An Almost Ideal Demand System

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Ever since Richard Stone (1954) first estimated a system of demand equations derived explicitly from consumer theory, there has been a continuing search for alternative specifications and functional forms. Many models have been proposed, but perhaps the most important in current use, apart from the original linear expenditure system, are the Rotterdam model (see Henri Theil, 1965, 1976; Anton Barten) and the translog model (see Laurits Christensen, Dale Jorgenson, and Lawrence Lau; Jorgenson and Lau). Both of these models have been extensively estimated and have, in addition, been used to test the homogeneity and symmetry restrictions of demand theory. In this paper, we propose and estimate a new model which is of comparable generality to the Rotterdam and translog models but which has considerable advantages over both. Our model, which we call the Almost Ideal Demand System (*AIDS*), gives an arbitrary first-order approximation to any demand system; it satisfies the axioms of choice exactly; it aggregates perfectly over consumers without invoking parallel linear Engel curves; it has a functional form which is consistent with known household-budget data; it is simple to estimate, largely avoiding the need for non-linear estimation; and it can be used to test the restrictions of homogeneity and symmetry through linear restrictions on fixed parameters. Although many of these desirable properties are possessed by one or other of the Rotterdam or translog models, neither possesses all of them simultaneously.

In Section I of the paper, we discuss the theoretical specification of the *AIDS* and justify the claims in the previous paragraph. In Section II, the model is estimated on postwar British data and we use our results to test the homogeneity and symmetry restrictions. Our results are consistent with earlier findings in that both sets of restrictions are decisively rejected. We also find that imposition of homogeneity generates positive serial correlation in the errors of those equations which reject the restrictions most strongly; this suggests that the now standard rejection of homogeneity in demand analysis may be due to insufficient attention to the dynamic aspects of consumer behavior. Finally, in Section III, we offer a summary and conclusions. We believe that the results of this paper suggest that the *AIDS* is to be recommended as a vehicle for testing, extending, and improving conventional demand analysis. This does not imply that the system, particularly in its simple static form, is to be regarded as a fully satisfactory explanation of consumers' behavior. Indeed, by proposing a demand system which is superior to its predecessors, we hope to be able to reveal more clearly the problems and potential solutions associated with the usual approach.

I. Specification of the *AIDS*

In much of the recent literature on systems of demand equations, the starting point has been the specification of a function which is general enough to act as a second-order approximation to any arbitrary direct or indirect utility function or, more rarely, a cost function. For examples, see Christensen, Jorgenson, and Lau; W. Erwin Diewert (1971); or Ernst Berndt, Masako Darrough, and Diewert. Alternatively, it is possible to use a first-order approximation to the demand functions themselves as in the Rotterdam model, see Theil (1965, 1976); Barten. We shall follow these approaches in terms of generality but we start, not from some arbitrary preference

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ordering, but from a specific class of preferences, which by the theorems of Muellbauer (1975, 1976) permit exact aggregation over consumers: the representation of market demands as if they were the outcome of decisions by a rational representative consumer. These preferences, known as the *PIGLOG* class, are represented via the *cost* or *expenditure function* which defines the minimum expenditure necessary to attain a specific utility level at given prices. We denote this function $c(u,p)$ for utility u and price vector p, and define the *PIGLOG* class by

$$\log c(u,p) = (1-u)\log\{a(p)\} + u\log\{b(p)\} \tag{1}$$

With some exceptions (see the Appendix), u lies between 0 (subsistence) and 1 (bliss) so that the positive linearly homogeneous functions $a(p)$ and $b(p)$ can be regarded as the costs of subsistence and bliss, respectively. The Appendix further discusses this general model as well as the implications of the underlying aggregation theory.

Next we take specific functional forms for $\log a(p)$ and $\log b(p)$. For the resulting cost function to be a flexible functional form, it must possess enough parameters so that at any single point its derivatives $\partial c/\partial p_i$, $\partial c/\partial u$, $\partial^2 c/\partial p_i \partial p_j$, $\partial^2 c/\partial u \partial p_i$, and $\partial^2 c/\partial u^2$ can be set equal to those of an arbitrary cost function. We take

$$\log a(p) = a_0 + \sum_k \alpha_k \log p_k + \frac{1}{2}\sum_k \sum_j \gamma^*_{kj} \log p_k \log p_j \tag{2}$$

$$\log b(p) = \log a(p) + \beta_0 \prod_k p_k^{\beta_k} \tag{3}$$

so that the *AIDS* cost function is written

$$\log c(u,p) = \alpha_0 + \sum_k \alpha_k \log p_k + \frac{1}{2}\sum_k \sum_j \gamma^*_{kj} \log p_k \log p_j + u\beta_0 \prod_k p_k^{\beta_k} \tag{4}$$

where α_i, β_i, and γ^*_{ij} are parameters. It can easily be checked that $c(u,p)$ is linearly homogeneous in p (as it must be to be a valid representation of preferences) provided that $\Sigma_i \alpha_i = 1, \Sigma_j \gamma^*_{kj} = \Sigma_k \gamma^*_{kj} = \Sigma_j \beta_j = 0$. It is also straightforward to check that (4) has enough parameters for it to be a flexible functional form provided it is borne in mind that, since utility is ordinal, we can always choose a normalization such that, at a point, $\partial^2 \log c/\partial u^2 = 0$. The choice of the functions $a(p)$ and $b(p)$ in (2) and (3) is governed partly by the need for a flexible functional form. However, the main justification is that this particular choice leads to a system of demand functions with the desirable properties which we demonstrate below.

