Lecture 20

Tuesday, December 06, 2011
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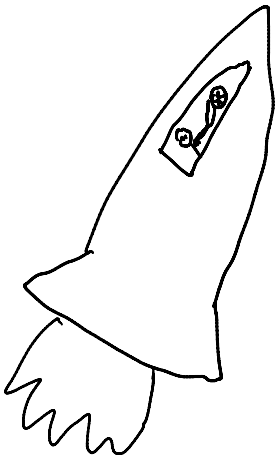
Feedback and Control



Physical System:

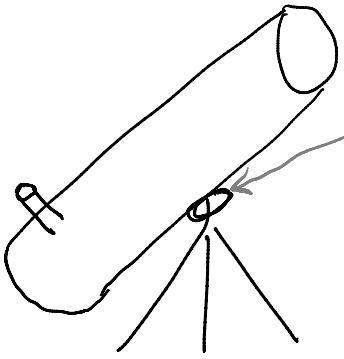
Inputs : Thrusters
Output : Location, speed, direction.

Very unstable
Can't be controlled well manually.

Desired Control System

Input : Path

Build this.



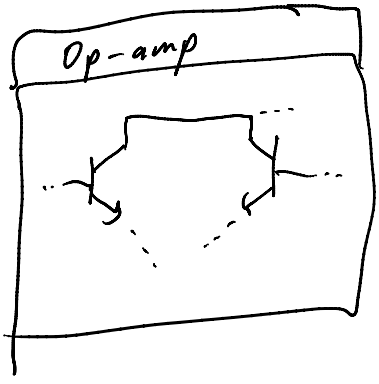
Physical System

Input : Voltages to Motors
Outputs : Direction

Desired System (Automatic Tracking)

Input : Celestial Object to track

Don't want in to shake much



Output characteristics are not ideal.

- Non-linear
- Poor spectral properties

Desired System

- Linearity and flat frequency response (i.e. quick response time)

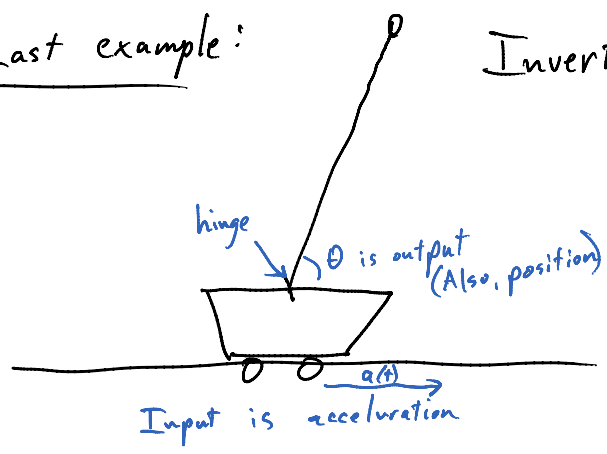
- Cost: Lower gain.

Black (1927, Bell Labs)

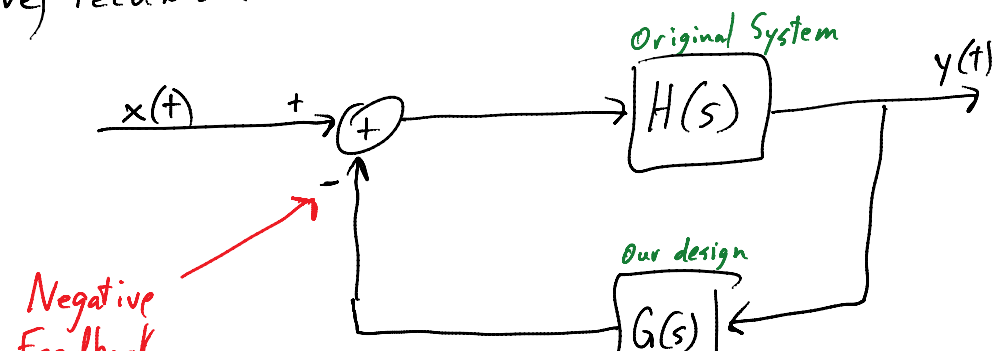
History: Positive Feedback
- Oscillator (Signal generator)
- HP Audio Oscillator http://www.ohio.edu/people/postr/bapix/HP200C.htm

