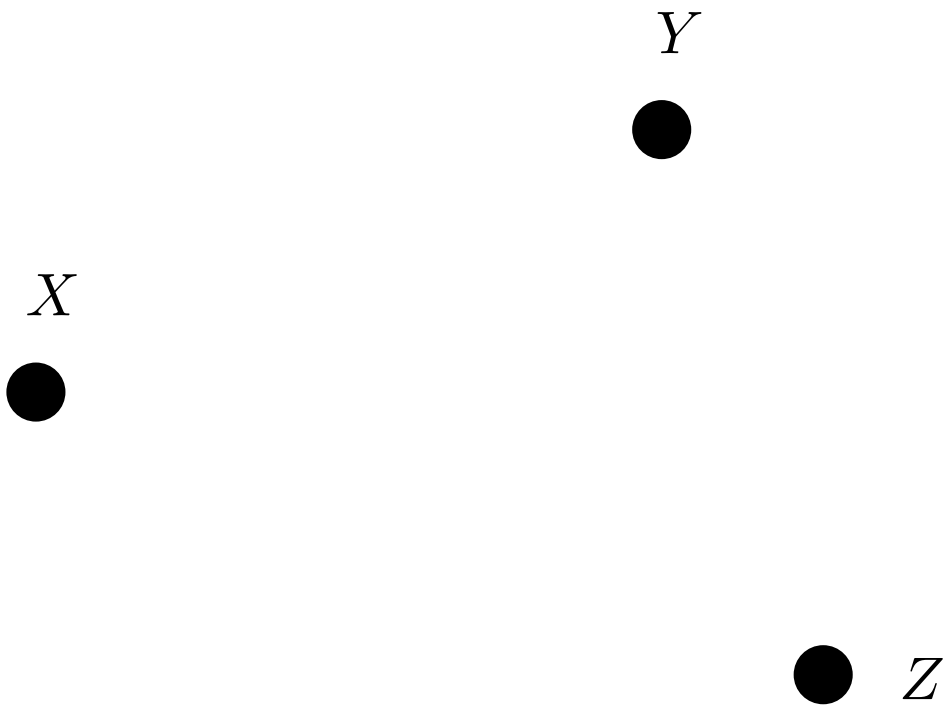
Coordination via Communication

Paul Cuff
(Tom Cover and Haim Permuter)

Stanford University

School of Information Theory
June 5, 2008

Coordination Defined by a Joint Distribution



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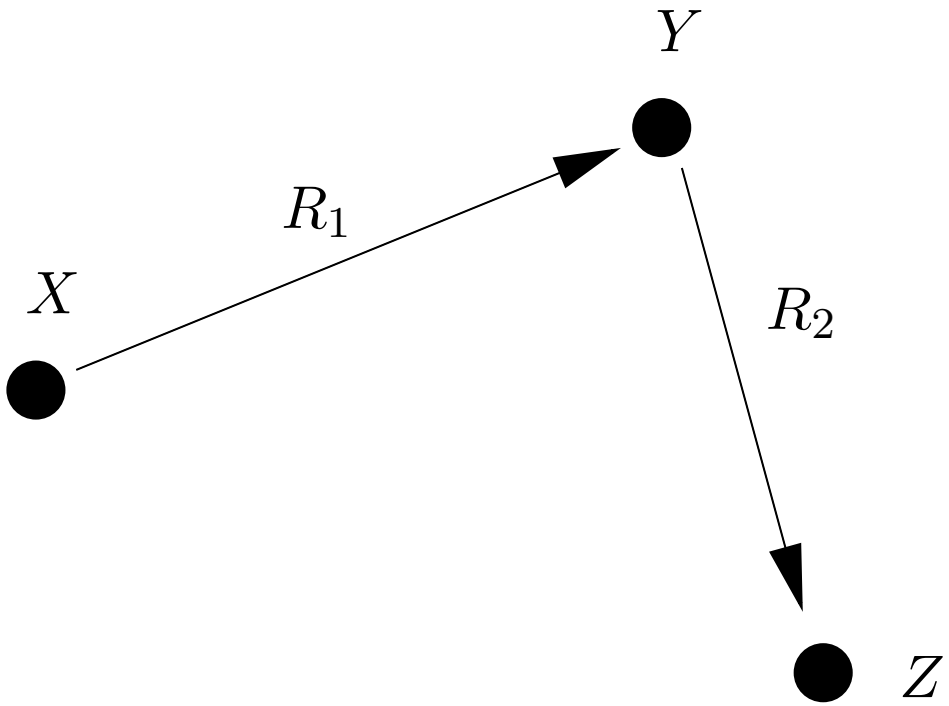


$$\{p(x, y, z)\}$$

Two Definitions of Coordination

A distribution $\hat{p}(x,y)$ is achievable if for $\forall \epsilon > 0$, $\exists n$ such that,