The demand functions can be derived directly from equation (4). It is a fundamental property of the cost function (see Ronald Shephard, 1953, 1970, or Diewert's 1974 survey paper) that its price derivatives are the quantities demanded: $\partial c(u,p)/\partial p_i = q_i$. Multiplying both sides by $p_i/c(u,p)$ we find

$$\frac{\partial \log c(u,p)}{\partial \log p_i} = \frac{p_i q_i}{c(u,p)} = w_i \tag{5}$$

where w_i is the budget share of good i. Hence, logarithmic differentiation of (4) gives the budget shares as a function of prices and utility:

$$w_i = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i u \beta_0 \prod p_k^{\beta_k} \tag{6}$$

where

$$\gamma_{ij} = \frac{1}{2}(\gamma^*_{ij} + \gamma^*_{ji}) \tag{7}$$

For a utility-maximizing consumer, total expenditure x is equal to $c(u,p)$ and this equality can be inverted to give u as a function of p and x, the indirect utility function. If we do this for (4) and substitute the result into (6) we have the budget shares as a function of p and x; these are the *AIDS* demand functions in budget share form:

$$w_i = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log\{x/P\} \tag{8}$$

where P is a price index defined by

$$(9) \quad \log P = \alpha_0 + \sum_k \alpha_k \log p_k + \frac{1}{2} \sum_j \sum_k \gamma_{kj} \log p_k \log p_j$$

The restrictions on the parameters of (4) plus equation (7) imply restrictions on the parameters of the *AIDS* equation (8). We take these in three sets

$$(10) \quad \sum_{i=1}^{n} \alpha_i = 1 \quad \sum_{i=1}^{n} \gamma_{ij} = 0 \quad \sum_{i=1}^{n} \beta_i = 0$$

$$(11) \quad \sum_j \gamma_{ij} = 0$$

$$(12) \quad \gamma_{ij} = \gamma_{ji}$$

Provided (10), (11), and (12) hold, equation (8) represents a system of demand functions which add up to total expenditure ($\Sigma w_i = 1$), are homogeneous of degree zero in prices and total expenditure taken together, and which satisfy Slutsky symmetry. Given these, the *AIDS* is simply interpreted: in the absence of changes in relative prices and "real" expenditure (x/P) the budget shares are constant and this is the natural starting point for predictions using the model. Changes in relative prices work through the terms γ_{ij}; each γ_{ij} represents 10^2 times the effect on the ith budget share of a 1 percent increase in the jth price with (x/P) held constant. Changes in real expenditure operate through the β_i coefficients; these add to zero and are positive for luxuries and negative for necessities. Further interpretation is best done in terms of the claims made in the introduction.

A. *Aggregation Over Households*

The aggregation theory developed in Muellbauer (1975, 1976, of which the main relevant points are summarized in the Appendix) implies that exact aggregation is possible if, for an individual household h, behavior is described by the generalization of (8):

$$(8') \quad w_{ih} = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log\{x_h / k_h P\}$$

The parameters k_h can be interpreted as a sophisticated measure of household size which, in principle, could take account of age composition, other household characteristics, and economies of household size; and which is used to deflate the budget x_h to bring it to a "needs corrected" *per capita* level. This allows a limited amount of taste variation across households. The share of aggregate expenditure on good i in the aggregate budget of all households, denoted $\bar{w}_i$ is given by

$$\sum_h p_i q_{ih} \Big/ \sum x_h \equiv \sum_h x_h w_{ih} \Big/ \sum x_h$$

which, when applied to (8′) gives

$$(8'') \quad \bar{w}_i = \alpha_i + \sum_j \gamma_{ij} \log p_j - \beta_i \log P + \beta_i \left\{ \sum_h x_h \log(x_h / k_h) \Big/ \sum x_h \right\}$$

Define the aggregate index k by

$$(13) \quad \log(\bar{x}/k) \equiv \sum_h x_h \log(x_h / k_h) \Big/ \sum x_h$$

where $\bar{x}$ is the average level of total expenditure x_h. Hence (8″) becomes

$$(8''') \quad \bar{w}_i = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log(\bar{x}/kP)$$

This is identical in form to (8′) and this confirms that under these assumptions aggregate budget shares correspond to the decisions of a rational representative household whose preferences are given by the *AIDS* cost function (4) and whose budget is given by $\bar{x}/k$, the "representative budget level."

The index k has an interesting interpretation. If each household had the same tastes ($k_h = 1$, all h), k would be an index of the equality of the distribution of household budgets. In fact, this index is identical to Theil's (1972) entropy measure of equality Z deflated by the number of households H, where $\log Z = -\Sigma(x_h/X)\log(x_h/X)$ and X is the aggregate budget; Z reaches its maximum level of H when there is perfect equality so that $x_h = \bar{x}$, all h. Therefore as inequality increases, $k = Z/H$ decreases and the representative budget level increases. When k_h differs across households, for example, because of differences in household composition, the index k reflects not only the distribution of budgets but the demographic structure. Ideally, one might attempt to model the variation of k_h with household characteristics in a cross-section study and, given time-series data on the joint distribution of household budgets and characteristics, construct a series for k for use in fitting (8‴). Data limitations have prevented us from carrying out this proposal in the empirical application below. To the extent that k is constant or uncorrelated with $\bar{x}$ or p, no omitted variable bias arises from our procedure of omitting k and redefining $\alpha_i^* = \alpha_i - \beta_i \log k^*$ where k^* is the constant or sample mean value of k.

When the distribution of household budgets and household characteristics is invariant except for equiproportional changes in household budgets, k is constant. In this case there is considerable extra scope for taste variations in the individual demand functions without altering the validity of the representative consumer hypothesis embodied in (8‴). Indeed, it turns out that not only α_{ih}, all i, but also γ_{ijh}, all i,j, can differ over households. The α_i and γ_{ij} parameters in (8‴) are then weighted averages of the micro parameters.

B. *Generality of the Model*

The flexible functional form property of the *AIDS* cost function implies that the demand functions derived from it are first-order approximations to any set of demand functions derived from utility-maximizing behavior. The *AIDS* is thus as general as other flexible forms such as the translog or the Rotterdam models. However, if maximizing behavior is not assumed but it is simply held that demands are continuous functions of the budget and of prices, then the *AIDS* demand functions (8) (without the restrictions (11) and (12)) can still provide a first-order approximation. In general, without maximizing assumptions, we can think of the budget shares w_i as being unknown functions of $\log p$ and $\log x$. From (8) and (9), the *AIDS* has derivatives $\partial w_i/\partial \log x = \beta_i$ and $\partial w_i/\partial \log p_j = \gamma_{ij} - \beta_i \alpha_j - \beta_i \Sigma \gamma_{jk} \log p_k$ so that, at any point, β and γ can be chosen so that the derivatives of the *AIDS* will be identical to those of any true model. Given that the α parameters act as intercepts, the *AIDS* can thus provide a local first-order approximation to any true demand system, whether derived from the theory of choice or not. This property is important since it means that tests of homogeneity of symmetry are set within a maintained hypothesis which makes sense and would be widely accepted in its own right.

