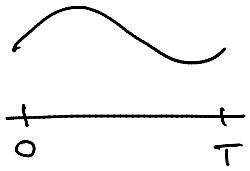
Wednesday, October 19, 2011
3:08 PM

- Mini (Re/Over) View (CTFS DTFS CTFT DTFT)
- Where does the DFT come from?
- Sampling
- Aliasing

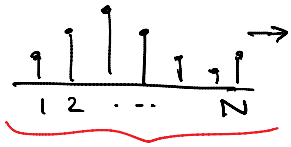
Pretend you don't know about the DFT

"real time domain"



→ $\boxed{\text{wavread}}$ →

"matlab time domain"



→ $\boxed{\text{Something Fourieresque}}$ →

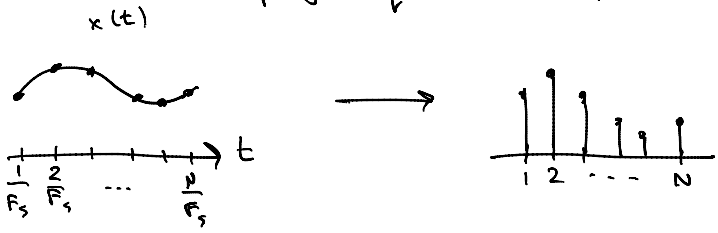
"matlab frequency domain"

① how does this relate to "real time domain"?

② how do we define this?

how does this relate to "real frequency domain"?

① Need to know sampling freq F_s: $F_s = \frac{N}{T}$ $\frac{\text{samples}}{\text{sec}}$

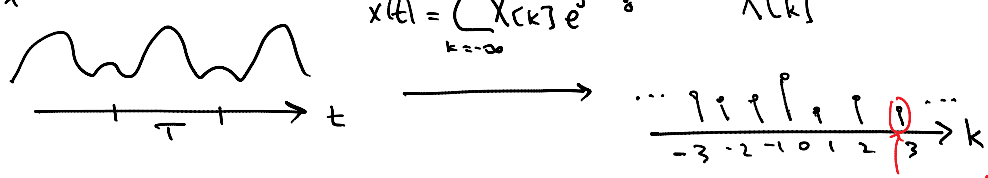


② We need a way to take the Fourier transform of a finite vector and end up with a finite vector

CTFS

$x(t)$ $\quad f_0 = \frac{1}{T}$

$$x(t) = \sum_{k=-\infty}^{\infty} X[k] e^{j2\pi f_0 k t} \qquad X[k]$$



corresponds to $e^{j2\pi f_0 3 t}$ term $\Rightarrow$ freq $3f_0$

sum of sinusoids at frequency:

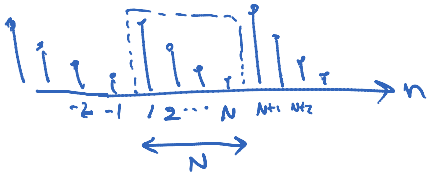
freqs: $f_0 k$, $k = -\infty, \ldots, \infty$

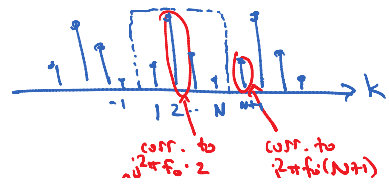
units: $\frac{\text{cycles}}{\text{sec}}$

DTFS

$x[n]$ $\quad$ period N $\quad f_0 = \frac{1}{N}$

$$x[n] = \sum_{k=1}^{N} X[k] e^{j2\pi f_0 k n} \qquad X[k]$$



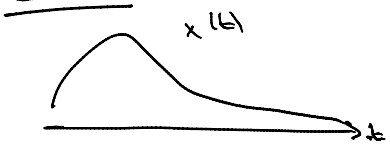


corr. to $e^{j2\pi f_0 \cdot 2}$ $\quad$ corr. to $e^{j2\pi f_0 (N+1)}$ $\quad$ $e^{j2\pi f_0 \cdot 1}$

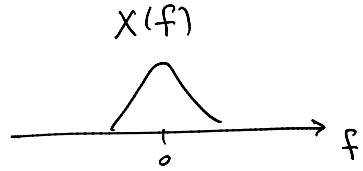
freqs: $f_0 k$, $k = 1, \ldots, N$

units: $\frac{\text{cycles}}{\text{sample}}$

CTFT

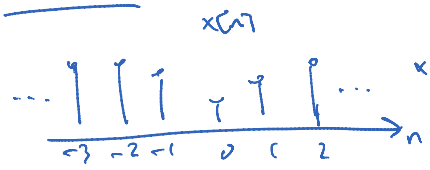


$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi f t} df \longrightarrow$$

