Lecture 4

Wednesday, February 11, 2015 11:50 AM

Continuous-time

E.g. $x(t)$ is current
$x(t)$ is speed

Discrete-time

Energy: $$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$ $$E = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

Power: $$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$ $$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

If periodic
one period is sufficient: $$P = \frac{1}{T} \int_0^T |x(t)|^2 \, dt$$ $$P = \frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2$$

period   period.

$|x(t)|^2$ is intantaneous power.

Example: $x(t) = \sin(t)$. Energy? ∞

Power? $$P = \frac{1}{2\pi} \int_0^{2\pi} \sin^2(t) \, dt$$
$$= \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{2}\left(1 - \cos(2t)\right) dt = \frac{1}{2}$$

In circuits, RMS voltage for AC $= \frac{V_p}{\sqrt{2}} \Rightarrow V_{rms}^2 = \frac{V_p^2}{2}$

Inner Product:

Let x and y be column vectors:

$$\langle x, y \rangle = x^T y^*$$ ("dot product", Euclidean inner product)

Let x and y be signals:

CT: $$\langle x, y \rangle = \int_{-\infty}^{\infty} x(t) y^*(t) \, dt$$

DT: $$\langle x, y \rangle = \sum_{n=-\infty}^{\infty} x[n] y^*[n]$$

Let x and y be periodic with period T (N for D.T.)

Notice that N must be an integer.

CT: $$\langle x, y \rangle_T = \frac{1}{T} \int_0^T x(t) y^*(t) \, dt$$

$$CT: \quad \langle x, y \rangle_T = \frac{1}{T}\int_0^T x(t)\, y^*(t)\, dt$$

$$DT: \quad \langle x, y \rangle_N = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, y^*[n]$$

Notice that N must be an integer. Is $\sin[n]$ periodic?

Notice that $E = \langle x, x \rangle = \|x\|^2$

$P = \langle x, x \rangle_T = \|x\|^2_{(T)}$

Orthogonal: $\{x_i\}$ orthogonal if $\langle x_i, x_j \rangle = 0 \quad \forall i \neq j$

Orthonormal: Orthogonal and $\langle x_i, x_i \rangle = 1 \quad \forall i$

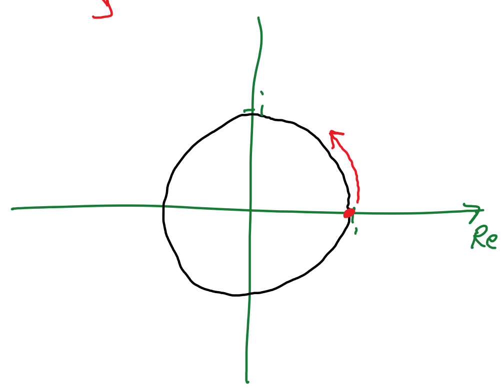
i.e. $\langle x_i, x_j \rangle = \mathbb{1}_{i=j}$

Fourier Series: C.T. $c_K = \frac{1}{T}\int_0^T x(\tau)\, e^{-i 2\pi \frac{K}{T}\tau}\, d\tau$

Assume $x(t)$ has period T.

$$x(t) = \sum_{K=-\infty}^{\infty} c_K\, e^{i 2\pi \frac{K}{T} t}$$

$e^{i 2\pi f t}$



D.T. Assume $x[n]$ has period N.

$$c_K = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-i 2\pi \frac{K}{N} n}$$

$$x[n] = \sum_{K=0}^{N-1} c_K\, e^{i 2\pi \frac{K}{N} n}$$

Complex exponentials

http://demonstrations.wolfram.com/TheComplexExponential/

Transforms:

Taylor Series: (t_0)

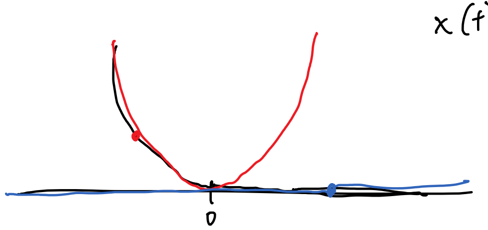
$$x(t) \longrightarrow (a_0\ a_1\ a_2\ \ldots), \text{ and } t_0$$

$$a_k = \frac{1}{k!}\frac{d^k}{dt^k}x(t)\Big|_{t_0}$$

$$x(t) = a_0 + a_1(t-t_0) + a_2(t-t_0)^2 + \ldots$$

Non-analytic: Works for "analytic" function (smooth).

$$x(t) = \begin{cases} t^2, & t \le 0 \\ 0, & t > 0 \end{cases}$$



Transforms may be linear (these ones are).

$$x \to \mathcal{X}, \quad y \to \mathcal{Y}$$

$$\alpha x + y \to \alpha\mathcal{X} + \mathcal{Y} \qquad \forall x, y \text{ and scalars } \alpha.$$