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SPECIFICATION AND TESTING IN APPLIED DEMAND ANALYSIS[1]

For rather more than two decades, applied econometricians have been estimating systems of demand equations which have been explicitly or implicitly derived from the theory of utility maximisation. For the most part, such studies have used the traditional specification of utility as a function of quantities consumed and have derived demand functions by manipulation of the first-order conditions for utility maximisation. For recent examples of a large literature, see the books by Theil (1975, 1976) and the review paper by Barten (1977). However, over the same period, demand theorists have made increasing use of duality tools, so that the direct utility function is now frequently discarded as an analytical tool. The considerable power of the new methods in economic theory can be gauged from the recent paper by Gorman (1976); there, a wide range of problems is handled with great elegance and simplicity using the tools of duality. These methods have had a good deal less impact on applied work and although there have been exceptions (see, for example, Houthakker (1960) and the recent work on the translog model by, for example, Christensen, Jorgenson and Lau (1975)), even these make only limited use of the crucial concepts. The purpose of the first part of this paper is to lay out the central concepts of duality in demand analysis as they are relevant to the applied econometrician. The approach is informal (the interested reader may consult Diewert (1974) for a more formal treatment) but is I hope informative. The use of duality allows a considerable increase in the flexibility with which empirical demand equations can be specified and permits a much more intimate relationship between theory and practice. I shall pursue this by means of a simple example.

However, specification is only half the task and an enhanced ability to generate new, plausible models only renders more acute the problem of discriminating between them. To date, hypothesis testing in demand analysis has been confined to testing specialisations of particular models (see, for example, Barten (1969), Deaton (1974*a*), or Jorgenson and Lau (1975)). But this leaves a crucial problem untouched; how is one to adjudicate between competing specifications, between the translog models and the Rotterdam system, or between the linear expenditure system and the indirect addilog model, and so on? The basic problem is that these models are not "nested" within one another; it is not possible to derive one from the other by the imposition of suitable restrictions. In some cases it might be possible to combine non-nested models into a composite hypothesis against which each of the original models can be tested as specialisations. But this is not possible in demand analysis. A composite of the Rotterdam and translog models would have so many parameters as to make estimation impossible and would have

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little interest in its own right. Instead, it is now possible to use tests explicitly designed for non-nested hypotheses. These tests are due to Cox (1961, 1962) and were adapted and introduced into the economics literature by Pesaran (1974) and extended by Pesaran and Deaton (1978). Although the available references are somewhat technical, the basic idea of these tests is extremely simple. Part 2 of this paper shows how they can be applied in demand analysis to complement traditional techniques of testing. Use is made of the example derived in Part 1 and the new model is tested both against an unrestricted version of itself and against the linear expenditure system, with which it is non-nested.

1. DUALITY AND THE SPECIFICATION OF EMPIRICAL HYPOTHESES

If utility is written as a function of quantities consumed, and if the utility function is strictly quasi-concave, monotone increasing and differentiable, empirical demand functions can be derived from the first-order conditions of utility maximisation. This is much more easily said than done; these conditions frequently cannot be solved explicitly for the demand functions, and when they can, the resulting equations may be difficult or impossible to estimate. The linear expenditure system provides an exception to these problems, and although it has been much used by econometricians and planners, as a model it has proved remarkably difficult to follow. Minor generalisations have frequently been proposed, but even more than 20 years after Stone (1954) first estimated the parameters of an explicit utility function, there are only a few direct utility functions, other than those closely related to the L.E.S., which lead to demand functions which are interesting enough to be worth the sometimes considerable effort of estimation. Hence, apart from the linear expenditure system form, the direct utility function has been remarkably unproductive as a tool of empirical analysis.