Jointly Typical Objective

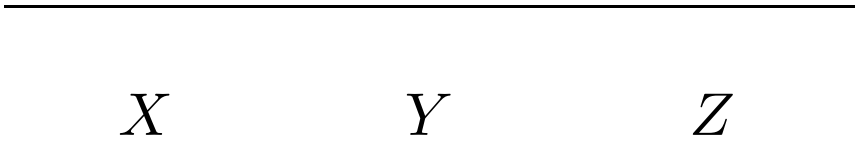
$$E\left|\left|P_{X^n,Y^n}(x,y) - \hat{p}(x,y)\right|\right|_{TV} < \epsilon.$$

$$P_{X^n,Y^n}(x,y) \triangleq \frac{1}{n}\sum_{i=1}^{n} \mathbf{1}((X_i,Y_i) = (x,y)).$$

Jointly Random Objective

$$\left|\left|\left| p(x^n,y^n) - \prod_{i=1}^{n} \hat{p}(x_i,y_i) \right|\right|\right|_{TV} < \epsilon.$$

Example: Assign 3 tasks to 3 people



Example: Assign 3 tasks to 3 people

1	2	3
X	Y	Z

Example: Assign 3 tasks to 3 people

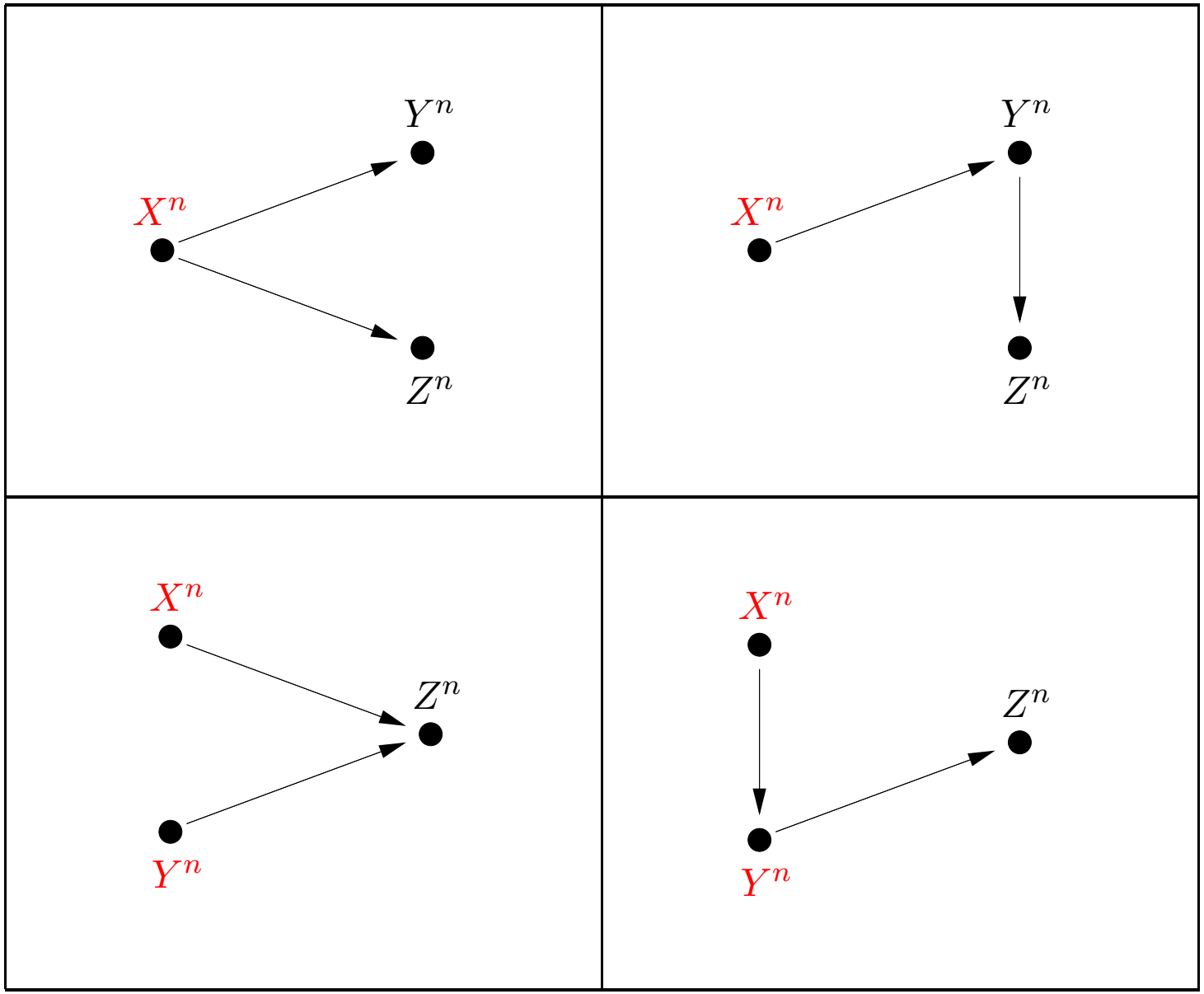
3	1	2
1	2	3
X	Y	Z

Example: Assign 3 tasks to 3 people

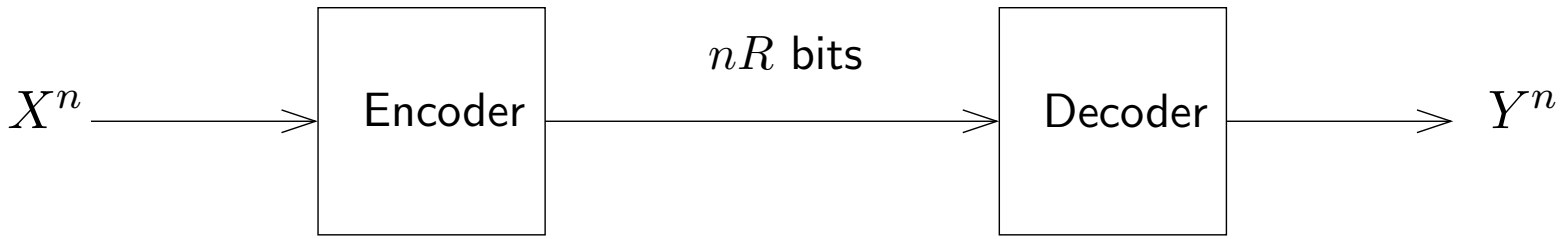
1	2	3
2	3	1
3	1	2
1	2	3
2	3	1
3	1	2
1	2	3
X	Y	Z

$$p(x,y,z) = \{1/3, (1,2,3) \\ 1/3, (2,3,1) \\ 1/3, (3,1,2)\}$$

Three Node Network Settings



Two Node Question for Jointly Random Coordination



Minimum communication rate needed for $p(x, y)$?

- Jointly typicality—$I(X;Y)$.
- What about jointly random coordination?

Properties of Efficient Encoding

- Randomization is required in the encoding/decoding process.
- An efficient scheme requires some randomization at the encoder and further randomization at the decoder.

