Embedded Coordination in Signals

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A Cooperative Game

- Players 1 and 2 simultaneously guess uniform random bits
- One point if they both guess correctly
- Player 1 will be given all the answers ahead of time
  - "Penny Matching" [Gossner et. al. 03]

Coordination

Design n, f, and g.

Empirical coordination:

$P_S P_{Y,Z|S}$ is achievable if

empirical distribution ⟶ $P_{SYZ} \approx P_S P_{Y,Z|S}$

with high probability

Coordination = Distortion

- Distortion:
  - Design system to minimize average distortion $d(s_i,y_i)$ or $d(s_i,z_i)$
    - Maximize score in a game: $\pi(s_i,y_i,z_i)$
  - The coordination capacity region completely characterizes distortion
    - Achievable distortion is $E_P\, d(S;Z)$

[C.-Permuter-Cover 10]

Results

- Non-causal encoder and decoder

$$I(S;Y,Z) \leq H(Y)$$

Converse

- $S^n - Y^n - Z^n$
- $I(S^n; Y^n, Z^n) \leq H(Y^n)$
- $I(S_T; Y_T, Z_T) \leq H(Y_T)$
  where T is a random time index

Results

- Strictly causal decoder: (penny game)

$$I(Y;Z) + I(S;Y,Z) \leq H(Y)$$

[Gossner et. al.] [Larrousse-Lacaulse-Bloch]

Results

- Causal decoder:

$$S-(Y,V)-Z$$

$$I(Y;V) + I(V;Y,Z) \leq H(Y)$$

Results

- Strictly causal encoder:

$$S \perp Y$$

$$I(S;Y,Z) \leq H(Y)$$

Results

- Causal encoder:

$$S \perp U$$
$$H(Y|S,U) = 0$$
$$I(S;Y,Z|U) \leq H(Y)$$

Results

- Both causal:

$$S-Y-Z$$

Applications of Concept

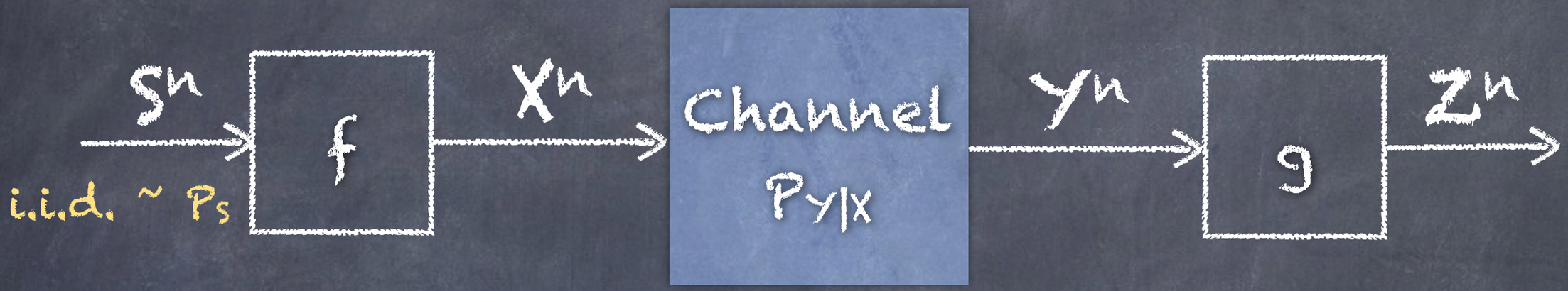
- Cribbing encoders
  [Willems, Van der Meulen, Permuter, Asnani, Weissman, Lapidoth, Bross, Bracher, Steinberg...]
- Relay channel (hybrid codes)
  [Minero, Lim, Kim]
- State Amplification
  [Kim, Sutivong, Cover, Choudhuri, Mitra, Tian, Merhav, Shamai, Chiang...]
- Information Embedding and Control
  [Witsenhausen, Grover, Sahai, Park, Wagner...]
- Delay constrained source coding
- Joint source-channel coding