The major reasons are clear enough in retrospect. There are two immediately obvious forms for the utility function, either

$$u = \Sigma f_i(q_i) \tag{1}$$

or

$$u = (\mathbf{q}^* - \mathbf{a})' \mathbf{A}(\mathbf{q}^* - \mathbf{a}), \tag{2}$$

where $\mathbf{q}^*$ is some transformation of $\mathbf{q}$; for example,

$$q_i^* = \frac{q_i^\alpha - 1}{\alpha}, \tag{3}$$

so that (2) encompasses quadratic utility and the translog forms. As for (1), the form

$$f_i(q_i) = \alpha_i \frac{(q_i - \gamma_i)^{\beta_i} - 1}{\beta_i}, \tag{4}$$

with $\alpha_i > 0$, and $\beta_i < 1$, covers most of the important cases; with $\beta_i = 0$ we get the linear expenditure system, with $\gamma_i = 0$ Houthakker's (1960) addilog, and with $\beta_i = \beta$ independent of i we get what Pollak (1971) calls "generalised Bergson functions with minima". The whole class (1) has well-known and highly undesirable properties and is too restrictive for most empirical work (see Deaton, 1974*b*). Besides, apart from the case when β_i is independent of i, it is not possible to derive explicit demand functions and this poses (not necessarily insuperable) problems for estimation. On the other hand, the quadratic form (2) is as general as the additive form (1) is restrictive, and the large number of parameters involved prevents estimation of the model for all but academic exercises involving half-a-dozen or so commodities. It turns out to be remarkably difficult to find something intermediate between (1) and (2). The search is also much hampered by the inability to move easily from the utility function to the demand functions and back. The econometrician has exactly the situation of the child's puzzle where there are a dozen strings leading to a dozen destinations and, although the loose ends are visible, everything is in an impossible tangle in the middle. This causes difficulties in both directions. On the one hand, one has very little idea, in general, what effect the specification of utility has on the demand functions, especially when it is difficult to discover precise analytical forms. More important, perhaps, is the curtain that is drawn between statistical inference and theoretical specification. We know quite a lot about behaviour, we know what shapes Engel curves are and are not, we know what we expect about expenditure and price elasticities, and yet it is extremely difficult to say what implications this evidence has for the shape of the direct utility function.

The duality approach solves many of these problems. We can go from preferences to behaviour in one step and we can go from behaviour to preferences in one step. The construction of preference-consistent demand functions becomes straightforward, and a clear route is open for the incorporation of empirical evidence into a knowledge of preferences.

Given the language and apparent difficulty which sometimes surrounds theoretical treatments of duality, it is surprising how simple are the basic tools and how familiar. It is also perhaps surprising that although duality in demand analysis and duality in mathematical programming are conceptually identical, in practice they are very different. In mathematical programming, the formulation of a dual involves the stating of an entirely new problem; a minimising problem if the original was a maximising problem and a maximising problem if the original was a minimising problem. This new problem is defined over different variables; the so-called dual variables. It is this aspect of duality, the change of variables, which is central to consumer demand analysis; the dual problem exists, but it is not of central importance. Indeed, the crucial concepts involve very little explicit duality theory and are widely used in production theory with no consciousness of duality being involved.

The central concept is the *cost function* (sometimes called the *expenditure* function). If we label indifference curves by some arbitrarily scaled utility indicator u, so that higher indifference curves always have a higher u value, then we can measure the *minimum* cost of attaining that indifference curve (i.e. utility u) at any prices $\mathbf{p}$. We write this function $c(u, \mathbf{p})$. It is very easy to show that $c(u, \mathbf{p})$ has the following properties (see Diewert (1974) for proofs):

(i) If the consumer is actually maximising utility he will be minimising costs so that his total expenditure at any time will be the current value of $c(u, \mathbf{p})$.

(ii) $c(u, \mathbf{p})$ is increasing in u and $\mathbf{p}$; it costs more to be better-off and it costs more to be as well-off if prices rise.

(iii) $c(u, \mathbf{p})$ is homogeneous of degree 1 in $\mathbf{p}$; a doubling of prices doubles the cost of staying on the same indifference curve.

(iv) $c(u, \mathbf{p})$ is *concave* in $\mathbf{p}$; this implies that an increase in one (or more) price(s) will cause no more than proportionate, and usually less than proportionate increases in costs. This is because the consumer *minimises* costs; he always chooses the best way of producing utility so that he will substitute, as far as possible, away from goods which become relatively expensive. This is a very general property of the cost function, and it in no way depends on the convexity of indifference curves. This is a significant advance on Lagrangian methods which rest crucially on convexity.