Negative Feedback (Black, 1927, Bell Labs)
- Stability
- Less sensitive to the characteristic of the physics system.
- Better response time
- Linearity
- (Less gain)

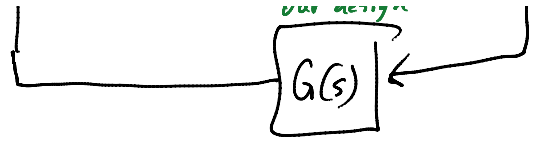
Last example: Inverted Pendulum



(Negative) Feedback:

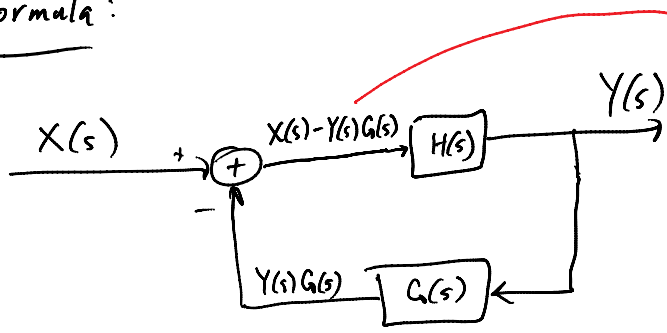


Negative Feedback



Let $Q(s)$ be the transfer function (Laplace or Z-transform) of the "closed-loop" feedback system above.

Black's Formula:



$$Y(s) = (X(s) - Y(s)G(s))\,H(s)$$

$$\Rightarrow \quad Y(s)\left(1 + H(s)\,G(s)\right) = X(s)\,H(s)$$

$$\Rightarrow \quad Q(s) = \frac{Y(s)}{X(s)} = \frac{H(s)}{1 + H(s)\,G(s)}$$

Assume Causal.

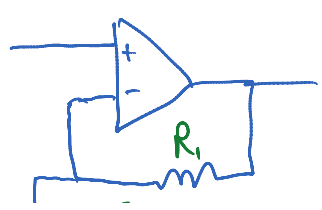
Notice a couple of things:

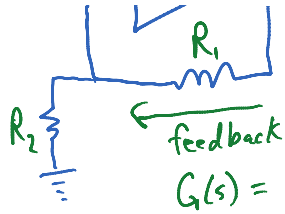
(this is common)

1.) If $|H(s)\,G(s)|$ is very large (i.e. $|H(s)\,G(s) \gg 1|$)

then $Q(s) \approx \dfrac{1}{G(s)}$

This is the op-amp configuration, at least within the operational frequency range.





feedback loop

$G(s) = \frac{R_2}{R_1+R_2}$

Assume $H(s)$ is very large

$$Q(s) \simeq \frac{R_1+R_2}{R_2}$$

flat frequency response, but in really, the $|H(s)G(s)| >> 1$ assumption won't hold for all s.

2.) If $|H(s)G(s)| << 1$ then $Q(s) \simeq H(s)$

(It's as if you have no feedback)

Look at stability of $Q(s)$ (i.e. poles)

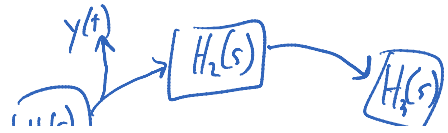
$$\frac{H(s)}{1+H(s)G(s)} = \frac{\frac{(s-z_{H1})(s-z_{H2})}{(s-p_{H1})(s-p_{H2})}}{1+\frac{(s-z_{H1})(s-z_{H2})(s-z_{G1})(s-z_{G2})}{(s-p_{H1})(s-p_{H2})(s-p_{G1})(s-p_{G2})}}$$