Cooperating Encoders

Cribbing Encoders

Hybrid Codes

Noisy transmission

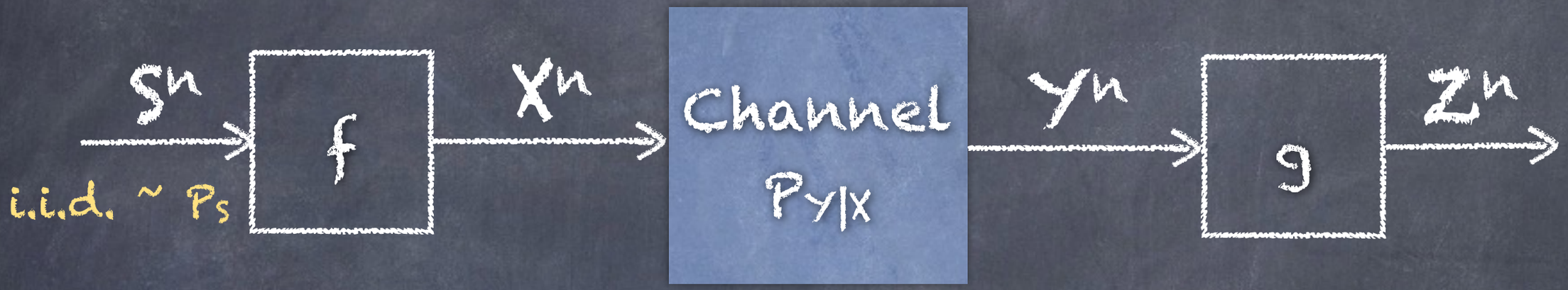


This is a source-channel coding problem.

Coordination $P_S P_{X,Y,Z|S}$

(We care to make the channel input be correlated with the source)

Noisy transmission

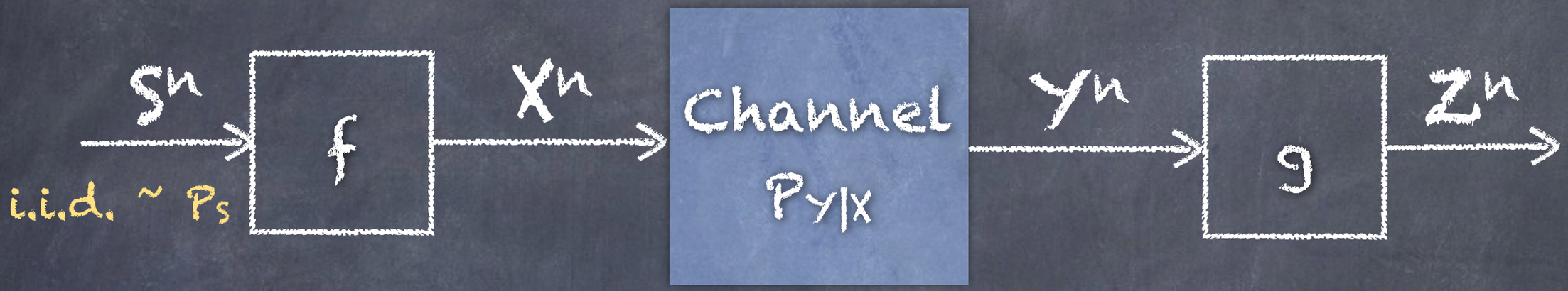


- Non-causal encoder and decoder:

$$(S,U)-X-Y$$
$$(S,X)-(U,Y)-Z$$
$$I(S;U) \leq I(Y;U)$$

- Use hybrid codes
- May not be optimal

Noisy transmission



- Strictly causal encoder:

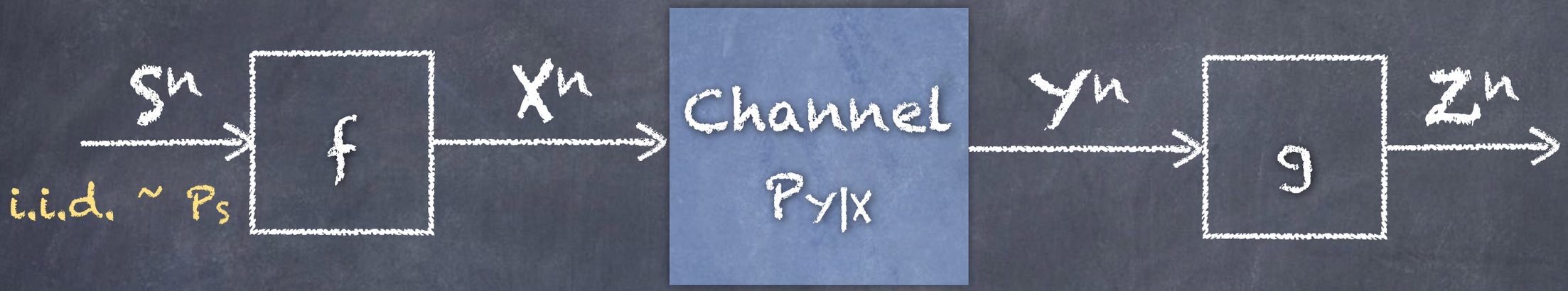
$$S \perp X$$

$$(S,V)-X-Y$$

$$S-(V,X,Y)-Z$$

$$I(S;X,V) \leq I(Y;X,V)$$

Noisy transmission



- Causal encoder:

