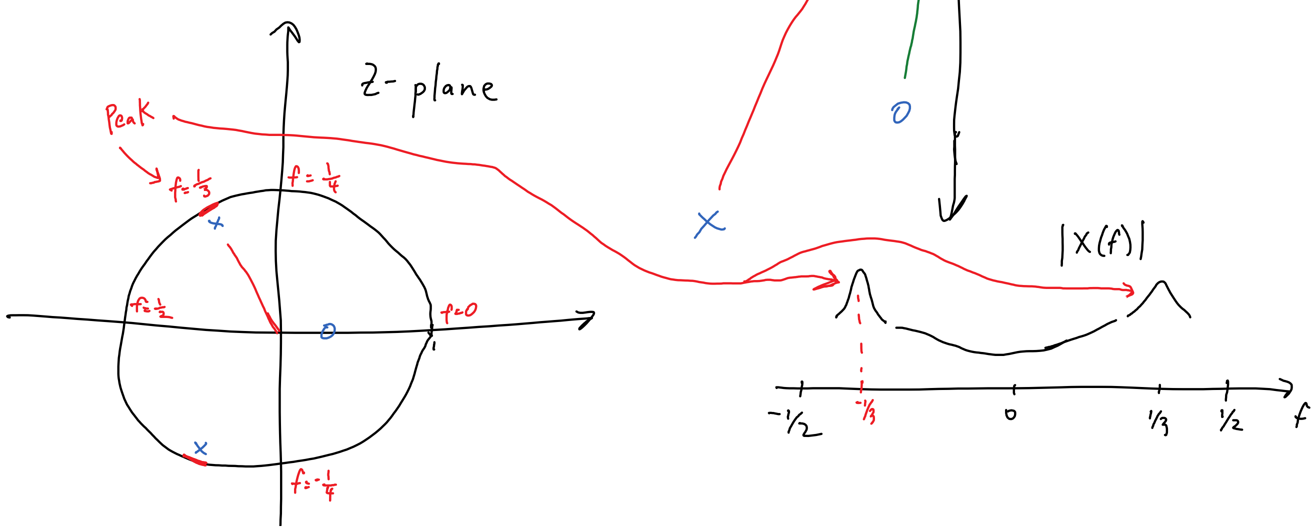
Suppose system is stable.

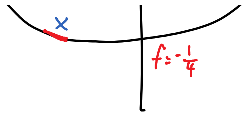
Interpret $X(f)$ from poles and zeros:

See Butterworth filter.

2.) <u>Time domain</u>: Each pole one term.



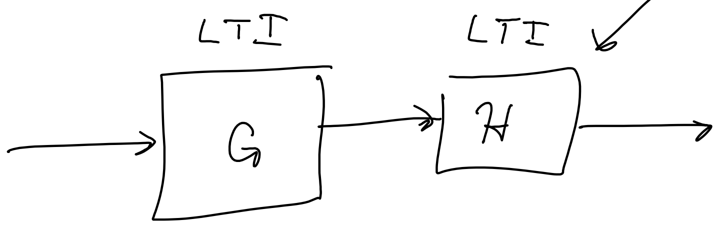




Control :

Unstable

Rocket
Aircraft



Possible for zeros of G to cancel poles of H.

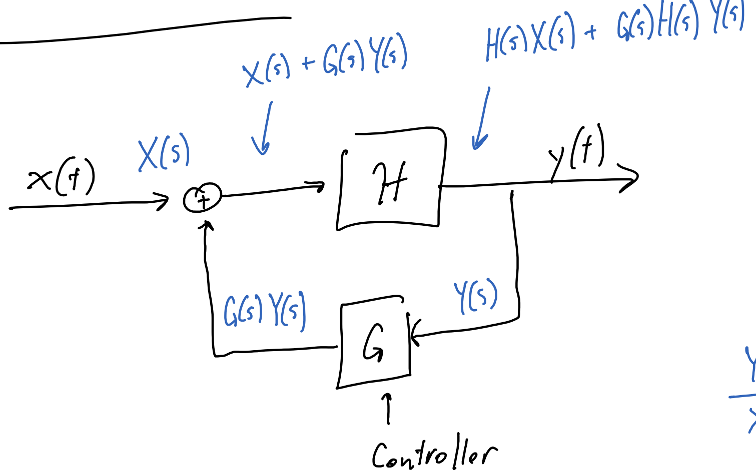
Combine poles and zeros.