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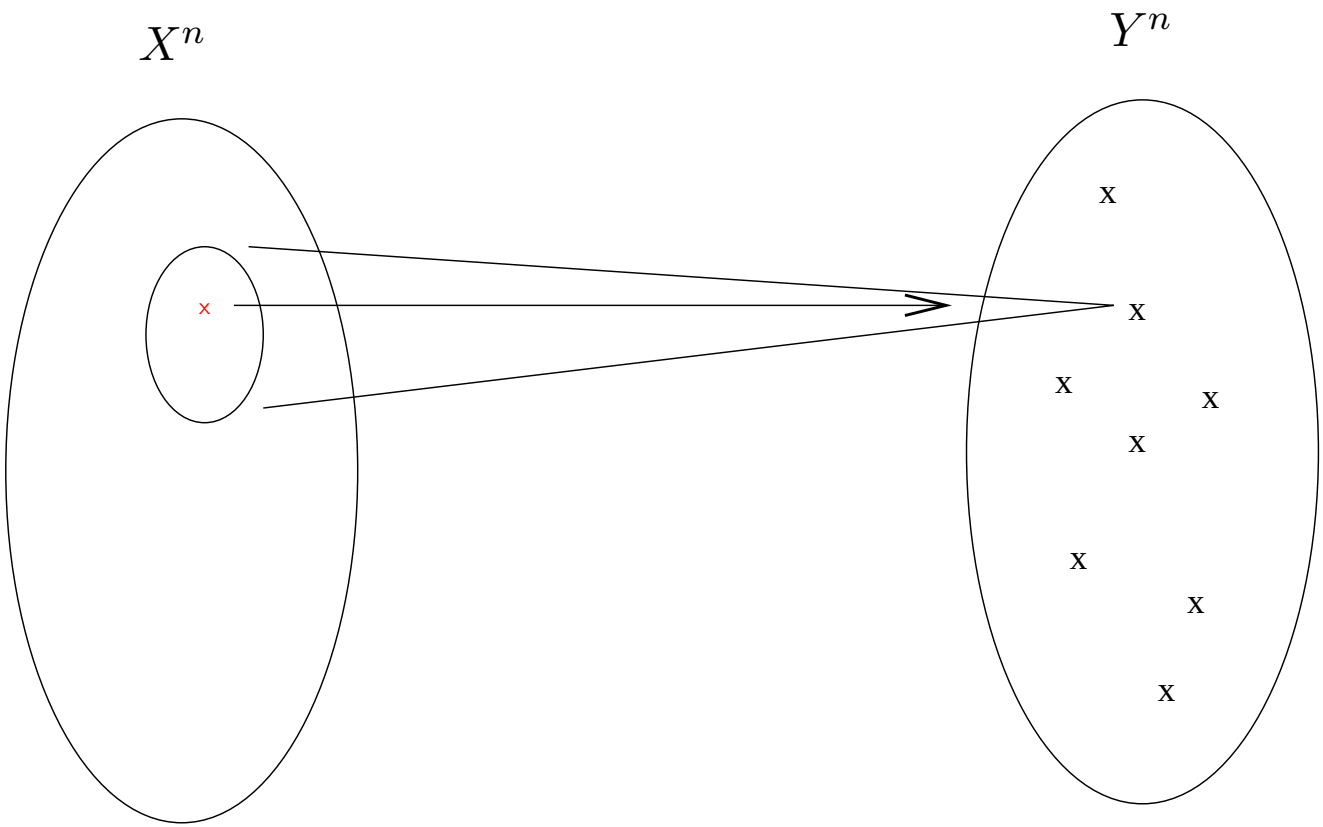