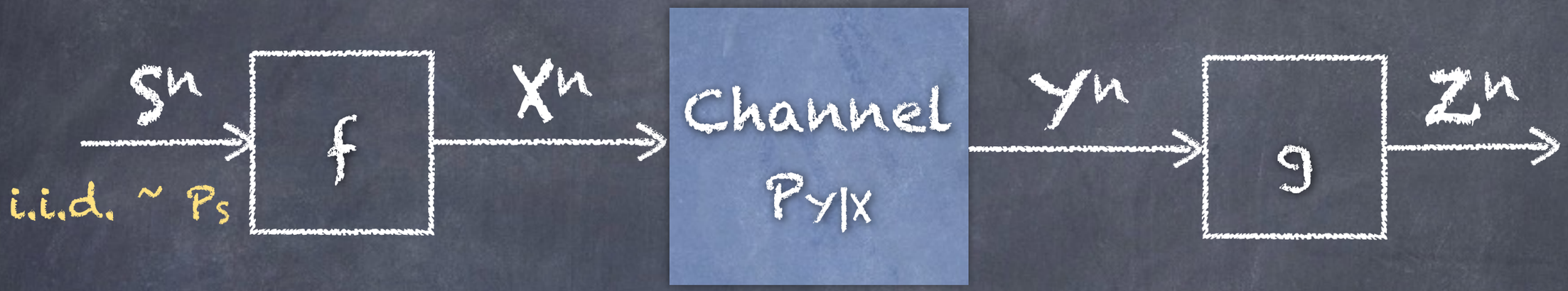
$$S \perp U$$

$$V-(S,U)-X-Y$$

$$(S,X)-(U,V,Y)-Z$$

$$I(S;U,V) \leq I(Y;U,V)$$

Noisy transmission

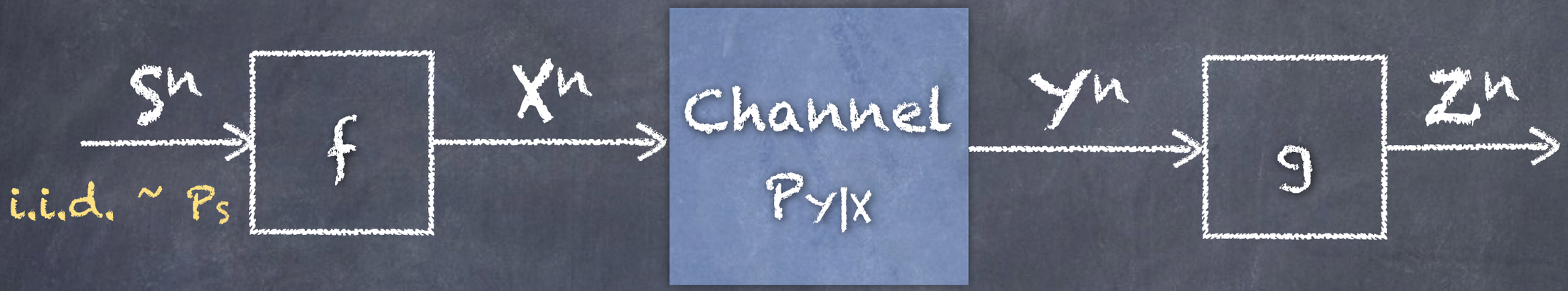


- Strictly causal decoder [Truest]:

$$(S,U,Z)-X-Y$$

$$I(Y;Z) + I(S;U,Z) \leq I(Y;U,Z)$$

Noisy transmission



- Causal decoder [Truest]:

$$(S,U,V)-X-Y$$
$$(S,U,X)-(V,Y)-Z$$
$$I(Y;V) + I(S;U,V) \leq I(Y;U,V)$$

Special case

- Systematic transmission
- Demand that Z=X
  (i.e. transmission is the compressed signal)
- Result: $I(S;X) \leq I(X;Y)$ is necessary and sufficient
  - Gaussian w. squared error: no penalty
  - Binary w. hamming distortion: penalty