Generality is not without its problems, however. There is a large number of parameters in (18) and on most data sets these are unlikely to be all well determined. It is thus important that there should exist some straightforward procedure for eliminating unnecessary parameters without untoward consequences for the properties of the model. In the *AIDS*, this can be done by placing whatever restrictions on γ_{ij} parameters are thought to be empirically or theoretically plausible. As we shall see below, in many cases it will be possible to impose these restrictions on a single equation basis. One obvious restriction is that for some pairs $(i,j), \gamma_{ij}$ should be zero; for such pairs the budget share of each is independent of the price of the other if (x/P) is held constant. It can be shown that γ_{ij} has approximately the same sign as the compensated cross-price elasticity and this is also useful in suggesting prior restrictions. We should not however expect *all* the γ_{ij}s to be zero; the resulting model, $w_i = \alpha_i + \beta_i \log(x/P)$ is extremely restrictive and has been tested and rejected by Deaton (1978).

C. *Restrictions*

If we start from equations (8) and (9) as our maintained hypothesis, we can examine the effects of the restrictions (10)–(12) which are required to make the model consistent with the theory of demand. The conditions (10) are the *adding-up* restrictions; as can easily be checked from (8), these ensure that $\Sigma w_i \equiv 1$. *Homogeneity* of the demand functions requires restriction (11) which can be tested equation by equation. Slutsky *symmetry* is satisfied by (8) if and only if the symmetry restriction (12) holds. As is true of other flexible functional forms, *negativity* cannot be ensured by any restrictions on the parameters alone. It can however be checked for any given estimates by calculating the eigenvalues of the Slutsky matrix s_{ij}, say. In practice, it is easier to use not s_{ij} but $k_{ij} = p_i p_j s_{ij}/x$, the eigenvalues of which have the same signs as those of s_{ij} and which are given by

$$(14) \quad k_{ij} = \gamma_{ij} + \beta_i \beta_j \log \frac{x}{P} - w_i \delta_{ij} + w_i w_j$$

where δ_{ij} is the Kronecker *delta*. Note that apart from this negativity condition, all the restrictions are expressible as linear constraints involving only the parameters and so can be imposed globally by standard techniques.

D. *Estimation*

In general, estimation can be carried out by substituting (9) in (8) to give

$$(15) \quad w_i = (\alpha_i - \beta_i \alpha_0) + \sum_j \gamma_{ij} \log p_j + \beta_i \left\{ \log x - \sum_k \alpha_k \log p_k - \frac{1}{2} \sum_k \sum_j \gamma_{kj} \log p_k \log p_j \right\}$$

and estimating this non-linear system of equations by maximum likelihood or other methods with and without the restrictions (11) and (12). (Note that since the data add up by construction, (10) is not testable.) Equation (15) is not particularly difficult to estimate since the first-order conditions for likelihood maximization are linear in α and γ given β and vice versa so that "concentration" allows iteration on a subset of the parameters (see for example, Deaton, 1975, pp. 46–49). Although all the parameters in (15) are identified given sufficient variation in the independent variables, in many examples the practical identification of α_0 is likely to be problematical. This parameter is only identified from the α_is in (15) by the presence of these latter inside the term in braces, originally in the formula for *log P*, equation (9). However, in situations where individual prices are closely collinear, *log P* is unlikely to be very sensitive to its weights so that changes in the intercept term in (15) due to variations in α_0 can be offset in the αs with minimal effect on *log P*. This can be overcome in practice by assigning a value to α_0 a priori. Since the parameter can be interpreted as the outlay required for a minimal standard of living when prices are unity (usually in the base year; see the Appendix), choosing a plausible value is not difficult.

However, in many situations, it is possible to exploit the collinearity of the prices to yield a much simpler estimation technique. Note from (8) that if P were known, the model would be linear in the parameters α, β, and γ, and estimation (at least without cross-equation restrictions such as symmetry) can be done equation by equation by *OLS* which, in this case and given normally distributed errors, is equivalent to maximum likelihood estimation for the system as a whole. The adding-up constraints (10) will be automatically satisfied by these estimates. In situations where prices are closely collinear, it may well be adequate to *approximate* P as proportional to some known index P^*, say. One obvious candidate in view of (8) and (9) is Stone's (1953) index $\log P^* = \Sigma w_k \log p_k$. If $P \simeq \phi P^*$ say, then (8) can be estimated as

$$(16) \quad w_i = (\alpha_i - \beta_i \log \phi) + \sum_j \gamma_{ij} \log p_j + \beta_i \log\left(\frac{x}{P^*}\right)$$

Note that in this framework the α_i parameters are identified only up to a scalar multiple of β_i; if we write $\alpha_i^* = \alpha_i - \beta_i \log \phi$, it is easily seen that $\Sigma\alpha_k^* = 0$ is still required for adding up, since $\Sigma\,\beta_k = 0$.

In the empirical results below we shall estimate both (15) and (16), and show that the latter is an excellent approximation to the former. However, it must be emphasized that (16) exists only as an approximation to (15) and will only be accurate in specific circumstances, albeit widely occurring ones in time-series estimation. Note finally that if single equation estimation is used to investigate likely restrictions amongst the γ parameters, as is suggested in Part B above, the *constrained OLS* estimates will no longer automatically be maximum likelihood, efficient, or satisfy adding up. Hence, once the restrictions have been selected, (15) should be used to reestimate the whole system.

E. *Relationship with Budget Studies and with the Rotterdam Model*

The Engel curves corresponding to (8) take the form $p_i q_i = \xi_i x + \beta_i x \log x$ for appropriate functions of prices ξ_i. These are clearly not linear except in the proportional case when $\beta_i = 0$. The model thus allows a possible reconciliation between time-series models, which have to date required linearity of Engel curves for aggregation with cross-section results, which typically find evidence of nonlinearity. Indeed, the *PIGLOG* Engel curve $w_i = \xi_i + \beta_i \log x$ was used as early as 1943 by Holbrook Working and has recently been recommended by Claus Leser (1963, 1976) as providing an excellent fit to cross-section data in a wide range of circumstances.

