Fourier Series as Freq. Components:

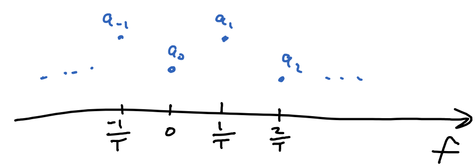
Cont.-time:

Period T

Coef.	Frequency	Component:
$\vdots$		
a_{-2}	$f_{-2} = \frac{-2}{T}$	$a_{-2}\, e^{i2\pi f_{-2} t}$
a_{-1}	$f_{-1} = \frac{-1}{T}$	$a_{-1}\, e^{i2\pi f_{-1} t}$
a_0	$f_0 = 0$	$\vdots$
a_1	$f_1 = \frac{1}{T}$	
a_2	$f_2 = \frac{2}{T}$	
a_3	$f_3 = \frac{3}{T}$	
$\vdots$		

Plot

Two dimension (eg. real, imag mag., phase)



Discrete-time:

Period N

a_k are also periodic:

Aliasing

$$a_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-i2\pi \frac{k}{N} n} = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-i2\pi \frac{k+N}{N} n}$$

$$= a_{k+N}$$

DFT returns $a = (a_0, a_1, \ldots, a_{N-1})$

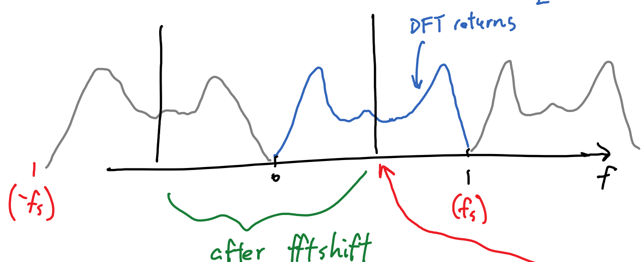
$$\overbrace{[a_0 \ \ldots \ a_{\frac{N}{2}-1} \ \ \underbrace{a_{\frac{N}{2}} \ldots \ a_{N-1}}]}^{a}$$

FFT shift:

$$[a_{\frac{N}{2}} \ \ldots \ a_{N-1} \ a_0 \ \ldots \ a_{\frac{N}{2}-1}]$$

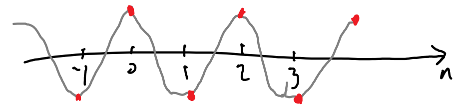
$$= [a_{-\frac{N}{2}} \ \ldots \ a_{-1} \ a_0 \ \ldots \ a_{\frac{N}{2}-1}]$$

Plot:



> **Real Signals:**
>
> $a_k = a_{-k}^*$
>
> $\Rightarrow$ neg. frequencies are redundant

What is the highest DT frequency? $f = \frac{1}{2}$ $\cos(2\pi \frac{1}{2} n) = \cos(\pi n) = (-1)^n$



Real life signals like sound: DT freq. is cycles/sample

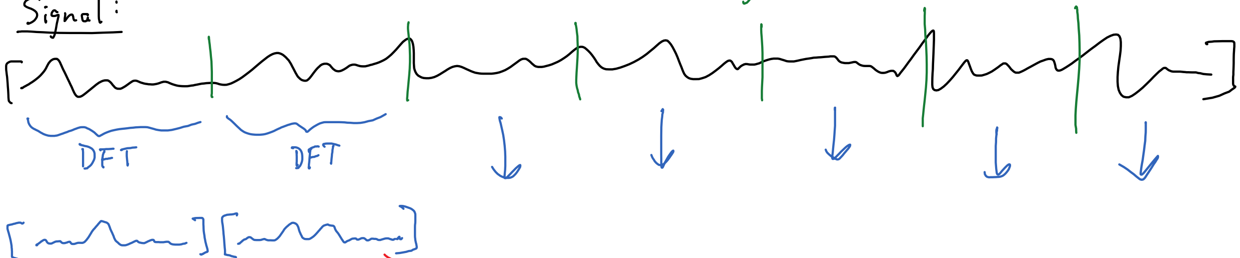
If the DT signal is samples of time, then f_s is samples/second.

$\Rightarrow$ Frequency axis is scaled by f_s

$\underbrace{\text{Short-time } \overbrace{\text{Fourier Transform}}^{\text{DFT}}}$

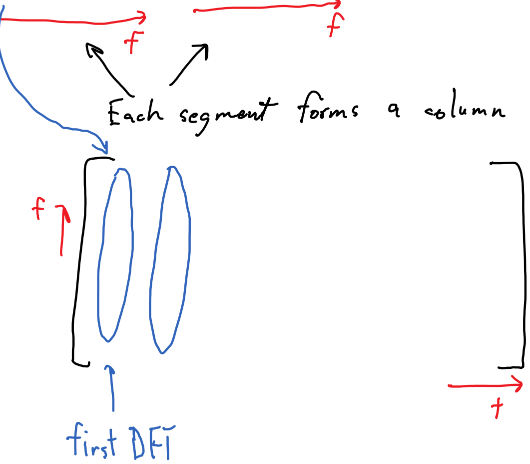
Divide into short segments

Signal:



Spectrogram:

Each segment forms a column



Plot log-magnitude

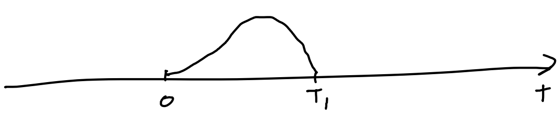
Fourier Transform:

So far: CT Fourier Series

Periodic → DT Fourier Series $\xleftarrow{\text{equivalent}}$ DFT

Spectrogram

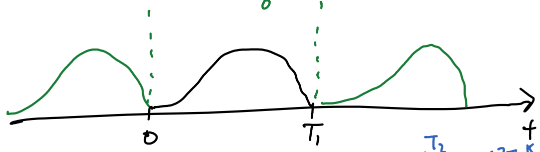
Fourier Transform from Fourier Series:



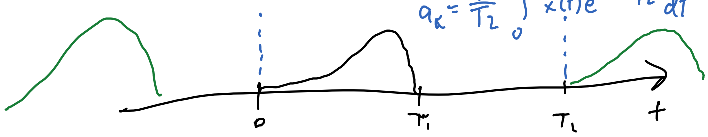
Fourier?

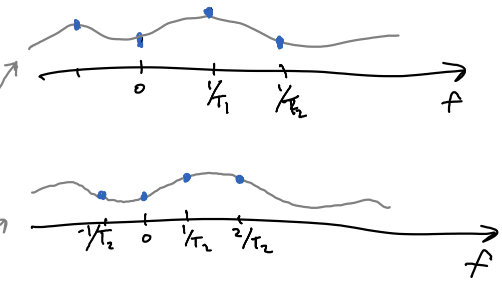
$a_k = \frac{1}{T_1}\int_0^{T_1} x(t)\, e^{-i2\pi \frac{k}{T_1} t}\, dt$

Try series



$a_k = \frac{1}{T_2}\int_0^{T_2} x(t)\, e^{-i2\pi \frac{k}{T_2} t}\, dt$





Let $T_2 \to \infty$

Fouries Transform formula:

CT: $$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-i2\pi f t}\, dt$$

$$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{i2\pi f t}\, df$$