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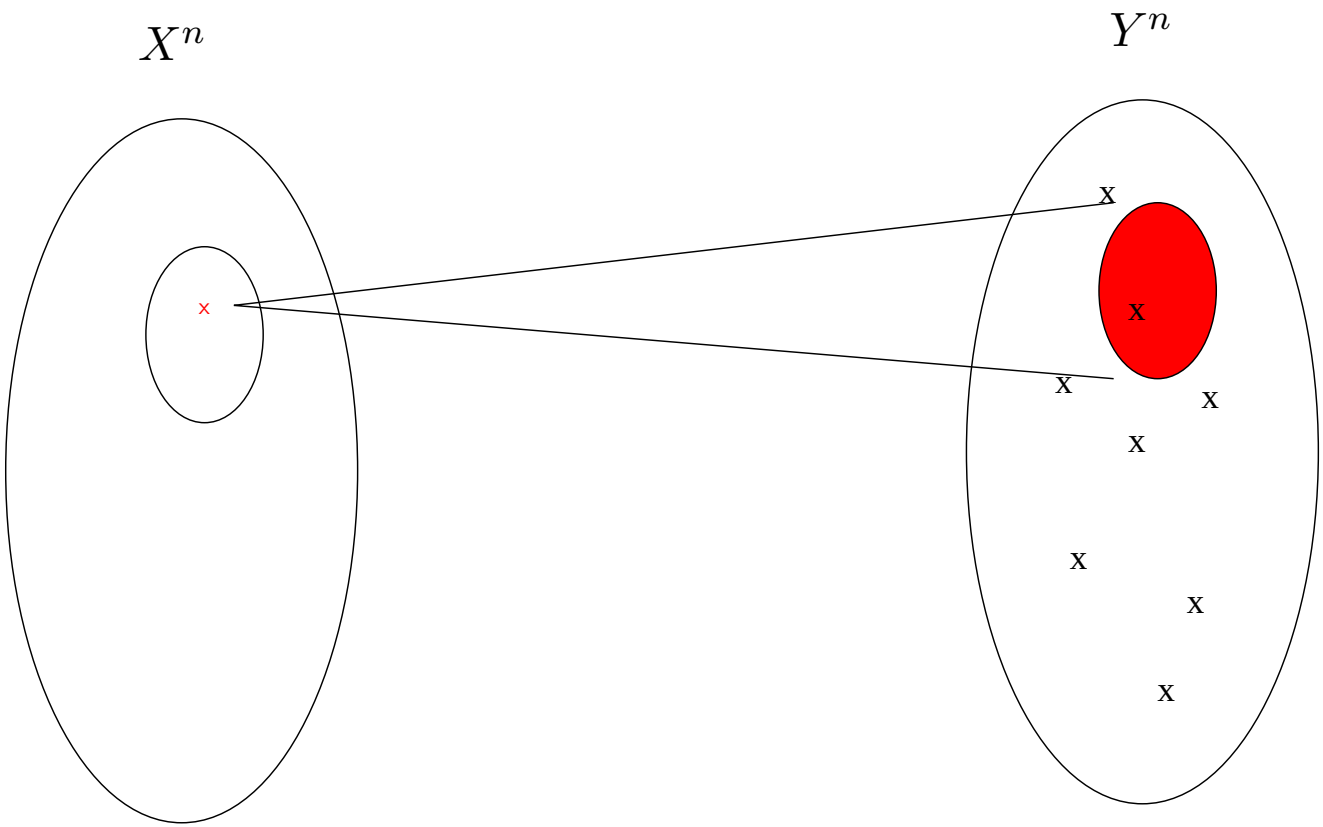