In the time-series context, the *AIDS* has a close relationship to the Rotterdam model of Theil (1965, 1976) and Barten. The first difference form of (8) is

$$(17)\quad \Delta w_i = \beta_i \Delta \log\left(\frac{x}{P}\right) + \sum_j \gamma_{ij} \Delta \log p_j$$

which no longer involves the α parameters except through $\Delta \log P$. This dependence can be seen by writing (17) in full, i.e.,

$$(18)\quad \Delta w_i = \beta_i \left\{ \Delta \log x - \sum_k \alpha_k \Delta \log p_k - \frac{1}{2} \sum_k \sum_j \gamma_{kj} \Delta(\log p_j \log p_k) \right\} + \sum_j \gamma_{ij} \Delta \log p_j$$

Again, all the parameters (except α_0) are theoretically identified, but in practice the substitutability between γ_{ij}s and $\beta_i \alpha_j$s in fitting (18) if prices are nearly collinear means that, in such cases, the only practical way of estimating (17) is to replace $\Delta \log P$ by some index, for example, $\Delta(\Sigma w_k \log p_k)$ as before or by its approximation $\Sigma w_k \Delta \log p_k$. In the latter case, the right-hand side of (17) becomes identical to the right-hand side of the Rotterdam model which is

$$(19)\quad w_i \Delta \log q_i = b_i \{\Delta \log x - \Sigma w_k \Delta \log p_k\} + \sum_j c_{ij} \Delta \log p_j$$

The dependent variable is different in the *AIDS*; instead of $w_i \Delta \log q_i$ we have Δw_i or $w_i \Delta \log w_i$. Thus, by replacing the dependent variable $w_i \Delta \log q_i$ in the Rotterdam model by $w_i \Delta \log w_i$, an addition of $w_i \Delta \log(p_i/x)$, we generate the first-difference form of the *AIDS*. The similarity between the two models is quite striking in this form; both are effectively linear and both can be used to test homogeneity and symmetry with only linear restrictions on constant parameters. Note however that the parameters have quite different interpretations in the two models so that, for example, the negativity condition applies directly to the matrix of price effects in the Rotterdam model which is not the case for the *AIDS*. The crucial difference between the two models is that (17), unlike (19), is derived from explicit demand functions, (8), and an explicit characterization of preferences, (4). For the prediction of demand this difference may not be vitally important, but in many other contexts, for example in calculating cost-of-living indices, household equivalence scales, or optimal tax rates, the ability to link estimated parameter values to preferences themselves becomes of great significance.

II. An Application to Postwar British Data

In this section we estimate the model using annual British data from 1954 to 1974 inclusive on eight nondurable groups of consumers' expenditure, namely, food, clothing, housing services, fuel, drink and tobacco, transport and communication services, other goods, and other services. As discussed in Section I, Part A above, if we assume that the index k in (8‴) is either constant or that its deviations are independently distributed from those of the average budget $\bar{x}$ and of prices, no biases result from its omission. In particular, we allow the intercepts in (8‴) to absorb the $-\beta_i \log k$ terms. We then proceed by first following the strategy outlined in Section I, Part D, setting $\log P^* = \sum w_k \log p_k$ for each year and estimating equation (16) for each good separately by *OLS*. The system is then reestimated, equation by equation, and again using P^*, in order to test the homogeneity condition. Equation (11) is imposed by substitution so that instead of (16), we estimate

$$(20) \quad \bar{w}_i = \alpha_i^* + \sum_{j=1}^{n-1} \gamma_{ij} \log\left(\frac{p_j}{p_n}\right) + \beta_i \log\left(\frac{\bar{x}}{P^*}\right)$$

At this stage, F-ratios are calculated equation by equation to test the validity of the restriction.

The next stage is to impose symmetry of γ, at this point replacing P^* by the "correct" price index (9) with α_0 set to some appropriate value. Since symmetry, unlike homogeneity or the unrestricted model, involves cross-equation restrictions, the variance-covariance matrix of the residuals for the first time plays a part in the estimation. Since this is unknown a priori, normal practice would be to replace it by its maximum likelihood estimate. However, with only twenty-one observations, this is not practicable for equation (15) since, with so many parameters in each equation, the likelihood can be made arbitrarily large by making any *one* equation fit perfectly.[1] This difficulty can only be resolved by assuming a particular structure for the variance-covariance matrix of the residuals. Following Deaton (1975, p. 39), we assume $V = \sigma^2(I - ii')$, where V is the variance-covariance matrix of the residuals, σ^2 is a (positive) parameter to be estimated, I is an $n \times n$ identity matrix and i is a vector each of the elements of which is $(n)^{-1/2}$. In this case, maximum-likelihood estimation reduces to least squares so that instead of minimizing the *determinant* of the matrix of residual cross products, we minimize its *trace*. The likelihood values quoted below are calculated on this assumption. Once again, maximum use is made of substitution in the estimation so that, under symmetry, (15) is estimated using only the fourteen independent α_i^*s and β_is and the twenty-eight parameters forming the upper right-hand triangle of γ with its final row and column deleted. We now check that P^* and P are sufficiently close to allow comparison of likelihoods both by direct evaluation of both indices and by reestimation of the unrestricted and homogeneous models using P as evaluated from the symmetric estimates. It is also possible to check concavity at this stage by using the symmetric parameter estimates to calculate the eigenvalues of the matrix in (14). Finally, the whole process is repeated with the model written in first differences, that is, equation (17) with the addition of intercepts. Collinearity prevented any successful attempt to link P to the parameter estimates in these regressions; instead, the value of P calculated from the symmetric estimation in levels was used throughout.

Note that we choose to test symmetry whether or not homogeneity is rejected. This procedure has been criticized by Grayham Mizon who suggests that optimal inference requires that further testing be abandoned as soon as a rejection is encountered. Mizon's criticism would be correct if we were certain of our maintained hypothesis, but to some extent this is a matter of choice. Many economists would choose not to test

[1] We are grateful to Teun Kloek for pointing this out to us.

