Feedback and Control

Physical System:

Inputs: Thrusters
Output: Location, speed, direction.

Desired Control System
Input: Path
Build this.

Physical System

Input: Voltages to Motors
Outputs: Direction

Desired System (Automatic Tracking)
Input: Celestial Object to track
Don't want in to shake much

Op-amp

Output characteristics are not ideal.
- Non-linear
- Poor spectral properties

Desired System
- Linearity and flat frequency response
  (i.e. quick response time)
History: Positive Feedback
- Oscillator (Signal generator)
- HP Audio Oscillator

Negative Feedback (Black, 1927, Bell Labs)
- Stability
- Less sensitive to the characteristic of the system.
- Better response time
- Linearity
- (Less gain)

Last example: Inverted Pendulum

(Negative) Feedback:

Original System

Our design

Negative Feedback: y(t)

Input is acceleration

θ is output (Also, position)
Negative Feedback

Let \( Q(s) \) be the transfer function (Laplace or \( z \)-transform) of the "closed-loop" feedback system above.

**Black's Formula:**

\[
Y(s) = (X(s) - Y(s)G(s))H(s)
\]

\[
\Rightarrow Y(s)(1 + H(s)G(s)) = X(s)H(s)
\]

\[
\Rightarrow Q(s) = \frac{X(s)}{Y(s)} = \frac{H(s)}{1 + H(s)G(s)}
\]

Assume Causal.

Notice a couple of things:

1. If \( |H(s)G(s)| \) is very large (i.e. \( |H(s)G(s)| \gg 1 \))

   then \( Q(s) \approx \frac{1}{G(s)} \)

   This is the op-amp configuration, at least within the operational frequency range.
Assume \( H(s) \) is very large

\[
Q(s) = \frac{R_1 \cdot R_2}{R_2}
\]

Flat frequency response, but in reality, the \( |H(s) G(s)| \gg 1 \) assumption won't hold for all \( s \).

2.) If \( |H(s) G(s)| \ll 1 \) then \( Q(s) \approx H(s) \)

(Its as if you have no feedback)

Look at stability of \( Q(s) \) (i.e. poles)

\[
\frac{H(s)}{1 + H(s) G(s)} = \frac{(s-z_{Hi})(s-z_{H2})(s-p_{Hi})(s-p_{H2})}{(s-p_{Hi})(s-p_{H2})(s-p_{G1})(s-p_{G2})} + \frac{(s-\bar{z}_{Hi})(s-\bar{z}_{H2})(s-\bar{z}_{G1})(s-\bar{z}_{G2})}{(s-p_{Hi})(s-p_{H2})(s-p_{G1})(s-p_{G2})}
\]

I've assumed two poles and zeros in each system, arbitrarily.

Notice that the poles depend only on closed-loop gain, \( H(s) G(s) \).

To illustrate this further,
The closed-loop system stability only depends on the product $H_1(s) H_2(s) H_3(s) H_4(s) H_5(s)$.

However, the actual transfer function depends on the products $H_2(s) H_3(s)$ and $H_1(s) H_5(s) H_4(s)$ separately.

**First-order system**
(1 pole)

Suppose $H(s) = \frac{1}{s-p}$

**Step response:**

Consider:

$$Q(s) = \frac{H(s)}{1 + K H(s)} = \frac{1}{K + (s-p)} = \frac{1}{s - (p-K)}$$

**Faster response**
Two poles:

\[ H(s) = \frac{1}{(s-p_1)(s-p_2)} \]

\[ Q(s) = \frac{1}{1 + \frac{K}{(s-p_1)(s-p_2)}} \]

\[ = \frac{1}{(s-p_1)(s-p_2) + K} \]

Root-Locus Method:

For general rational \( H(s) \) and \( G(s) \).

Poles of \( Q(s) \) are roots of \( (KN(s) + D(s)) \),
where \( H(s)G(s) = \frac{N(s)}{D(s)} \).

\[ \Rightarrow \quad KN(s) + D(s) = 0 \]
\[ \Rightarrow \quad \frac{N(s)}{D(s)} + \frac{1}{K} = 0 \]
\[ \frac{N(s)}{D(s)} \text{ is real} \]

Poles of $H(s)G(s)$

(in this case, let's assume no zeros)

Points that evaluate to real numbers.