(v) If $c(u, \mathbf{p})$ is differentiable, or where it is differentiable, its derivatives with respect to price are the quantities demanded:

$$\frac{\partial c(u, \mathbf{p})}{\partial p_i} = h_i(u, \mathbf{p}) = q_i. \tag{5}$$

It is now clear why it is so easy to generate demand functions. All we need is to think of a function $c(u, \mathbf{p})$ which is increasing, homogeneous, concave and differentiable, and we can use (5) to write down the demand functions. Note, however, that the demand functions given by (5) are functions of utility and prices, the so-called *Hicksian* demand functions. In practice we have observations on total expenditure, not utility, so that we must be able to write the quantities as functions of the former. This is simple enough. Since we know the value of $c(u, \mathbf{p})$ at any time, i.e. total expenditure, we can invert the function and write utility, u, as a function of total expenditure and prices:

$$u = \psi(x, \mathbf{p}), \tag{6}$$

where we use x, i.e. total expenditure, for $c(u, \mathbf{p})$ to emphasise that it is no longer a function. Substitution of (6) into the Hicksian demand function (5) leads immediately to the *Marshallian* demand functions.

$$q_i = h_i[\psi(x, \mathbf{p}), \mathbf{p}] = g_i\{x, \mathbf{p}\}. \tag{7}$$

These can be estimated in the usual way.

For empirical work, equation (5) is doubly important, not only because it can be used to generate demand functions, but also because it offers a direct link in the other direction, from demand back to preferences. This can best be illustrated by a simple example.

In work with Engel curves, it has been suggested (see, for example, Working, 1943; Leser, 1976) that a model in which budget shares are linearly related to the logarithm of total outlay provides an excellent fit to the data. In other words,

$$w_i = \alpha_i + \beta_i \log x, \tag{8}$$

where $w_i = p_i q_i / x$. This form is consistent with adding-up provided $\Sigma\alpha_k = 1$ and $\Sigma\beta_k = 0$, and is of further interest since it allows perfect aggregation over consumers as defined by Muellbauer (1975, 1976*a*). We take this as a starting-point, and in our example attempt to show how the prior knowledge that (8) is a sensible functional form can be incorporated into a full system of demand equations.

Equation (8) makes no allowance for prices since these are usually assumed to be constant across households. Making the necessary modification, the full demand system underlying (8) can be written

$$w_i = \alpha_i(\mathbf{p}) + \beta_i(\mathbf{p}) \log x, \tag{9}$$

where we try to choose $\alpha_i(\mathbf{p})$ and $\beta_i(\mathbf{p})$ to ensure econometric simplicity and so as to integrate into a cost function. From the first consideration we might, for example, choose α and β so that

$$w_i = \alpha_i + \beta_i \log x + \Sigma\gamma_{ij} \log p_j, \tag{10}$$

where the α, β and γ parameters are now constant.

This model is of considerable interest in its own right: it is very general; it can be regarded as an arbitrary first-order approximation to any set of demand functions; and it can be estimated equation by equation by ordinary least squares. Results of doing so are presented in Table 1 below and will be discussed in the next section.

Meanwhile, let us explore the utility implications of (10). Since, from (5)

$$\frac{\partial \log c(u, \mathbf{p})}{\partial \log p_i} = w_i, \tag{11}$$

equation (10) can be written

$$\frac{\partial \log c}{\partial \log p_i} = \alpha_i + \beta_i \log c + \Sigma\gamma_{ij} \log p_j, \tag{12}$$

which is a system of partial differential equations defining c as a function of $\mathbf{p}$. This system has a solution only in the restricted case where, for some θ,

$$\gamma_{ij} + \beta_i \alpha_j = \theta\beta_i \beta_j. \tag{13}$$

If (13) is satisfied, the solution to (12) is

$$\log c(u, \mathbf{p}) = a_0 + \Sigma a_k \log p_k + u \Pi p_k^{\beta_k}, \tag{14}$$

where $a_0 = -\theta$, and $a_i = \alpha_i - \theta\beta_i$.