TABLE 1—THE UNCONSTRAINED PARAMETER ESTIMATES AND TESTS OF HOMOGENEITY
(t-Values in Parentheses)

Commodity i	α_i^*	β_i	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	γ_{i5}	γ_{i6}	γ_{i7}	γ_{i8}	$\sum_j \gamma_{ij}$	S.E.E. (10^{-2})	R^2	D.W.
Food	1.221	−0160	0.186	−0.077	−0.013	−0.020	−0.058	0.032	0.015	−0.098	−0.033	.113	0.999	2.33
	(7.4)	(−6.1)	(9.8)	(−4.3)	(−0.8)	(−1.1)	(−6.2)	(1.3)	(0.7)	(−4.2)	(−4.4)	.180	0.998	1.74
Clothing	−0.482	0.091	0.033	0.016	−0.024	−0.026	−0.029	0.014	0.033	−0.049	−0.032	.106	0.984	2.29
	(−3.1)	(3.7)	(1.8)	(1.0)	(−1.6)	(−1.5)	(−3.3)	(0.6)	(1.6)	(−2.2)	(−4.5)	.171	0.955	1.55
Housing	0.793	−0.104	−0.082	−0.9	0.088	0.9	0.033	−0.055	−0.030	0.098	0.051	.086	0.999	1.89
	(6.3)	(−5.1)	(−5.6)	(−0.7)	(7.2)	(0.7)	(4.7)	(−2.9)	(−1.8)	(5.5)	(9.1)	.241	0.992	1.29
Fuel	−0.159	0.033	−0.042	0.010	−0.011	0.037	−0.004	0.022	0.007	−0.031	−0.010	.140	0.883	2.25
	(−0.8)	(1.0)	(−1.8)	(0.4)	(−0.5)	(1.6)	(−0.3)	(0.7)	(0.3)	(−1.1)	(−1.1)	.141	0.870	2.03
Drink and	−0.043	0.028	−0.043	0.034	−0.027	−0.020	0.056	0.005	−0.018	0.014	0.001	.099	0.969	2.96
Tobacco	(−0.3)	(1.2)	(−2.6)	(2.2)	(−1.9)	(−1.2)	(6.9)	(0.2)	(−0.9)	(0.7)	(0.0)	.095	0.969	2.93
Transport and	−0.061	0.029	−0.022	−0.012	−0.002	0.011	0.060	−0.023	−0.024	0.053	0.040	.047	1.000	2.24
Communication	(−0.9)	(2.6)	(−2.7)	(−1.6)	(−0.3)	(1.4)	(15.2)	(−2.2)	(−2.6)	(5.3)	(13.1)	.184	0.992	1.36
Other Goods	−0.038	0.022	0.001	−0.003	−0.001	−0.006	−0.030	0.007	0.032	−0.006	−0.005	.108	0.885	1.92
	(−0.2)	(0.9)	(0.0)	(−0.2)	(−0.0)	(−0.3)	(−3.4)	(0.3)	(1.5)	(−0.2)	(0.7)	.106	0.880	1.91
Other Services	−0.231	0.060	−0.032	0.041	−0.011	0.014	−0.028	−0.003	−0.015	0.019	−0.014	.107	0.843	2.27
	(−1.5)	(2.4)	(−1.8)	(2.4)	(−0.7)	(0.8)	(−3.1)	(−0.1)	(−0.7)	(0.9)	(−2.0)	.119	0.788	1.98

homogeneity, treating absence of money illusion as a maintained hypothesis; the test of symmetry would then be the interesting one. Even if the maintained hypothesis turns out to be false, tests based on it are not necessarily without interest. Few if any tests in econometrics are carried out within the framework of maintained hypotheses which are even widely accepted, let alone of unchallengeable validity.

Table 1 reports the first-stage estimates of (16) using P^* and without any constraints on the parameters save (10) which are automatically and costlessly satisfied. The estimates of β classify food and housing as necessities while the other goods are luxuries. A large number of γ coefficients are significantly different from zero; twenty-two out of sixty-four have t-values absolutely larger than 2. Even so, none of the variables considered have any detectable effect on the value share for fuel and very few have influence in the other goods or other services equations. Similarly, the prices of fuel, of transport and communication, and of other services have little or no effect anywhere (except, of course, through P^* and the value share itself), while the prices of food, drink and tobacco, and of other services appear with considerable regularity. The total expenditure and own-price elasticities are shown in the first two columns of Table 2 and, although food has an (insignificant) positive price elasticity, these numbers appear both credible and in line with other studies. Note the general price inelasticity of demand; only transport and communication appear to be price elastic.

Table 1 also shows, in the column headed $\Sigma\gamma_{ij}$, the row sums of the unconstrained γ_{ij} matrix; this number shows 10^2 times the absolute effect on each value share of a 1 percent increase in all prices and total expenditure. Under homogeneity, this should be zero and the bracketed numbers given are t-tests of the significance of the deviation from zero. These numbers are, of course, identical to the square roots of the F-ratios obtained by comparing the residual sums of squares of equations (16) and (20). Hence, a proportional increase in prices and expenditure will decrease expenditure on food and on clothing, and increase expenditure on housing and transport and communication. These are also the commodities for which the elasticities suffer the largest changes between columns 1 and 2 and columns 3 and 4 of Table 2. Other deviations from homogeneity appear not to be significant. The final columns of Table 1 give equations standard errors, the R^2 and Durbin-Watson ($D.W.$) statistics for free and restricted estimation. Note that for the four commodity groups where homogeneity

TABLE 2—TOTAL EXPENDITURE AND OWN-PRICE ELASTICITES

	Levels Model				First-Differences Model			
	Unconstrained		Homogeneous		Unconstrained		Homogeneous	
	e_i	e_{ii}	e_i	e_{ii}	e_i	e_{ii}	e_i	e_{ii}
Food	0.21	0.07	0.04	−0.01	0.04	0.22	0.17	−0.00
Clothing	2.00	−0.92	1.51	−0.48	2.83	−0.94	2.92	−0.94
Housing	0.30	−0.31	0.79	−0.16	0.04	−0.31	−0.02	−0.30
Fuel	1.67	−0.28	1.37	0.10	1.00	0.00	0.86	−0.08
Drink and Tobacco	1.22	−0.60	1.22	−0.62	1.37	−0.67	1.36	−0.68
Transport and Communication	1.23	−1.21	1.73	−0.92	1.14	−1.23	1.05	−1.17
Other goods	1.21	−0.72	1.15	−0.77	2.03	−0.52	1.92	−0.47
Other services	1.40	−0.93	1.28	−0.78	1.03	−0.78	1.06	−0.74

is rejected, the *D.W.* statistic shows a sharp fall in each case.