Transfer function $\underbrace{G(s)\,H(s)}$ ← Poles and zeros can cancel.

If a pole gets cancelled, ROC expands.

"Open loop control" . Not robust.

Feedback Control ("closed loop")



$$Y(s) = H(s)X(s) + G(s)H(s)Y(s)$$

$$\frac{Y(s)}{X(s)} = \frac{H(s)}{1 - G(s)H(s)}$$

Poles of H(s) cancel.
Stability no longer depends directly on poles of H(s).

This gives robust control.

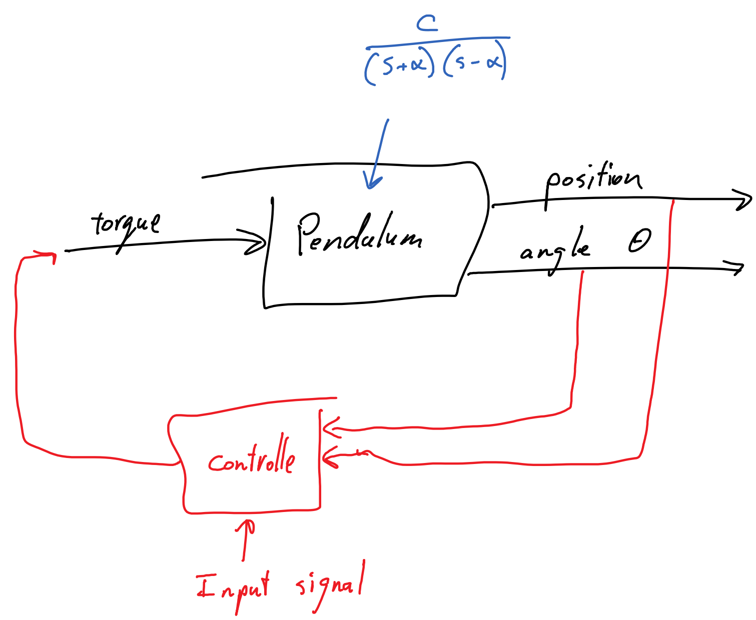
System Inverse :

$x(t) \to \boxed{H} \to \boxed{W} \to x(t)$

In Laplace (z) domain, $W(s) = \frac{1}{H(s)}$ is the system inverse.

$\Rightarrow$ Poles become zeros and zeros become poles.

Inverted Pendulum Demo:



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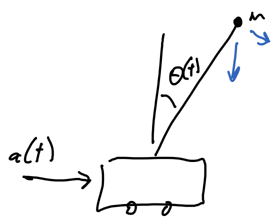

$$g\sin(\theta(t)) - a(t)\cos(\theta(t)) = L\frac{d^2}{dt^2}\theta(t)$$

Linearization:

For small $\theta(t)$: $\sin(\theta(t)) \simeq \theta(t)$

$\cos(\theta(t)) \simeq 1$

$$g\,\theta(t) - a(t) = L\frac{d^2}{dt^2}\theta(t)$$

$$g\,\Theta(s) - A(s) = L s^2\,\Theta(s) \Rightarrow \frac{\Theta(s)}{A(s)} = \frac{1}{g - Ls^2}$$

Y(s)

X(s)

$$= \frac{-\frac{1}{L}}{}$$

$$\left(s+\sqrt{\tfrac{g}{L}}\right)\left(s-\sqrt{\tfrac{g}{L}}\right)$$

Poles: $\pm\sqrt{\frac{g}{L}}$