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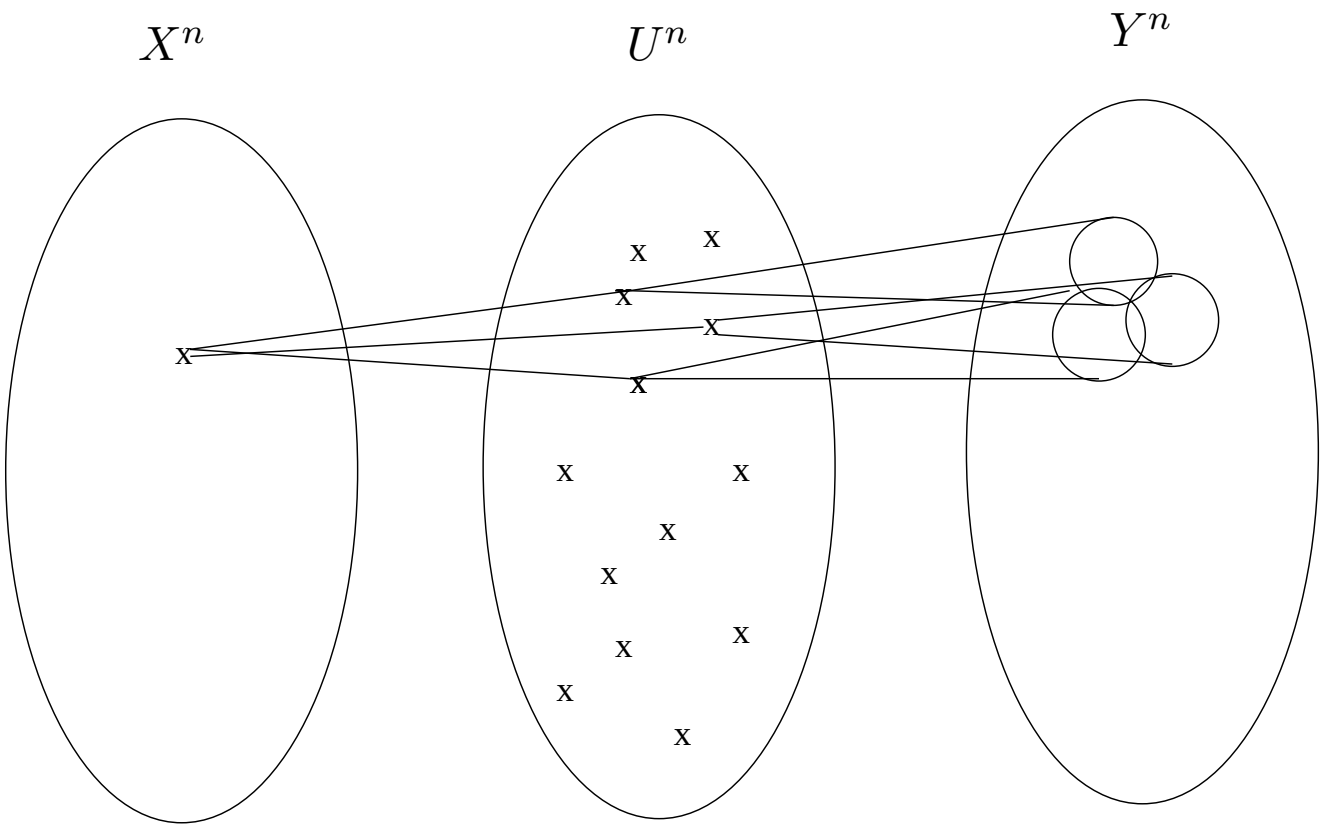
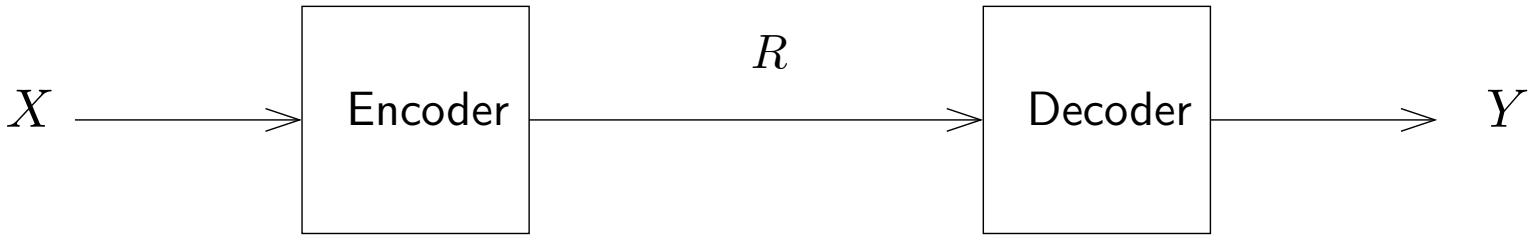