The failure of homogeneity is not a new result (see, for example, Barten; Ray Byron; Deaton, 1974a), and can be ascribed to a number of possible causes. However, as far as we are aware, the introduction of serial correlation through the imposition of homogeneity is a result which has not been previously remarked, although it may have been implicit in earlier work. There are a number of plausible explanations for this phenomenon. For example, expenditure on several items may be relatively inflexible in the short run; housing is the obvious case here. The explanation of such items may require other variables such as stocks, lagged dependent variables, or time trends which can perhaps be proxied by the absolute price level. The omission of such variables will thus lead to a rejection of homogeneity associated with an introduction of serial correlation in the residuals of the restricted equations. In principle one could easily include such conditioning variables in the *AIDS* cost function, for example by allowing the αs to vary linearly with them, and this is likely to be an important topic for future research. A second explanation is the omission of price expectations—the argument advanced by Deaton (1977a) would suggest that factors such as the frequency of purchase for different goods will be relevant in assessing the response of expenditures to changes in price, especially when there is rapid relative or absolute price change. A third possibility, suggested by the discussion of aggregation in Section I, Part A above, is that it may be incorrect to assume that k, the index reflecting the distribution of household budgets and demographic structure, is independent of the average budget and the price vector. Finally, the assumption of weak intertemporal separability of nondurable goods in the intertemporal utility function, which is required to justify the conventional static utility-maximizing model, may be inappropriate. It is not difficult to construct other models which produce the result and without extensive further empirical work it is extremely difficult to discriminate between them.

In moving to the symmetric estimates (not reported here), in which P^* is replaced by P, we must first check the closeness of the approximation. Table 3 reproduces the two series, $P^* = \exp\{\Sigma w_k \log p_k\}$ and P scaled to be unity in the base year, i.e., $\exp\{\Sigma \alpha_k \log p_k + \frac{1}{2}\Sigma\Sigma\gamma_{kj} \log p_k \log p_j\}$ evaluated at the symmetric parameter estimates. Both series are based on unity in 1970. Clearly the differences are small; the absolute magnitude of the difference is never greater than .008.

The reestimation of the unconstrained and homogeneous models using P rather than P^* confirmed the empirical unimportance of the difference. Both sets of likelihoods are given in Table 4. It must be reemphasized that these findings are conditional on the kind of relative price movements that took place in our sample. However, even if relative price changes had been greater, the results suggest that the proce-

TABLE 3—COMPARISON OF PRICE INDICES

	P^*	P		P^*	P		P^*	P
1954	0.566	0.571	1961	0.684	0.686	1968	0.894	0.888
1955	0.587	0.595	1962	0.712	0.715	1969	0.946	0.944
1956	0.611	0.617	1963	0.729	0.730	1970	1.000	1.000
1957	0.631	0.636	1964	0.754	0.758	1971	1.084	1.084
1958	0.648	0.653	1965	0.793	0.797	1972	1.161	1.161
1959	0.655	0.661	1966	0.827	0.830	1973	1.271	1.279
1960	0.663	0.666	1967	0.851	0.852	1974	1.465	1.461

TABLE 4—COMPARATIVE VALUES OF 2 *LOG* LIKELIHOOD

	Levels		First Differences	
	Using P^*	Using P	Using P^*	Using P
Unrestricted	1722.5	1723.8	1560.0	1560.3
Homogeneous	1579.6	1585.1(7)	1546.6	1547.9(7)
Symmetric	–	1491.0(21)	–	1508.8(21)

Note: Number of restrictions in parentheses. Numbers can only be compared within columns, not between levels and first differences.

dure of starting with P^*, calculating *OLS* regressions, computing a new P, and repeating will be a computationally efficient way of obtaining good estimates of the full nonlinear system.

Symmetry, unlike homogeneity, cannot be tested on an equation-by-equation basis and we must rely on a large-sample likelihood-ratio test for the system as a whole. For comparison, twice the logarithm of the likelihood is 1722.5 for the unconstrained system, 1579.6 for the homogeneous model (a fall which reflects the individual restrictions) and this falls further to 1491.0 under symmetry. Since symmetry embodies twenty-one constraints over and above the seven of homogeneity, the restriction is rejected on an asymptotically valid χ^2-test whether or not the maintained hypothesis is taken to contain homogeneity. Once again, this is consistent with earlier results although rejection of symmetry *given* homogeneity is not always clear-cut in the studies cited above. The interpretation of the rejection is not clear without some convincing explanation of the lack of homogeneity. Without this, it is impossible to know whether or not we should expect symmetry to hold. For example, it is possible to introduce habits into demand functions so that, if they are allowed for, symmetry can be expected to hold, while if ignored, symmetry will be destroyed.

The full set of symmetric parameter estimates are not included here for reasons of space. The most interesting property of these, apart from symmetry, is in their implications for negativity. To assess this, the K matrix of equation (14) was evaluated for each year in the sample and its eigenvalues calculated. One of these is identically zero and, for concavity, the others should be negative. Contrary to this, we found one positive eigenvalue for the early part of the period, increasing to two by the end. The most obvious symptom of nonconcavity in the symmetric estimates was an estimated positive compensated own-price elasticity for fuel throughout the sample period. This may seem to be of limited importance given that the symmetric homogeneous model has already been rejected; if the cost function doesn't exist, why worry about its concavity? However, for several reasons it would be extremely useful to have parameter estimates for a reasonably general concave homogeneous cost function. For example, we frequently wish to calculate price and quantity index numbers or to use optimal taxation formulae to derive numerical values for tax rates. All such calculations require numerical estimates of cost functions and, if they are to make any sense at all, these cost functions must be both homogeneous and concave. Consequently, in cases where empirical estimates of demand equations have been used in applied welfare analysis, the linear expenditure system has invariably been used; see, for example, Anthony Atkinson and Joseph Stiglitz, Muellbauer (1974), or Deaton (1977b). With the linear expenditure system, the model is so restrictive that concavity of the cost function is virtually guaranteed provided inferior goods do not appear. But this restrictiveness is also known to be empirically false (see, for example, Deaton, 1974b or 1978) so that it would be of considerable value to have estimates of a concave cost function which allowed considerably more substitution than does the linear expenditure system. Consequently, it will be of considerable interest in future work to attempt to restrict the parameters further so that the estimated cost function is concave.