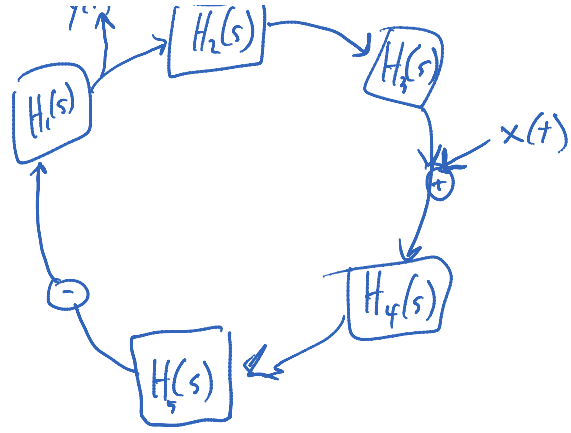
$$= \frac{(s-z_{H1})(s-z_{H2})(s-p_{G1})(s-p_{G2})}{(s-p_{H1})(s-p_{H2})(s-p_{G1})(s-p_{G2}) + (s-z_{H1})(s-z_{H2})(s-z_{G1})(s-z_{G2})}$$

I've assumed two poles and zeros in each system, arbitrarily.

Notice that the poles depend only on closed-loop gain, $H(s)G(s)$.

To illustrate this further.



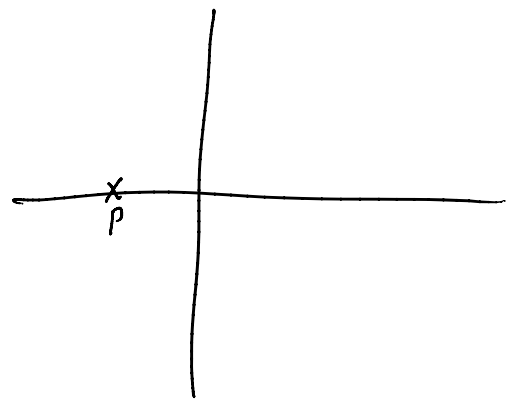


The closed-loop system stability only depends on the product $H_1(s)H_2(s)H_3(s)H_4(s)H_5(s)$.

However, the actual transfer function depends on the products $H_2(s)H_3(s)$ and $H_1(s)H_5(s)H_4(s)$ seperately.

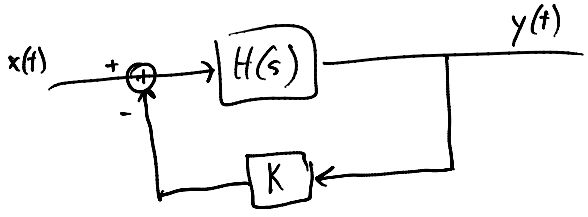
First-order system
(1 pole)

Suppose $H(s) = \dfrac{1}{s-p}$



step response:

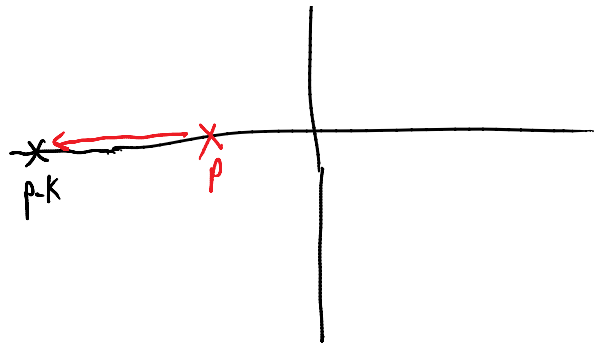
Consider:



$$Q(s) = \frac{H(s)}{1 + K\,H(s)} = \frac{1}{K + (s-p)}$$

$$= \frac{1}{s - (p-K)}$$

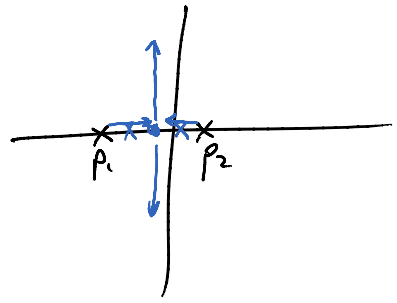
Faster response

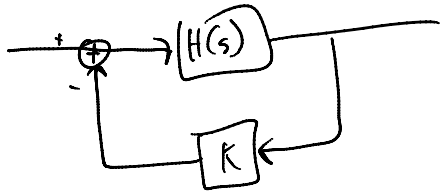


- Faster response (or stability)
- $H(0) \approx \frac{1}{K}$ if K is large. (p becomes irrelavent)