It is a simple matter to check that (14) is a cost function of Muellbauer's (1975) PIGLOG form, and indeed, this model has already been derived by Muellbauer (1976*b*) and Carlevaro (1976) in a different, but conceptually similar situation. The indirect utility function $\psi(x, \mathbf{p})$ is solved from (14) to give

$$u = (\log x - a_0 - \Sigma a_k \log p_k) / \Pi p_k^{\beta_k}. \tag{15}$$

This is as far back as we can go in this case since it is not possible to derive the direct utility function explicitly. But this is of no importance; the existence of (14) means that a utility function exists which will give rise to the demand functions so that these latter will have all the usual properties.

The demand functions corresponding to (14) are, by differentiation with respect to $\log p_i$ followed by substitution of u from (15),

$$w_i = a_i + \beta_i(\log x - a_0 - \Sigma a_k \log p_k), \tag{16}$$

or alternatively

$$w_i = a_i + \beta_i \log (x/P), \tag{16'}$$

where $P = e^{a_0}\Pi p_k^{a_k}$, is a price index. Equation (16′) establishes the model as a member of the Fourgéaud and Nataf (1959) class, in which demands are a function of real income and real price alone, while (16), with its constant term and a real income term, is reminiscent of the linear expenditure system. (Indeed, the substitutions which serve to integrate the linear expenditure system (see Samuelson, 1947–8) serve equally well in this case.)

Although this example serves very well to illustrate the ease with which empirical evidence can be used to generate new preference-consistent demand models, the final result in this case is somewhat disappointing. The original equation (10) is very general and allows a wide range of price response. After imposing the constraints (13) the utility consistent version (16) is extremely restrictive containing only $(2n-1)$ independent parameters, all but one of which are identified without changes in prices. Further research would thus discard this version and examine other possible specifications for the functions $\alpha_i(\mathbf{p})$ and $\beta_i(\mathbf{p})$.

Nevertheless, for illustrative purposes, it is worth showing how we may proceed to test a new model such as this. There are two possible approaches, both of considerable interest. In the first, the final model (16) can be tested as a specialisation of (10). This will tell us whether the integrability conditions (equations 13) do or do not hold in practice. (This of course tells us nothing about whether, in general, demand functions are integrable since, by assuming (10), we have selected a very special functional form.) The second test is to compare the model with an alternative utility-based formulation, such as the linear expenditure system. Since the linear expenditure system and (16) are not nested either one within the other, special techniques must be used. It is to these we now turn.

2. TESTING ALTERNATIVE HYPOTHESES

The normal procedure of statistical inference is that we have some basic framework for the test and within this we wish to examine various specialisations. The framework, or maintained hypothesis, is never itself challenged within the procedure. The example in this paper is the general equation (10) within which we wish to test the special case (16) which is attained by imposing the restrictions (13). If we adopt a maximum-likelihood approach to estimation, this can be turned naturally to the problem of hypothesis testing. For if L_F is the maximum attainable likelihood without restriction, say of equation (10), and L_R is the maximum attainable with the restriction, equation (16), then the ratio L_R/L_F forms a natural basis for the test (see, for example, Silvey, 1970). If the ratio is less than some critical level, k say, the restrictions are rejected. In some problems (for example, single equation linear regression) k can be set to give an exact test with a preset known significance level. In more complicated cases, such as the present, it is necessary to rely on asymptotic results, particularly that which states that, if the restricted hypothesis is true, minus twice the logarithm of the ratio is asymptotically distributed as χ^2.

All this rests crucially on the nesting or asymmetry between the two hypotheses and on the fact that, if the restrictions are valid, the likelihood ratio must approach unity as the sample size increases. When the two hypotheses are non-nested, none of this happens. There is now no question of a maintained hypothesis which is never challenged; one or both of the models may be false. And even if one of the models is true, we can no longer assert anything definite about the likelihood ratio. It is even possible, although generally unlikely, that the true model may predict that the false model has the higher likelihood. To deal with this situation, Cox (1961, 1962) derived a statistic which modifies the straightforward likelihood ratio, essentially by removing its expectation. However, since there are two possible models being considered, the likelihood ratio has two possible expectations. But this turns out to be a natural feature of the problem. If we assume each hypothesis to be true in turn, as 'working hypotheses' as it were, we may derive two modified likelihood ratio tests, one for each hypothesis. This allows us four possibilities: rejecting one or other of the models, rejecting neither, or rejecting both.

