Lecture 6
ELE 301: Signals and Systems

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Outline

- LTI System Response
- Filtering
Transfer Function

- Response to LTI system $h$.

  Continuous time: $e^{st} \rightarrow^h H_c(s)e^{st}$,
  Discrete time: $z^n \rightarrow^h H_d(z)z^n$.

- We are interested in the cases $s = i2\pi f$ and $z = e^{i2\pi f}$.

  Continuous time: $y(t) = \sum_{k=-\infty}^{\infty} a_k H_c(i2\pi f_0 k) e^{i2\pi f_0 k t}$,
  Discrete time: $y[n] = \sum_{k=-\infty}^{\infty} a_k H_d(e^{i2\pi f_0 k}) e^{i2\pi f_0 kn}$.

  where $a_k$ are the Fourier Series coefficients of the input with period $T = 1/f_0$.

Intuitive Visualization

Note: Plots aren’t technically accurate because complex numbers are not one-dimensional.
Intuitive Visualization

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Filtering Example

\[ h(t) = e^{-t}u(t), \]
\[ H(i2\pi f) = ?. \]
First-order low-pass filter

\[ H(i2\pi f) = \frac{1}{1 + i2\pi f}. \]

Filtering example - running average

\[ h[n] = \frac{1}{3}(\delta[n] + \delta[n - 1] + \delta[n - 2]), \]

\[ H(e^{i2\pi f}) = ?. \]
Filtering example - Differentiator

What is the impulse response of a differentiator?

\[ h(t) = ? \]
Another invented pseudo-function

Conceptually the derivative of the Dirac delta function

Properties

\[ \delta' \ast f = f' \]
\[ f(t)\delta'(t - t_0) = -f'(t_0)\delta(t - t_0) \]
\[ \delta'(-t) = -\delta'(t) \]

Differentiator

\[ h(t) = \delta'(t), \]
\[ H(i2\pi f) = ?. \]
High-pass filter (Differentiator)

\[ H(i2\pi f) = i2\pi f. \]

Filtering example - discrete difference

\[ h[n] = \frac{1}{2}(\delta[n] - \delta[n - 1]), \]

\[ H(e^{i2\pi f}) = ?. \]
Discrete Difference

\[ H(i2\pi f) = e^{-i\pi f} \sin(\pi f). \]