Finally, we turn to the estimation of the model in first-difference form. Here we use equation (17) plus intercepts, i.e.,

$$(21)\quad \Delta w_i = \eta_i + \beta_i \Delta \log\left(\frac{\bar{x}}{P}\right) + \sum_k \gamma_{ik} \Delta \log p_k$$

where the constants η_i are introduced primarily for econometric reasons but, if significant, would imply time trends in the original model which expresses the variables in levels. The P is taken as in Table 3. In these regressions homogeneity is only rejected for food and for transport and communication; clothing and housing, which rejected homogeneity in the earlier regressions, now yield insignificant F-ratios. Closer inspection reveals that for both these cases, the constant term η_i, which is insignificant without constraints, becomes significant when homogeneity is imposed. Similarly, for transport and communication η_i becomes significant when homogeneity is imposed, but in this case the F-ratio remains significant. This would support our earlier conjectures as to the possible role of time trends, stocks, or other omitted variables in explaining nonhomogeneity. Likewise, in Table 2 the expenditure elasticities from the first-difference model tend to be higher than the levels estimates when stock effects are likely to be important (clothing, other goods) and lower when one would expect short-run total expenditure effects to be limited (food, housing, transport). Otherwise, the first-difference parameter estimates, homogeneous or unconstrained, are rather close to the values originally obtained. As with levels, tests of concavity with the first-difference model revealed several violations. The likelihoods for the two models are summarized in Table 4; the fact that homogeneity cannot be rejected overall at the 5 percent level reflects the importance of the time trends in the housing, clothing, and transport and communication equations. Note too that in this case, from the last column of the table, symmetry is only just rejected given homogeneity. Hence, if we make some allowance for the asymptotic nature of the test, these final results would suggest that the introduction of (arbitrary) time trends removes much of the conflict between the data and the hypothesis of a representative consumer maximizing a conventional static utility function.

III. Summary and Conclusions

In this paper we have introduced a new system of demand equations, the *AIDS*, in which the budget shares of the various commodities are linearly related to the logarithm of real total expenditure and the logarithms of relative prices. The model is shown to possess most of the properties usually thought desirable in conventional demand analysis, and to do so in a way not matched by any single competing system. Fitted to postwar British data, the *AIDS* is capable of explaining a high proportion of the variance of the commodity budget shares but, unless allowance is made for omitted variables by the arbitrary use of time trends, does so in a way which is inconsistent with the hypothesis of consumers making decisions according to the model's demand functions governed by the conventional static budget constraint. These results suggest that influences other than current prices and current total expenditure must be systematically modelled if even the broad pattern of demand is to be explained in a theoretically coherent and empirically robust way. Whether these developments generalize the static framework by including stock effects, errors in price perceptions, or by going beyond the assumption of weak intertemporal separability on which the static model rests, we believe that the *AIDS*, with its simplicity of structure, generality, and conformity with the theory, offers a platform on which such developments can proceed.

APPENDIX: *AIDS* IN THE CONTEXT OF AGGREGATION THEORY

In Muellbauer (1975, 1976), a definition of the existence conditions for a representative consumer is given which allows more general behavior than the parallel linear Engel curves which are required if average demands are to be a function of the average budget. We know that in general, the average budget share

$$\bar{w}_i = \sum_h p_i q_{ih} / \sum_h x_h \equiv \sum_h x_h w_{ih} / \sum_h x_h$$

is a function of prices and the complete distribution vector $(x_1, x_2, \ldots, x_H)$. A representative consumer exists in Muellbauer's sense if each $\bar{w}_i$ can be written as a function of prices and the same single scalar x_0, itself a function of prices and the distribution vector. This scalar, which can be thought of as marking a position in the distribution of xs, is the representative budget level. Muellbauer shows that for an x_0 to exist such that

(A1)
$$\sum_h x_h w_{ih}(x_h, p) / \Sigma x_h = w_i\{x_0(x_1, \ldots, x_H, p), p\}$$

the individual budget share equations must have the "generalized linear" (*GL*) form:

(A2)
$$w_{ih}(x_h, p) = v_h(x_h, p) A_i(p) + B_i(p) + C_{ih}(p)$$

where v_h, A_i, B_i, and C_{ih} are functions satisfying $\Sigma_i A_i = \Sigma_i C_{ih} = \Sigma_h C_{ih} = 0$ and $\Sigma B_i = 1$. Clearly, (A1) goes beyond the usual formulation of $x_0 = \bar{x}$, and, as we shall see below, allows us to incorporate into the demand functions features of the expenditure distribution other than the mean.

Of particular interest is the case where x_0 is independent of prices, depending only on the individual xs. This occurs if, and only if, the v_h function in (A2) restricts to

$$\text{(A3)} \quad v_h(x_h, p) = \{1 - (x_h/k_h)^{-\alpha}\}^{\alpha^{-1}}$$

where α is a constant and k_h, although not a function of x_h, and p is free to vary from household to household. In this case, the budget shares are said to have the "price-independent generalized linear" form (*PIGL*). Note the special case of (A3) as $\alpha \to 0$, i.e.,

$$\text{(A4)} \quad v_h(x_h, p) = log(x_h/k_h)$$

For obvious reasons, this is referred to as the *PIGLOG* case. By substituting (A3) in (A2), (A1) can be used to give an explicit form for x_0, viz.,

$$\text{(A5)} \quad x_0 = \left\{ \Sigma \left(\frac{x_h}{k_h} \right)^{-\alpha} \Big/ \Sigma x_h \right\}^{-1/\alpha}$$

If we assume that individual behavior is preference consistent, the cost function corresponding to *PIGL* takes the form

$$\text{(A6)} \quad \{c(u_h, p)/k_h\}^{\alpha} = (1 - u_h)\{a(p)\}^{\alpha} + u_h\{b(p)^{\alpha}\}$$

which as α tends to zero takes the *PIGLOG* form

$$\text{(A7)} \quad log\{c(u_h, p)/k_h\} = (1 - u_h) log\{a(p)\} + u_h log\{b(p)\}$$

where $a(p)$ and $b(p)$ are linear homogeneous concave functions, α is the constant parameter of (A3), and (with some exceptions discussed below) $0 \leq u \leq 1$. The quantity k_h can be used to allow for family composition effects within *PIGL*; for the standard or "reference" household k_h is unity.