Before showing how this works for two non-nested models of demand, it is worth beginning with a simple example to show how easy the test is to calculate and how intuitively acceptable it is. Pesaran (1974) considers the case of two competing single equation linear regressions. In the obvious notation

$$H_0: \mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}_0, \tag{17}$$

$$H_1: \mathbf{y} = \mathbf{Z}\boldsymbol{\gamma} + \boldsymbol{\epsilon}_1, \tag{18}$$

where $\mathbf{X}$ and $\mathbf{Z}$ are alternative sets of explanatory variables, although some variables may appear in both. The normally distributed errors $\boldsymbol{\epsilon}_0$ and $\boldsymbol{\epsilon}_1$ are assumed to be serially independent. The test statistic, T_0, is calculated by estimating (17) and (18) by ordinary least squares and estimating equation standard errors, $\hat{\sigma}_0^2$ and $\hat{\sigma}_1^2$. Next, we need an estimate of σ_{10}^2, which is the expectation of $\hat{\sigma}_1^2$ given the truth of H_0. It turns out that this can easily be calculated by regressing the *predicted* values from the H_0 equation on $\mathbf{Z}$ (i.e. the explanatory variables from the second regression) and estimating an equation standard error. This last is added to $\hat{\sigma}_0^2$ to give $\hat{\sigma}_{10}^2$. The statistic, T_0, is then given by the formula

$$T_0 = \frac{m}{2} \log \frac{\hat{\sigma}_{10}^2}{\hat{\sigma}_1^2}, \tag{19}$$

where m is the number of observations. This can be converted to an $N(0, 1)$ variable by dividing by the square root of its variance which is calculated by carrying out one more linear regression involving the residuals of the auxiliary regression described above (see Pesaran (1974) for details).

All this has been carried out on the basis of H_0 being true, so that T_0 is the logarithm of the corrected likelihood ratio on that assumption. A precisely similar calculation, *mutatis mutandis*, gives T_1 on the assumption that H_1 is true. It is easy enough to see why (19) makes a reasonable test statistic. The expression $\hat{\sigma}_{10}^2$ is the estimate of how well H_1 ought to fit if H_0 is true. If $\hat{\sigma}_1^2$ is large relative to this, H_1 fits much worse than it ought to if H_0 is true, so that H_0 must be rejected, although not favouring H_1. Similarly if $\hat{\sigma}_1^2$ is small relative to $\hat{\sigma}_{10}^2$, H_1 fits better than it ought, and again H_0 must be rejected, this time with some prejudice in favour of H_1. The calculation is then repeated for T_1 to give corresponding results for H_1. Indeed, the test can be used to give pairwise comparisons of any number of non-nested hypotheses, see Pesaran and Deaton (1978).

Demand equations are neither single-equation nor linear. If we stay with the general form of (16), we may write the two competing hypotheses in the form

$$H_0: w_{it} = f_{it}(x_t, \mathbf{p}_t, \boldsymbol{\beta}_0) + \epsilon_{0it}, \tag{20}$$

$$H_1: w_{it} = g_{it}(x_t, \mathbf{p}_t, \beta_1) + \epsilon_{1it}, \tag{21}$$

where f and g are some functions, and $\boldsymbol{\beta}_0$ and $\boldsymbol{\beta}_1$ are vectors containing all the parameters of each model. The errors ϵ_{0it}, ϵ_{1it} are assumed to be serially independent and to be multivariate normally distributed as $N(\mathbf{O}, \boldsymbol{\Omega}_0)$ and $N(\mathbf{O}, \boldsymbol{\Omega}_1)$ under H_0 and H_1 respectively. The calculations in this case are a good deal more complex but exactly the same principles apply as in the single-equation linear case. Once again, each equation system is estimated, this time by full information maximum likelihood. If e_{0it} and e_{1it} are the two sets of estimated residuals, maximum-likelihood estimates of $\boldsymbol{\Omega}_0$ and $\boldsymbol{\Omega}_1$ are given by

$$[\hat{\boldsymbol{\Omega}}_0]_{ij} = \frac{1}{m} \sum_t e_{0it} e_{0jt}, \tag{22}$$

$$[\hat{\boldsymbol{\Omega}}_1]_{ij} = \frac{1}{m} \sum_t e_{1it} e_{1jt}. \tag{23}$$