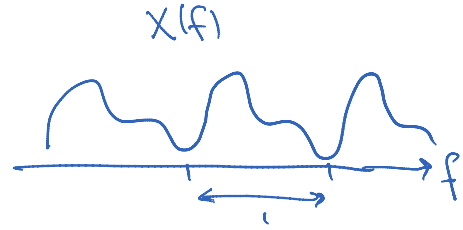


freq: all
units: cycles/sec

DTFT



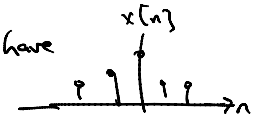
$$x[n] = \int_0^1 X(f) e^{j2\pi f n} \longrightarrow$$



freq: all
units: cycles/sample

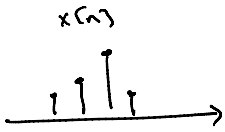
Possible candidates for modification: DTFS and DTFT

To use DTFS, we need periodic signal, but we just have



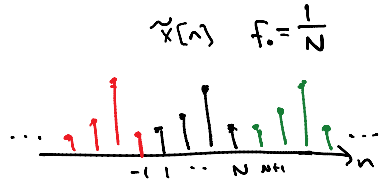
periodic extension:

matlab vector

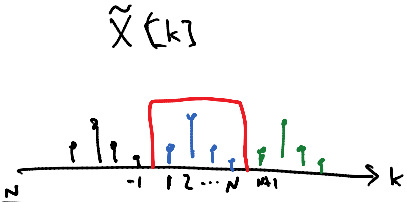


$\longrightarrow$

periodic extension



$\xrightarrow{\text{DTFS}}$



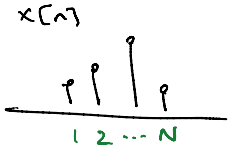
$$\tilde{X}[k] = \frac{1}{N}\sum_{n=1}^{N} x[n] e^{-j2\pi f_0 k n}, \quad k = -\infty, \ldots, \infty$$

So we might define DFT to be $\underline{\underline{x[k] = \frac{1}{N}\sum_{n=1}^{N} x[n] e^{-j2\pi \frac{1}{N} k n}}}$, $k = 1, \ldots, N$ $\leftarrow$ vector of length n.

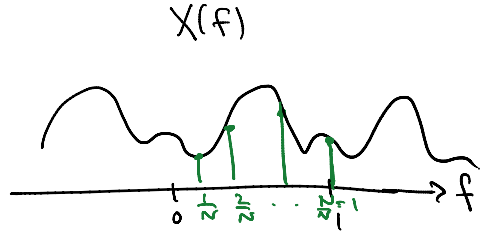
To use DTFT:

We don't need periodic extension, but the spectrum is continuous so we need to sample it

matlab vector



$\xrightarrow{\text{DTFT}}$



So we might define DFT to be

$$x[k] = X\left(\frac{k}{N}\right), \quad k = 1, \ldots, N$$

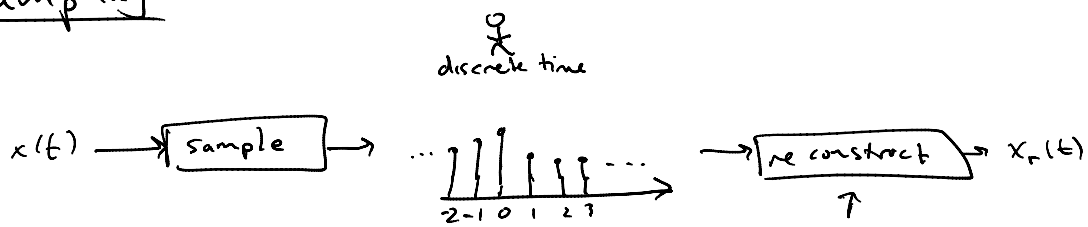
$$= \sum_{n=-\infty}^{\infty} x[n] e^{-j2\pi\left(\frac{k}{N}\right)n}$$

$$= \sum_{n=1}^{N} x[n] e^{-j2\pi \frac{k}{N} n}$$

(remember $X(f) = \sum_{n=-\infty}^{\infty} x[n] e^{-j2\pi f n}$)

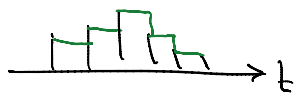
Exactly the same as when we used periodic extension + DTFS !!! (without $\frac{1}{N}$ factor, but who cares)

Sampling



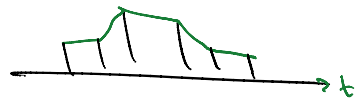
how do we do this so that $x(t) = x_r(t)$?

one attempt



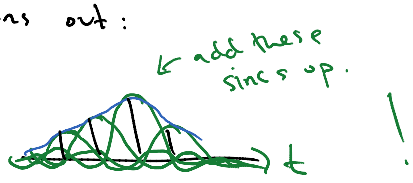
FAIL

another attempt:



FAIL

Turns out:

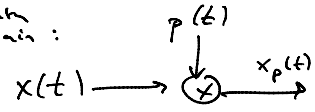


!