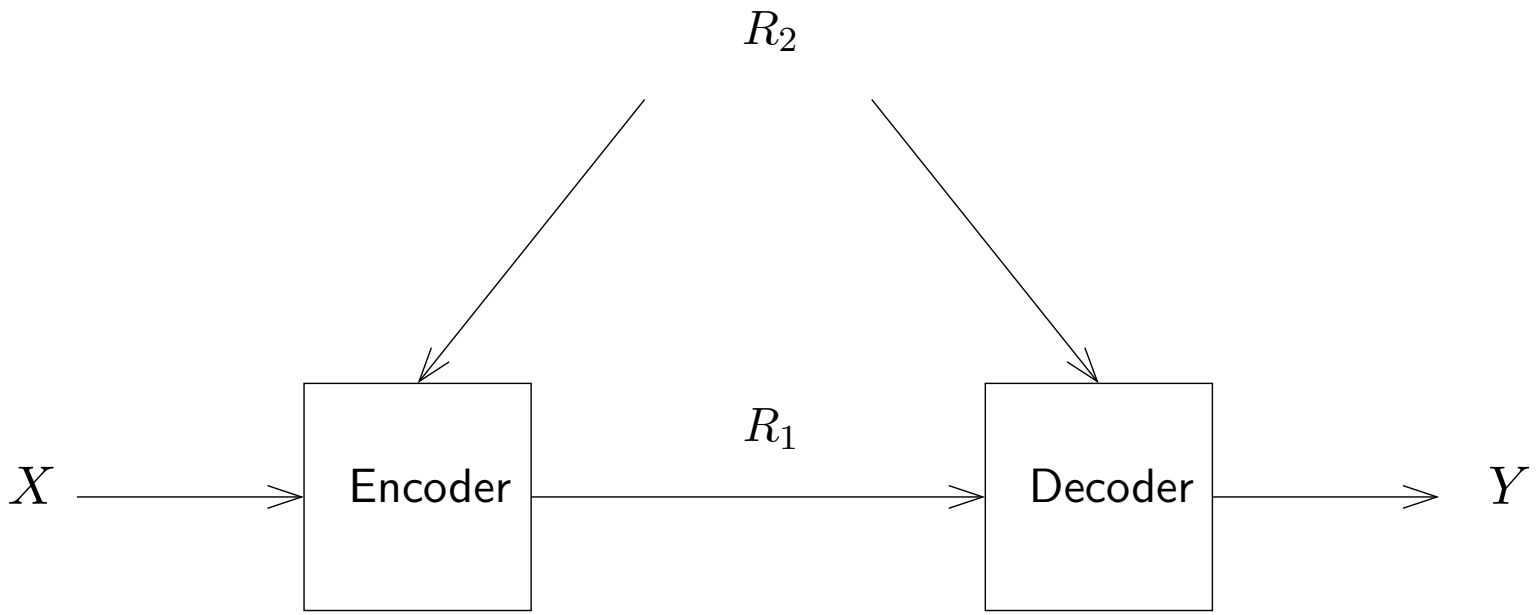
Jointly Random Coordination



Rate requirement is Wyner's Common Information

$$R \;>\; C(X;Y) = \min_{X-U-Y} I(X,Y;U)$$

Jointly Random Coordination with Common Randomness



Rate requirements

$$\begin{aligned} R_1 &> I(X;U) \\ R_1 + R_2 &> I(X,Y;U) \\ X- &\quad U \quad -Y \end{aligned}$$