As before, the next step is to estimate the matrix $\boldsymbol{\Omega}_{10}$, the expectation of $\hat{\boldsymbol{\Omega}}_1$ under H_0. The predicted values from H_0 are used in H_1 to compute another FIML regression and another variance–covariance matrix which is added to $\hat{\boldsymbol{\Omega}}_0$ to give $\hat{\boldsymbol{\Omega}}_{10}$. The T_0 statistic is then given by

$$T_0 = \frac{m}{2} \log \frac{\det \hat{\boldsymbol{\Omega}}_1}{\det \hat{\boldsymbol{\Omega}}_{10}}. \tag{24}$$

Since the determinant of the covariance matrix plays the same role in FIML estimation as does the equation standard error in single equation estimation, (24) is precisely analogous to (19) and has an identical interpretation. The variance of T_0 is calculated via a generalised least-squares regression again involving the residuals of the auxiliary regression (see Pesaran and Deaton for details).

There is one final complication, which is that demand systems such as (20) and (21) must satisfy adding-up without error. Thus the sums of w_{it} add exactly to unity under both H_0 and H_1 and hence the sums of the errors ϵ_{0it} and ϵ_{1it} must add exactly to zero. This means that $\boldsymbol{\Omega}_0$ and $\boldsymbol{\Omega}_1$ are singular, as are their estimates, $\hat{\boldsymbol{\Omega}}_0$ and $\hat{\boldsymbol{\Omega}}_1$, so that (24) is not defined. This is a reflection of a general problem in the maximum-likelihood estimation of demand systems and has been successfully treated by, for example, Barten (1969): see also Deaton (1975, chapter 3), for an exposition. In this context the problem can be solved by dropping one equation from the set or, more elegantly, by correcting the formulae (22)–(24) for the singularity. The details are not of general interest but an

Table 1

Parameter Estimates for General Model (10)

	α	β	$\gamma_{\cdot 1}$	$\gamma_{\cdot 2}$	$\gamma_{\cdot 3}$	$\gamma_{\cdot 4}$	$\gamma_{\cdot 5}$	$\gamma_{\cdot 6}$	$\gamma_{\cdot 7}$	$\gamma_{\cdot 8}$	R^2/D.W.
1. Food	1·2827	−0·1702	0·2329	−0·0531	−0·0047	−0·0103	−0·0294	0·0377	0·0240	−0·0503	0·9994
	(8·04)	(−6·70)	(12·20)	(−3·35)	(−0·30)	(−0·60)	(−3·25)	(1·62)	(1·18)	(−2·19)	2·38
2. Clothing	−0·5204	0·0974	0·0063	0·0027	−0·0290	0·0312	−0·0453	0·0105	0·0284	−0·0763	0·9852
	(−3·32)	(3·90)	(0·33)	(0·17)	(−1·90)	(−1·85)	(−5·10)	(0·46)	(1·42)	(−3·39)	2·25
3. Housing	0·7987	−0·1046	−0·0534	0·0065	0·0907	0·0153	0·0510	−0·0527	−0·0237	0·1273	0·9989
	(5·69)	(−4·68)	(−3·13)	(0·47)	(6·65)	(1·02)	(6·42)	(−2·56)	(−1·33)	(6·32)	1·84
4. Fuel	−0·1493	0·0319	−0·0502	0·0047	0·0109	0·0354	−0·0096	0·0221	0·0047	−0·0397	0·8815
	(−0·69)	(0·93)	(−1·92)	(0·22)	(−0·52)	(1·53)	(−0·78)	(0·70)	(0·17)	(−1·29)	2·21
5. Drink and tobacco	−0·0451	0·0287	−0·0509	0·0301	−0·0276	−0·0218	0·0513	0·0045	−0·0202	0·0057	0·9687
	(−0·30)	(1·19)	(−2·77)	(2·00)	(−1·87)	(−1·34)	(5·98)	(0·20)	(−1·04)	(0·26)	2·96
6. Transport and communication	−0·0760	0·0312	−0·0302	−0·0163	−0·0034	0·0091	0·0543	−0·0243	−0·0255	0·0437	0·9996
	(−1·08)	(2·79)	(−3·54)	(−2·33)	(−0·50)	(1·20)	(13·7)	(−2·36)	(−2·85)	(4·33)	2·29
7. Other goods	−0·0599	0·0258	−0·0064	−0·0064	−0·0026	−0·0072	−0·0345	0·0062	0·0312	−0·0129	0·8873
	(−0·37)	(0·99)	(−0·32)	(−0·40)	(−0·17)	(−0·41)	(−3·73)	(0·26)	(1·50)	(−0·55)	1·89
8. Other services	−0·2308	0·0599	−0·0481	0·0318	−0·0125	0·0107	−0·0379	−0·0041	−0·0189	0·0025	0·8372
	(−1·39)	(2·26)	(−2·38)	(1·92)	(−0·77)	(0·60)	(−4·02)	(−0·17)	(−0·89)	(0·11)	2·24