Makes alot more sense when we look at it in the freq. domain.

Sampling period T. Sampling Frequency $F_s = \frac{1}{T}$.

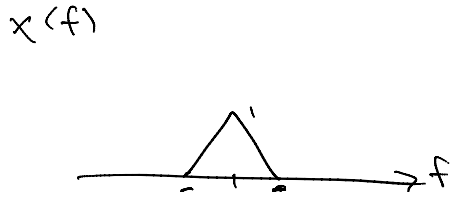
sample with impulse train:

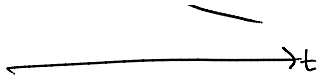


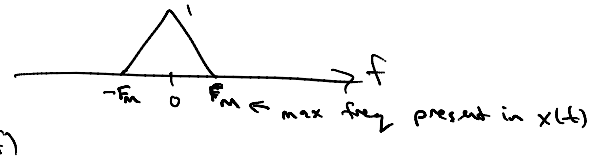
We don't actually sample this way, but let's go with it for now

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) \quad \xleftrightarrow{FT} \quad P(f) = F_s \sum_{k=-\infty}^{\infty} \delta(f - kF_s)$$

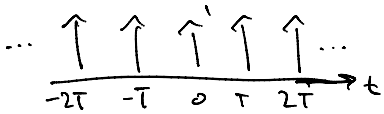
$$\begin{aligned} x_p(t) &= x(t) \cdot p(t) \\ &= x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT) \\ &= \sum_{n=-\infty}^{\infty} x(nT)\,\delta(t - nT) \end{aligned} \quad \xleftrightarrow{FT} \quad \begin{aligned} X_p(f) &= X(f) * P(f) \\ &= X(f) * F_s \sum_{k=-\infty}^{\infty} \delta(f - kF_s) \\ &= F_s \sum_{k=-\infty}^{\infty} X(f - kF_s) \end{aligned}$$



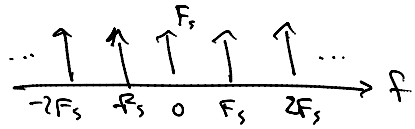




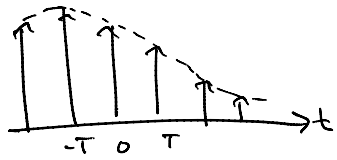
$p(t)$



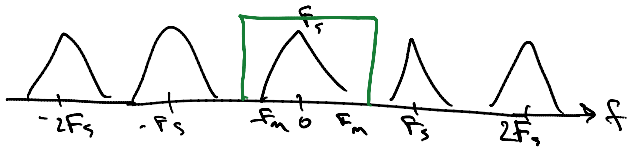
$P(f)$



$x_p(t)$

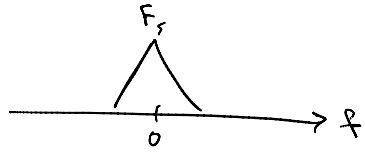


$X_p(f)$



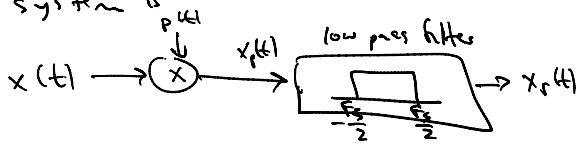
So how do we reconstruct $X(f)$?

LPF



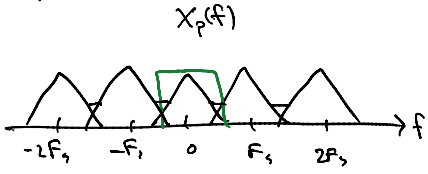
This is exactly $X(f)$!!!

So system is



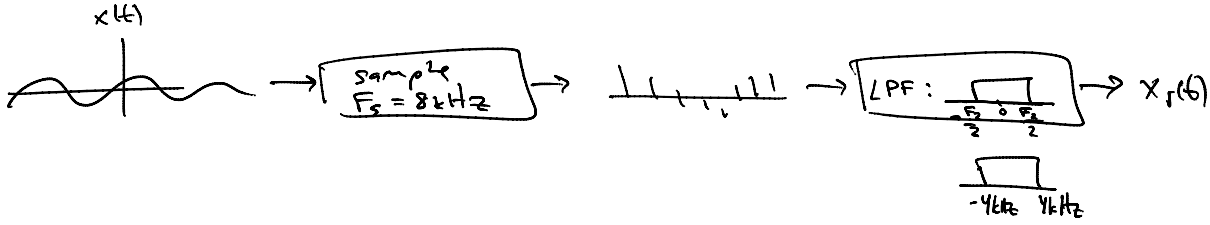
When does this work? (when does $x_r(t) = x(t)$?)