Since the *AIDS* is a member of the *PIGLOG* family, and hence of *PIGL*, we can achieve maximum generality by discussing some of the important properties of this class. If, omitting the household subscript, we write q_i for the quantity demanded of good i, then, by the derivative property of the cost function, $q_i = \partial c / \partial p_i$ so that $w_i = p_i q_i / x = \partial \log c / \partial \log p_i$. Hence from (A6), taking $k_h = 1$, and differentiating

$$\text{(A8)} \quad \alpha c^{\alpha} \frac{\partial \log c}{\partial \log p_i} = \alpha a^{\alpha} a_i (1-u) + u \alpha b^{\alpha} b_i$$

where $a_i = \partial \log a / \partial \log p_i$ and $b_i = \partial \log b / \partial \log p_i$. Hence, substituting x for c,

$$\text{(A9)} \quad w_i = (1-u)\left(\frac{a}{x}\right)^{\alpha} a_i + u\left(\frac{b}{x}\right)^{\alpha} b_i$$

where, from (A6), $u = (x^{\alpha} - a^{\alpha})/(b^{\alpha} - a^{\alpha})$ or from (A7), $u = (\log x - \log a)/(\log b - \log a)$. Similarly, when $\alpha = 0$,

$$\text{(A10)} \quad w_i = (1-u) a_i + u b_i$$

Equations (A9) and (A10) have attractive interpretations. Cost $c(u,p)$ is increasing in utility as long as $b(p)$ is greater than $a(p)$—note that this does not depend on the sign of α—so that as u increases from 0 to 1, $c(u,p)$ increases from $a(p)$ to $b(p)$ with w_i moving from a_i to b_i. Hence a total expenditure of $a(p)$ can be thought of as "poverty" expenditure with associated expenditure pattern a_i, while $b(p)$ is "affluence" expenditure with budget shares b_i. On this interpretation $x = a(p)$ and $x = b(p)$ are the equations of the tangents to the poverty and affluence indifference curves, respectively, $u = 0$ and $u = 1$. From (A6) therefore, we see that the tangent to the indifference curve actually attained is the mean of order α of poverty and affluence tangents, the weights depending on the welfare level or outlay of the household. This averaging is even more obvious in the value share equations (A9) and (A10). Since $(1-u)(a/x)^{\alpha}$ and $u(b/x)^{\alpha}$ sum to unity, as do $(1-u)$ and u, these equations give the actual budget shares as weighted averages of a_i and b_i.

Since the value shares of luxuries increase with total outlay and hence with u, we can characterize luxuries and necessities simply by whether b_i is greater than or less than a_i. Inferior goods are not excluded under *PIGL* and it is straightforward to construct examples from both (A9) and (A10).

Note finally that there are restrictions on the possible set of x and p over which the cost function and the associated demands are valid. One set of restrictions is implied by the necessity that, for all $i, 0 \leq w_i \leq 1$. The upper bound is relevant when $b_i > a_i$ and implies that $u = (x^{\alpha} - a^{\alpha})/(b^{\alpha} - a^{\alpha}) \leq \min_i[\{1 - a_i(a/x)^{\alpha}\}/\{(b/x)^{\alpha} b_i - (a/x)^{\alpha} a_i\}]$. The lower bound, relevant when $b_i < a_i$ requires similarly that $u = (x^{\alpha} - a^{\alpha})/(b^{\alpha} - a^{\alpha}) \geq \max_i[(a/x)^{\alpha} a_i / \{(a/x)^{\alpha} a_i - (b/x)^{\alpha} b_i\}]$. The second set of restrictions are those required to ensure that the cost function is concave. From (A6), we can see that a *sufficient* condition for concavity of $c(u,p)$ is that $a(p)$ and $b(p)$ be concave and that $0 \leq u \leq 1$. However, this is by no means necessary. If $b(p)$ is "more concave" than $a(p)$, then $c(u,p)$ is concave for $u > 0$ and for a range of $u > 1$. It can be shown that the *PIGL* cost function is concave for *all* $x > 0$, $p > 0$ if and only if $a_i = b_i$ for all i, and $a(p)$ and $b(p)$ are concave. In this not very interesting case, preferences are homothetic.

The practical application of the *PIGL* class requires selection of specific functional forms for the functions $a(p)$ and $b(p)$; those leading to the *AIDS* have been discussed in the text. However, the *PIGL* class is related to two other well-known models. Note first that if $\alpha = 1$, and $k_h = 1$, (A6) becomes the Gorman polar form. The *PIGL* class thus includes all models with linear Engel curves, for example, linear expenditure system, the quadratic utility function, as special cases. Perhaps less obviously, a weakly restricted form of the indirect translog is also *PIGLOG*. From Jorgenson and Lau the translog indirect utility function is

$$(A11)\quad u = a_0 + \sum_i \alpha_i \log\left(\frac{p_i}{x}\right) + \frac{1}{2}\sum_i\sum_j \beta_{ij}\log\left(\frac{p_i}{x}\right)\log\left(\frac{p_j}{x}\right)$$

where we can choose $\Sigma\alpha_i = -1$ and $\beta_{ij} = \beta_{ji}$ as arbitrary normalizations. Write $\Sigma_k\beta_{ki} = \beta_{Mi}$, then if we impose the additional restriction that $\Sigma_i\beta_{Mi} = 0$, (A11) solves explicity for $\log c(u,p)$ to

$$(A12)\quad \log c(u,p) = \frac{u - \frac{1}{2}\Sigma\Sigma\,\beta_{ij}\log p_i \log p_j - \Sigma\,\alpha_i \log p_i - a_0}{1 + \sum_i \beta_{Mi}\log p_i}$$

This is of the general form $\log c(u,p) = \log a(p) + u/\log h(p)$ for appropriate choice of $a(p)$ and $h(p)$. Using $\log h(p) = 1/\log\{b(p)/a(p)\}$ to define $b(p)$ and substituting, we see that (A12) is identical to (A7). Hence, in this case $(\Sigma_i\beta_{Mi} = 0)$ and in this case only, the indirect translog allows consistent aggregation. No such result holds for any interesting subcase of the direct translog.

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