2 log likelihood = 1814·4.
(*t* values in parentheses)

Appendix laying out the calculations will be made available by the author on request.

Both types of tests can be illustrated using the models discussed in this paper. We begin with the nested test of (16) against (10).

The results of estimating (10) on annual British data from 1954 to 1974 are given in Table 1. Value shares are notoriously hard to predict so that the model fits well, although it must be remembered that we have only 11 degrees of freedom per equation. The formulation of the model ensures that zero values for β and γ, i.e. failure to find significant effects, reduces the model to homotheticity and not to absurdity, as is often the case. Nevertheless, five out of eight commodities have total expenditure elasticities significantly different from 1; of these, clothing is relatively elastic and food and housing relatively inelastic. Given the large number of price terms, a very respectable proportion (37 out of 64) have t values greater than unity and these are well scattered throughout the matrix. From this evidence there appears to be a considerable degree of cross-price responsiveness among the commodities.

Most of this is suppressed by the restricted version of the model. The parameter estimates and other statistics corresponding to (16) are given in Table 2. The estimated values of $\boldsymbol{\beta}$, which are common to both models, are quite different from those in Table 1; indeed, this is not very surprising given the restrictions imposed on the γ_{ij} parameters, many of which were significant. The equations clearly fit a lot less well and twice the logarithmic likelihood value drops from 1814·4 to 1502·8. The difference between these two, i.e. 311·6, is asymptotically distributed as χ^2 with 56 degrees of freedom so that, even if we make some allowance for small sample problems, we are left in little doubt that the model is an unacceptable specialisation of the general form (10). We would thus conclude that the functional form chosen for (10) is an unfortunate one in that it requires integrability conditions which are much too stringent in practice.

At this point, the most obvious conclusion is to abandon the model (16) and, in practice, this would be correct. However, there are two possible arguments for retaining the model for further tests. The first is related to a view put forward by Phlips (1974, p. 55). Phlips argues that the truth of neoclassical consumer theory at the aggregate level must be taken as a maintained hypothesis so that a model which is inconsistent with the theory, e.g. (10) without the integrability conditions, is meaningless and cannot be sensibly interpreted. If one accepts this view then the only meaningful tests are those between different preference-consistent models; for example, between (16) and, say, the linear expenditure system. The present author would not accept this view, partly because integrability on aggregate data is a far-from-obvious desideratum, but also because it is hard to see why, if (16) is correct, it should be rejected by the data when compared with (10). The second argument is more substantial. In using the Cox procedure, even a false model can give us useful information about other models. For example, if the linear expenditure system were true, we would have certain expectations about what would happen when we estimate (16); these expectations can be checked with experience and may tell us something about the linear expenditure system, quite independently of the validity of (16).