Two-poles:

$$H(s) = \frac{1}{(s-p_1)(s-p_2)}$$



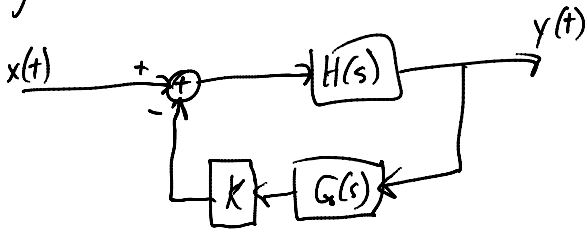


$$Q(s) = \frac{\frac{1}{(s-p_1)(s-p_2)}}{1 + \frac{K}{(s-p_1)(s-p_2)}}$$

$$= \frac{1}{(s-p_1)(s-p_2) + K}$$

Root-Locus Method:

For general rational $H(s)$ and $G(s)$.



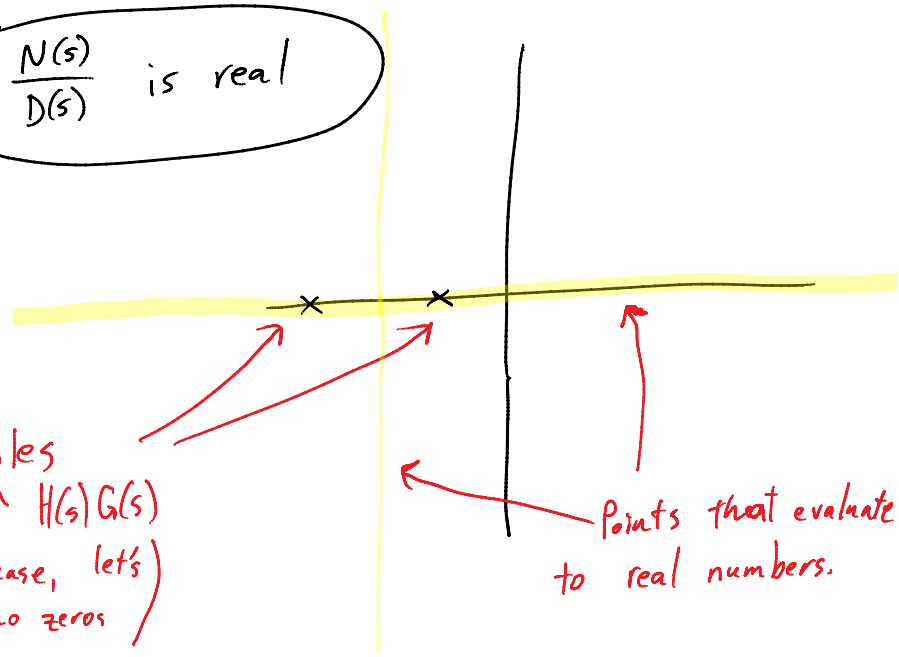
Poles of $Q(s)$ are roots of $(KN(s) + D(s))$,

where $H(s)G(s) = \frac{N(s)}{D(s)}$.

$$\Rightarrow \quad K\,N(s) + D(s) = 0$$

$$\Rightarrow \quad \frac{N(s)}{D(s)} + \frac{1}{K} = 0$$

$\Rightarrow$ $\boxed{\dfrac{N(s)}{D(s)} \text{ is real}}$



Poles of $H(s)G(s)$

(In this case, let's assume no zeros)

Points that evaluate to real numbers.