Problem if:



need $F_s > 2F_m$ (Nyquist rate!)

Aliasing Example

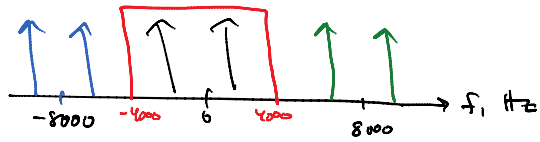


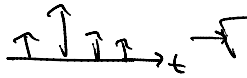
If $x(t) = \sin(2\pi \cdot 3000 \cdot t)$, $x_r(t) = \sin(2\pi \cdot 3000 t)$

If $x(t) = \sin(2\pi \cdot 7000 t)$, $x_r(t) = \sin(2\pi \cdot 1000 t)$

(mathematica demonstration)

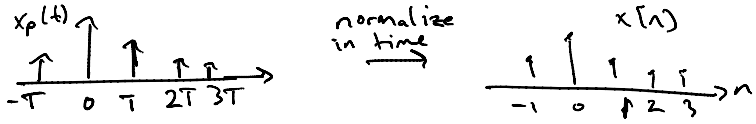
$X_p(f)$



So far, $x(t) \longrightarrow \overset{p(t)}{\otimes} \longrightarrow$  $\longrightarrow$ LPF $\longrightarrow x_r(t)$

But we want to work with discrete time

$x_s[n] = x(nT)$

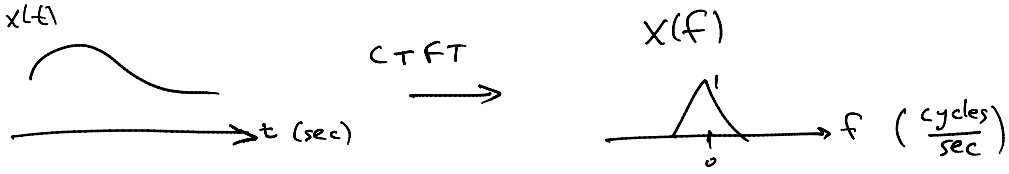


How does DTFT of $x_s[n]$ compare to CTFT of $x_p(t)$?

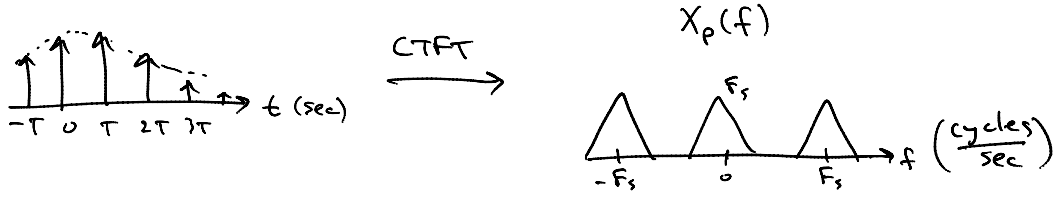
DTFT of $x_s[n]$: $X_s(f) = \sum_{n=-\infty}^{\infty} x_s[n] e^{-j2\pi fn} = \sum_{n=-\infty}^{\infty} x(nT) e^{-j2\pi fn}$

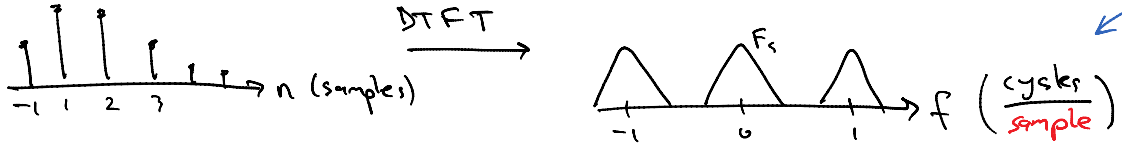
CTFT of $x_p(t)$: $X_p(f) = \sum_{n=-\infty}^{\infty} x(nT) e^{-j2\pi fnT}$

$$\Rightarrow X_s(f) = X_p\left(\frac{f}{T}\right) = X_p(f \cdot F_s)$$



"real frequency domain"





"matlab frequency domain"

$\left(\frac{\text{cycles}}{\text{sec}} \cdot \frac{1}{\text{samples}/\text{sec}}\right)$