Table 2

Parameter Estimates for Model (15) and the LES (20)

(Asymptotic t values in parentheses.)

	PIGLOG model			Linear expenditure system		
	a	β	R^2	γ	η	R^2
1. Food	0·2803 (185·6)	−0·2675 (−28·5)	0·9725	100·5 (92·5)	0·0614 (10·8)	0·9879
2. Clothing	0·1042 (140·4)	−0·0482 (−10·8)	0·8425	26·15 (26·9)	0·1008 (21·4)	0·8116
3. Housing	0·0991 (87·6)	0·1707 (24·5)	0·9632	22·79 (4·43)	0·2187 (17·2)	0·9608
4. Fuel	0·0473 (56·4)	0·0114 (2·31)	0·2006	15·34 (17·4)	0·0493 (14·8)	0·6841
5. Drink and tobacco	0·1389 (168·8)	−0·0269 (−5·59)	0·6011	45·05 (33·9)	0·1201 (33·8)	0·8155
6. Transport and communication	0·0866 (115·8)	0·1341 (28·9)	0·9733	13·15 (4·79)	0·2132 (46·3)	0·9648
7. Other goods	0·0990 (211·7)	0·0153 (5·60)	0·6028	28·65 (21·6)	0·1168 (35·5)	0·5662
8. Other services	0·1446 (306·4)	0·0110 (3·97)	0·4304	51·69 (29·16)	0·1198 (26·1)	−0·1560
	a_0 fixed* at 6·056; $2 \log L = 1502·8$			$2 \log L = 1530·5$		

* Technical note. The value of a_0 was fixed after failure to reach convergence with the parameters unconstrained. The value of 6·056 is theoretically plausible given (14) and is consistent with the evidence. Note that scale changes to price index numbers affect a_0 but, since the β_i parameters add to zero, do not affect the adding-up properties of the a_i parameters.

Table 2 gives estimates of the linear expenditure system. In order to ensure compatibility with the other models this is written in the form

$$w_{it} = \frac{p_{it}\gamma_i}{x_t} + \eta_i\left(1 - \sum_k \frac{p_{kt}\gamma_k}{x_t}\right), \qquad (25)$$

with parameters γ and η.

The equation-by-equation fit of the LES is similar to that of the PIGLOG Model (16) with some value shares predicted better by one and some by the other. Overall, the LES has the higher likelihood, 1530·5 as opposed to 1502·8 for the PIGLOG model. However, since the models are non-nested, we cannot conclude anything about the validity of either model from a comparison of likelihoods alone.

Moving to the non-nested testing procedure, and taking the LES as the first hypothesis, we find a test statistic of $-20{\cdot}86$; if the LES were true this should be a drawing from an $N(0, 1)$ distribution. In fact, such an extreme value occurs because PIGLOG fits much better than we would expect if the LES were true; hence we must reject the LES. Reversing the process, treating PIGLOG as a working hypothesis, the corresponding statistic is now $-18{\cdot}30$. Once again, the LES fits much better than is to be expected given the validity of PIGLOG; it too must thus be rejected.

This is a perfectly satisfactory result. It is quite obvious from the earlier test and from the restrictiveness of the integrability conditions that the PIGLOG model is false. The LES, although it fits better than PIGLOG, does not outshine it by enough to suggest that it is a serious contender. And this is exactly what the formal test tells us. Note, however, that the LES and the PIGLOG model reject one another without the intermediation of the general model and thus is an important property of the non-nested procedure.

One is now entitled to ask where one goes, having rejected both the models. In this particular case there are clearly better models available than either of those considered here. Nevertheless we might still, after further testing, be faced with a situation where *all* the models we can think of are rejected. In this author's view, there is nothing to suggest that such an outcome is inadmissable; it is perfectly possible that, in a particular case, economists are not possessed of the true model. If, in a practical context, *some* model is required, then a choice can be made by minimising some appropriate measure of loss, but such a choice in no way commits us to a belief that the model chosen is, in fact, true. Even so, it is too pessimistic to believe that we cannot build satisfactory models of demand. The examples given in this paper are not very serious contenders but, in my view, the methodology which they illustrate is likely to be an important element in further progress in the field.

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