Example

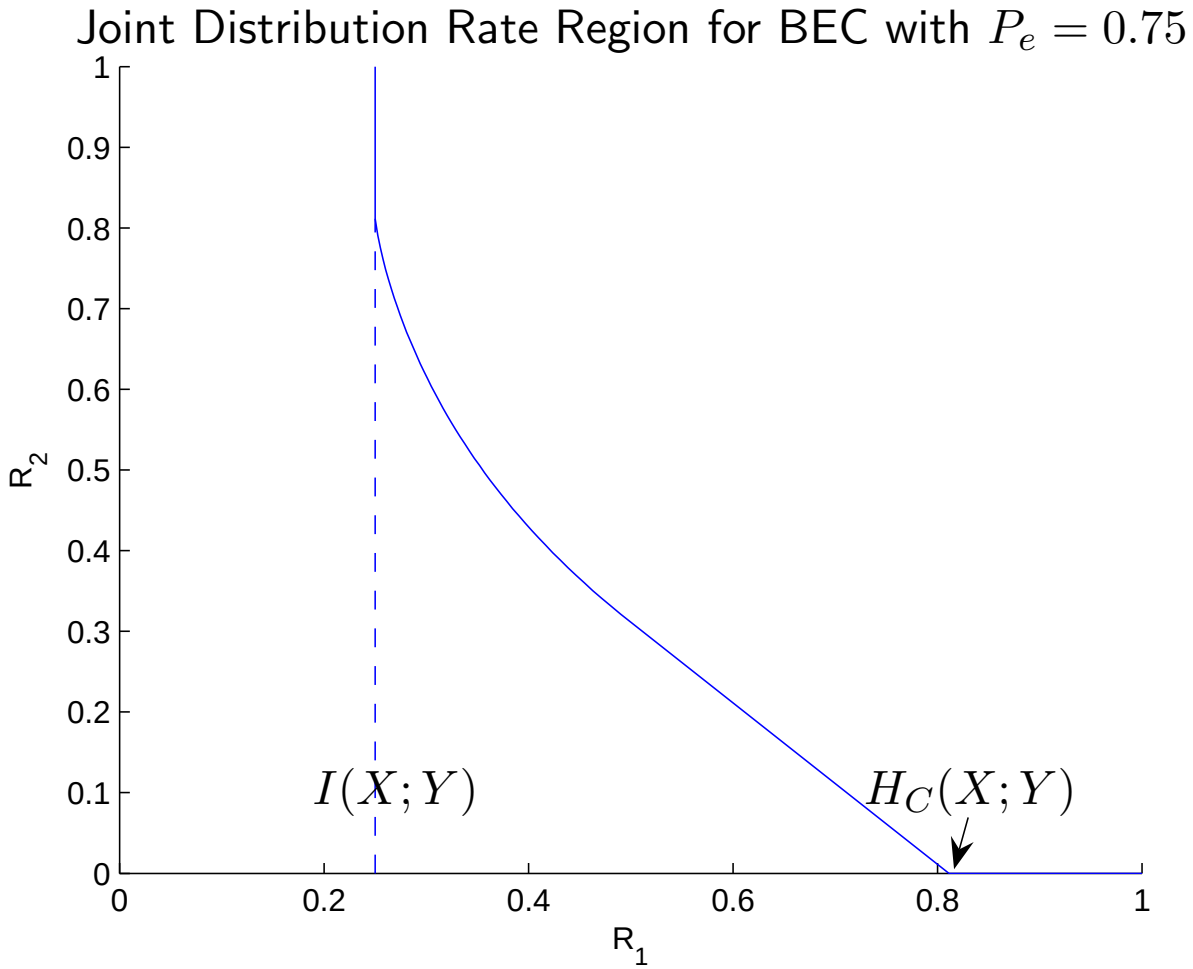
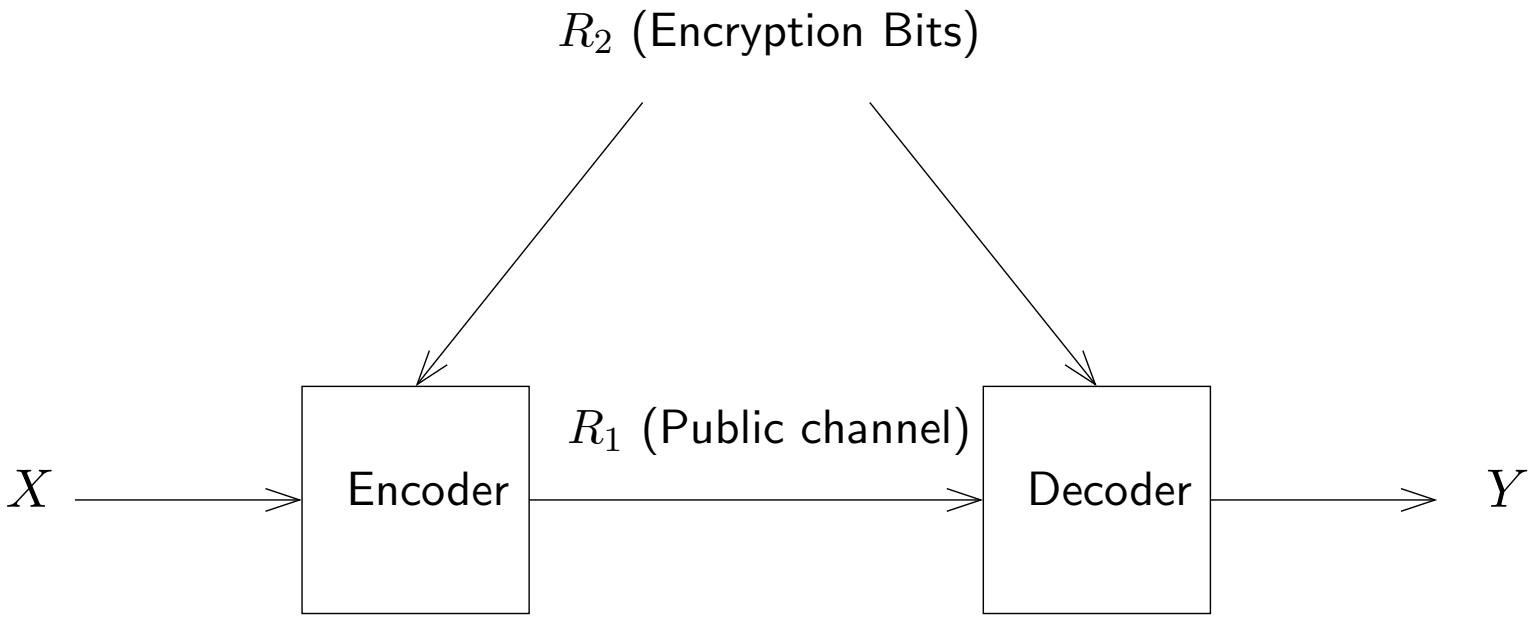


Figure: Boundary of the joint distribution rate region for a binary erasure channel with erasure probability $P_e = 0.75$ and a Bernoulli-half input.

Encryption over Public Channel



Rate requirements

$$\begin{aligned} R_1 &> I(X;U) \\ R_2 &> I(X,Y;U) \\ X- &\quad U \quad -Y \end{aligned}$$